

Response to Reviewer #3

We appreciate the constructive and insightful comments from the reviewer. Detailed replies to each of the reviewer's points (in blue italics) are provided below. Given the opportunity, we feel that incorporating the reviewer's comments will produce a substantially clearer and stronger paper as a result.

The authors present work on wind-farm layout optimization using RANS and gradient-based optimization using adjoints for the determination of the gradients. The work is interesting and merits publications. Nevertheless, I have a number of (relatively minor) comments that should be addressed first.

1. Page 5, Eq 1: formally the state u should be added under the min, so $\min_{m,u}$ (in this formulation, optimization is done over state and control, given the state and other constraints).

We agree, as explained more fully in the next response.

*2. page 6: the formalism for the derivation of the adjoint should be cleaned up a bit. First of all, there is a distinction between $J(u, m)$ and $\hat{J}(m) \equiv J(u(m), m)$, where $u(m)$ is defined by $F(u(m), m) \equiv 0$. The authors want to derive the gradient of $\hat{J}(m)$ (to m), not the gradient of J to m , which is simply $\partial J / \partial m$. In fact, the authors do not solve (1)-(4) directly, but rather a reduced formulation, i.e. $\min_m \hat{J}(m)$ (s.t. to all constraints except the state constraint, which is now already implicitly contained in $\hat{J}(m)$). This is a quite common approach in PDE-constrained optimization (cf. e.g. the book by Borzi and Schultz, *Computational Optimization of Systems Governed by Partial Differential Equations*). Please clarify your notations accordingly in the manuscript*

We appreciate the reviewer's suggestions concerning the optimization variables and use of a reduced functional. We will modify our formulation to include these changes, resulting in a cleaner description and additional rigor in the optimization formulation.

3. page 7: optimization problem should also include the boundary conditions + explicitly express that the turbine forces are function of m . Also similar to above, this formulation requires $\min_{m,u}$

Our boundary conditions are currently discussed in Section 3.3 and we agree that they should also be included in the optimization problem formulation. In particular, we will include the boundary conditions as well as make explicit the dependence of the turbine forces on m .

4. page 9: I find it very weird to talk about a smoothed thrust coefficient. This goes really against the normal definition of a thrust coefficient, which is a scalar value. Instead, please use the convention that the force is smoothed out over the RANS grid using a geometric smoothing function. This is the standard convention used in all Actuator Disk representations in literature (both RANS and LES – check any of the relevant papers)

We are in full agreement with the reviewer and we do in fact treat the power and thrust coefficients as scalars, indicated by the variable c' . Moreover, our smoothing kernel in Eq. 23 is exactly the geometric smoothing function suggested by the reviewer. Our smoothed field C' was introduced to simplify the objective function expression. In order to avoid confusion on this point and clarify the presentation in our revision, we will replace C' with the explicit use of the power and thrust coefficients and geometric smoothing kernel.

5. *page 10: given the domain size (height and width), please provide and discuss the blockage ratio*

In wind tunnel studies the blockage ratio is typically the ratio of the total tunnel cross-sectional area to the total rotor disk area. The total frontal area of our simulation domain is $8RD \times 30RD = 1.536 \times 10^6 \text{ m}^2$ for our turbines with 80 m rotor diameters. Our starting layouts contain a 4×4 grid of turbines, resulting in $4 \times \pi \times 40^2 = 2.01 \times 10^4 \text{ m}^2$, or 1.3% blockage effective blockage. A worst case scenario would have all 16 turbines arranged in a line perpendicular to the inflow direction, resulting in 5.2% blockage. We believe this blockage is sufficiently small to avoid requiring any correction effects based on a recent study which confirmed these corrections were only necessary above blockage ratios of 10% (Chen and Liou, Experimental Thermal and Fluid Sciences, 2011). We will include the blockage ratio and reference information in our domain description.

6. *page 12 , point 6: what optimization method is used?*

Please see next response.

7. *page 14, line 5. Please discuss in more detail what optimization method is used to solve the QP problems: Newton, quasi-Newton, what precise method (truncated, BFGS, thrust-region, ...)*

We have used SLSQP when we enforce an inter-turbine spacing constraint and L-BFGS-B when only enforcing site boundaries and not spacing constraints. The initial optimization studies we present are intended to provide the reader with an intuition about the flow physics that the optimizer uses, and consequently we used L-BFGS-B without inter-turbine spacings. The later annual energy production optimization results we present are intended to focus on more real-world optimization, and there we do enforce inter-turbine spacing constraints and switch to the SLSQP optimizer. We will clarify our selection of optimization algorithms in Section 3.4.

8. *page 14, and results section: you claim the use SQP, but then in the results section, you seem to mention that you do not include the distance constraint. What is the point then in using SQP? Please elaborate. Why did the distance constraint not work? And if not included, why not use a simple box-constrained quasi-Newton method?*

For all of the real-world wind rose results in Section 4.4 we imposed the turbine spacing constraint and used the SLSQP algorithm to optimize over the full annual energy production. The optimizations presented in Sections 4.2 and 4.3 were simplified test cases with idealized wind roses and no spacing constraints to help demonstrate the flow physics that produced optimal layouts. These simplified constraints allowed us to use the L-BFGS-B algorithm. We will clarify our selection of optimization algorithms in Section 3.4.

9. *page 15, figure 4 and discussion: Reference data (experiments or LES) should be added to the plot (in particular to Fig 4b and c). The porous disk is well documented experimentally as well as numerically (and has recently also been used in an intercomparison study Lignarolo 2016); without adding reference data, the later statement “Overall, the results presented here compare favorably to results reported elsewhere” is not verifiable. Given that the turbulence model used is really simple, there might be some differences with profiles from literature (which in itself is not a problem given that the authors develop a new approach) – this has to be properly discussed*

The reviewer is correct that, in this paper, we simply seek to introduce the coupled adjoint and flow solver optimization approach. As such, exact or quantitative flow solver agreement with prior results is not strictly required, and is likely to be strongly dependent on the choice of RANS flow model. Our intent in Section 4.1 was simply to provide confidence that the present flow solver results are *qualitatively* consistent with prior studies. In order to make this more clear, we will emphasize at the beginning and end of this section that the objective is to show that the present flow solver is a reasonable choice for introduction of the tool, and that more sophisticated RANS models or flow solvers can be implemented in the future.

10. Section 4.2: It would be interesting to also add a single wind direction + discussion. Even though such a case may not occur in reality, I believe it can yield extra theoretical insight

We will add a single inflow direction optimization to Figure 6.

11. Section 4.2: please provide relative efficiencies of the optimized layouts (compared to all inflow turbines).

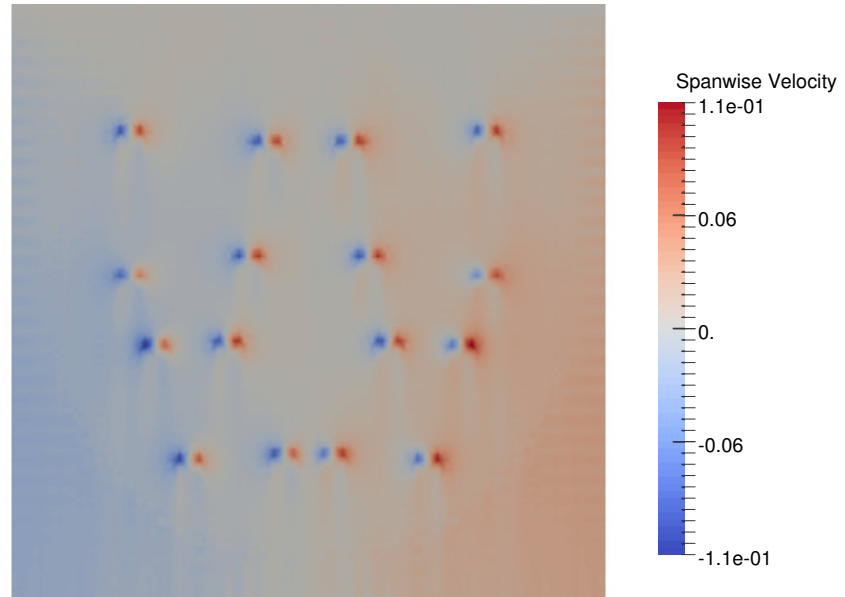
We will provide such a calculation in our revision.

12. Section 4.4: Simulations over different wind speeds should not be necessary (since, if I'm not mistaken, you work with a constant thrust coefficient per turbine, so you perform optimization for region II). Your simulations when normalized with wind speed, are independent of wind speed, so relative power doesn't change. Therefore, per wind direction, only wind-speed is necessary. Power will then just scale with cube of wind speed. Please reconsider approach in section 4.4 accordingly

We in fact had a similar idea during preparation of this manuscript and compared cases with and without constant wind speeds. From these tests, we found that the simulations actually do depend on wind speed, even when the thrust coefficient is constant. Our explanation for this is that, even when normalizing by the inflow hub height velocity, the simulation Reynolds number will change, and therefore so will the wake spreading and dissipation rates. We are happy to provide a comment regarding this point to the revised manuscript.

13. page 20 (and other locations earlier): the effect of flow-curvature is maybe a bit too much emphasized. It is certainly true that the current RANS approach allows flow curvature (e.g. not present in the Jensen model), but the authors do not substantiate the claim that flow curvature is an essential feature for optimal layout (i.e. an effect that makes a significant difference). Either substantiate (e.g. Based on detailed comparisons with other models that do not have the effect) or tune down the statement.

Our optimized layouts show that turbines behind the first row are placed in locations where streamlines are converging and flow is accelerating between turbines. Flow curvature and the resulting acceleration between turbines is the mechanism that underlies this optimization heuristic, which is not present in, e.g., the Jensen model. In order to emphasize the significance of the curvature effect in turbine placement, we will provide a new figure that shows the spanwise velocities (which are not present in a Jensen wake model) to demonstrate how the optimal turbine placement is driven by curvature and the resulting speedups. An example figure with the spanwise velocities for the layout in Figure 6d is shown below.



Sincerely, the authors.