Reply to the referees of “Joukowsky actuator disc momentum theory”

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Dear referees,

Thank you for your reviews and positive remarks about this paper. Please find here my reply to:

Referee 1:

- *I think the parenthesis in Eq. (7) are misplaced. As it is in the paper the right hand side is a vector and the left hand side a scalar.* Reply: I do not agree entirely. The right hand side is a scalar too as $\boldsymbol{v} \cdot \boldsymbol{\nabla}$ is a scalar operation at the scalar Bernoulli parameter H. However, the parenthesis are unnecessary, and thereby confusing. I have removed them.
- *One could perhaps say a few words that the swirl being a direct loss as shown in Eq. (15) is consistent with the classical Glauert optimum rotor that tries to minimize this rotation and thus the loss.* Reply: I agree and have added: This is consistent with the optimization of rotors according to Glauert’s theory [ref to the book of Jens Sørensen] which involves minimization of the swirl.
- On page 4 "is is" should be only "is"; Reply: corrected
- On page 11 "figures ?? and 5" should be "figures 4 and 5"; reply: corrected

Referee 2: Thanks for your positive remarks.

The proposed changes are considered to be minor. However, I have discussed the possibility for a major change with the editor of the paper, pending the results of the review. My arguments for a major change concerns two topics:

An addition: the momentum balance per stream-annulus. With the pressure at the boundaries of the annuli taken into the momentum balance, the non-uniformity of the axial velocity at the disc is reproduced accurately. This check per annulus is not included in the IOP-Torque2016 paper. I intended to include it in the WES contribution but did not manage to debug the computer code in time. The addition is more or less a stand-alone subsection, and has no effect on the other text.

A change: in section 3.2 the choice of the vortex core model is discussed, concluding that the choice for constant core diameter requires further investigation as it does not comply with the inviscid Euler equation. I was able to solve this problem after the submission. In the IOP-Torque2016 paper I integrated the pressure due to the swirl, to be included in the axial momentum balance. This was done in two planes perpendicular to the centreline, the far wake and the downstream side of the disc, from the vortex core boundary up to the stream-tube boundary. The integration from the centreline to the core radius was omitted as the limit of vanishing core size was taken. However, the pressure in the core is now shown to behave as a Delta function, so contributes for vanishing core size. When this contribution is added to the momentum balance, the results as presented in the WES draft paper are reproduced, but now irrespective of the choice of the vortex core model. This is new knowledge, not yet available when I submitted the WES draft paper. It is believed that with this addition the solution is complete, consistent and complies with the Euler flow equations. This change has no consequences for the mathematics, figures and results: all remain the same. However, some consequences for textual editing, abstract and conclusions are involved.

Comparison of the first version of the paper with the proposed second version shows the following changes, adding up to the changes mentioned above:

- New section 2.4 has been added, in which the Delta-function-characteristics of the vortex core are derived
- New section 4.3 has been added: the analysis per annulus. Old 4 is now 4.1, old 5 is 4.2
- In section 3.1 and figure contribution e is added, which is the contribution by the vortex core.
- Old section 3.2 "the choice of the vortex core model" is removed
- A part of old section 3.3 is merged with 3.1, a part is removed
- Abstract and conclusions have been adapted.
- Independent of the above: old equation 2 is removed as it was not used at all.

The two new sections 2.4 and 4.3 concern the major changes, the other changes are consequential.

I realize that it is quite unusual that an author asks for a major change. However, as this change resolves the question of the vortex core model it completes the paper. I hope you can take to look at my proposed changes, and look forward to your comments.

Gijs van Kuik