

1 **Assessing Spacing Impact on Coherent Features in a Wind**
2 **Turbine Array Boundary Layer**

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Abstract

As wind farms become larger, the spacing between turbines becomes a significant design element that imposes serious economic constraints. Effects of turbine spacing on the power produced and flow structure are crucial for future development of wind energy. To investigate the turbulent flow structures in a 4×3 Cartesian wind turbine array, a wind tunnel experiment was carried out parameterizing the streamwise and spanwise wind turbine spacing. Four cases were chosen spacing turbines by 6 diameters (D) or $3D$ in the streamwise, and $3D$ or $1.5D$ in the spanwise direction. Data were obtained experimentally using stereo particle-image velocimetry. Mean streamwise velocity showed maximum values upstream of the turbine with the spacing of $6D$ and $3D$, in the streamwise and spanwise direction, respectively. Fixing the spanwise turbine spacing to $3D$, variations in the streamwise spacing influence the turbulent flow structure and the power available to following wind turbines. Quantitative comparisons were made through spatial averaging, shifting measurement data and interpolating to account for the full range between devices to obtain data independent of array spacing. The largest averaged Reynolds stress is seen in cases with spacing of $3D \times 3D$. Snapshot proper orthogonal decomposition (POD) was employed to identify the flow structures based on the turbulence kinetic energy content. The maximum turbulence kinetic energy content in the first POD mode compared with other cases is seen for turbine spacing of $6D \times 1.5D$. The flow upstream of each wind turbine converges faster than the flow downstream according to accumulation of turbulence kinetic energy by POD modes, regardless of spacing. The streamwise-averaged profile of the Reynolds stress is reconstructed using a specific number of modes for each case; the case of $6D \times 1.5D$ spacing shows the fastest reconstruction. Intermediate modes are also used to reconstruct the averaged profile and show that the intermediate scales are responsible for features seen in the original profile. The variation in streamwise and spanwise spacing leads to changes in the background structure of the turbulence, where the color map based on barycentric map and anisotropy stress tensor provides a new perspective on the nature of the perturbations within the wind turbine array. The impact of the streamwise and spanwise spacings on power produced is quantified, where the maximum production corresponds with the case of greatest turbine spacing.

6 I. INTRODUCTION

7 Allowing insufficient space between wind turbines in an array leads to decreased perfor-
8 mance through wake interaction, decreased wind velocity and an increase in the accumulated
9 fatigue loads and intermittency events on downstream turbines (Viggiano et al. 2016, Ali
10 et al. 2016a). Wind turbine wakes lead to an average loss of 10-20% of the total potential
11 power output of wind turbine array (Barthelmie et al. 2007). Extensive experimental and
12 numerical studies focus on wake properties in terms of the mean flow characteristics used to
13 obtain estimates of power production (Chamorro and Porté-Agel 2009, 2011). Wake growth
14 depends on the shape and magnitude of the velocity deficit, surface roughness, flow above
15 the canopy and spacing between the turbines.

16 Although there are many studies dealing with the effect of the density of turbines on
17 the wake recovery, it is still a debated question. The actual spacing of wind turbines can
18 vary greatly from one array to another. For example, in the Nysted farm, spacing is 10.5
19 diameters (D) downstream by $5.8D$ spanwise at the exact row (ER). The wind direction at
20 the ER is 278° and mean wind direction can slightly offset from ER by $\pm 15^\circ$ (Barthelmie
21 et al. 2010). In the Horns Rev farm, spacing between devices is $7D$, although aligned with
22 the bulk flow direction spacing is as much as $10.4D$. Barthelmie and Jensen (2010) showed
23 that the spacing in the Nysted farm is responsible for 68-76% of the farm efficiency variation.
24 Hansen et al. (2012) pointed out that variations in the power deficit are almost negligible
25 when spacing is approximately $10D$ at the Horns Rev farm, in contrast to limited spacings
26 that present a considerable power deficit. González-Longatt et al. (2012) found that when
27 the streamwise and spanwise spacing increased, the wake coefficient, which represents the
28 ratio of total power output with and without wake effects, is increased. Further, the effect of
29 the incoming flow direction on the wake coefficient increased when the spacing of the array is
30 reduced. Meyers and Meneveau (2012) studied the optimal spacing in a fully developed wind
31 farm under neutral stratification and flat terrain. The results highlighted that, depending
32 on the ratio of land and turbine costs, the optimal spacing might be $15D$ instead of $7D$.
33 Stevens (2015) pronounced that the optimal spacing depends on the length of the wind farm
34 in addition to the factors suggested in Meyers and Meneveau (2012). Orography and wind
35 direction are relevant when deciding distance between turbines as well as layout as shown
36 by Romanic et al. (2018). Nilsson et al. (2015) performed large eddy simulations (LES)

37 of the Lillgrund wind farm, where pre-generated turbulence and wind shear were imposed
38 in the computational domain to simulate realistic atmospheric conditions. In the Lillgrund
39 wind farm, the actual spacing is $3.3D$ and $4.6D$ in the streamwise and spanwise directions.
40 A turbine is missing near to the center of the wind farm, demonstrating the effects of a
41 farm with limited spacing and one with sufficient spacing in otherwise identical operating
42 conditions. The results of Nilsson et al. (2015) are highly applicable in the current study,
43 although their foci are on turbulence intensity effects and yaw angle.

44 Further investigations in array optimization have been undertaken by changing the align-
45 ment of the wind farm, often referred to as staggered wind farms. Meyers and Meneveau
46 (2010) compared aligned versus staggered wind farms; the latter yielding a 5% increase in
47 extracted power. Yang et al. (2012) used LES to study the influence of the streamwise and
48 spanwise spacing on the power output in aligned wind farms under fully developed regime.
49 Their work confirmed that power produced by the turbines scales with streamwise spacing
50 more than with the spanwise spacing. Wu and Porté-Agel (2013) investigated turbulent flow
51 within and above aligned and staggered wind farms under neutral condition. Cumulative
52 wakes are shown to be subject to strong lateral interaction in the staggered case. In contrast,
53 lateral interaction is negligible in the aligned wind farm. Archer et al. (2013) quantified the
54 influence of wind farm layout on the power production, verifying that increasing the turbine
55 spacing in the predominant wind direction maximized the power production, regardless of
56 device arrangement in the wind farm. Stevens et al. (2016) investigated the power output
57 and wake effects in aligned and staggered wind farms with different streamwise and spanwise
58 turbine spacings. In the staggered configuration, power output in a fully developed flow de-
59 pends mainly on the spanwise and streamwise spacings, whereas in the aligned configuration,
60 power strongly depends on the streamwise spacing.

61 As wind farms become larger, the land costs and availability represent critical factors in
62 the overall value of the wind farm. Spacing between the turbines is an important design
63 factor in terms of overall wind farm performance and economic constraints. Investigation of
64 wind farms with limited spacing is important in order to quantify the effects of wind turbine
65 wake interaction on the power production. The current work compares the turbulent flow
66 in various configurations of the array, where the streamwise and spanwise spacings are var-
67 ied. The tunnel-scaled wind farm is, however, restricted to a flat surface and topographic
68 influences are not considered, although the inflow to the wind farm includes modifications

69 to more closely resemble an atmospheric boundary layer. The performance of the arrays
70 is characterized by analyzing the mean velocity, Reynolds shear stress, and power produc-
71 tion. Proper orthogonal decomposition (POD) is employed to identify coherent structures of
72 the turbulent wake associated with variations in spacing. The Reynolds stresses are recon-
73 structed from POD basis, demonstrating variation in rates of convergence according to wind
74 turbine spacing. Finally the anisotropy stress tensor is discussed to quantify the structure
75 of the stress tensor based on the invariant for the various spacings.

76 II. THEORY

77 A. Snapshot Proper Orthogonal Decomposition

78 POD is a mathematical tool that derives optimal basis functions from a set of measure-
79 ments, decomposing the flow into modes that express the most dominant features. The
80 technique, which was presented in the frame of turbulence by Lumley (1967), categorizes
81 structures within the turbulent flow depending on their energy content. Sirovich (1987)
82 presented the snapshot POD, that relaxes the computational difficulties of the classical or-
83 thogonal decomposition. POD has been used to describe coherent structures for different
84 flows, such as axisymmetric mixing layer (Glauser and George 1987), channel flow (Moin
85 and Moser 1989), atmospheric boundary layer (Shah and Bou-Zeid 2014), wake behind disk
86 (Tutkun et al. 2008), and wind turbine wake flow (Andersen et al. 2013, Bastine et al. 2014,
87 VerHulst and Meneveau 2014, Hamilton et al. 2015a, Ali et al. 2016b, 2017a).

88 The flow field, taken as the fluctuating velocity, can be represented as $u = u(\vec{x}, t^n)$, where
89 \vec{x} and t^n refer to the spatial coordinates and time at sample n , respectively. A set of the
90 orthonormal basis functions, ϕ , can be presented as

$$\phi = \sum_{n=1}^N A(t^n)u(\vec{x}, t^n), \quad (1)$$

91 where N is the number of snapshots. The largest projection can be determined using the
92 two point correlation tensor and Fredholm integral equation

$$\int_{\Omega} \frac{1}{N} \sum_{n=1}^N u(\vec{x}, t^n)u^T(\vec{x}', t^n)\phi(x')dx' = \lambda\phi(x), \quad (2)$$

93 where left hand side of the equation presents a spatial correlation between two points \vec{x}
94 and \vec{x}' , T signifies the transpose of a matrix, Ω is the physical domain, and λ are the
95 eigenvalues. To acquire the optimal basis functions, the problem is reduced to an eigenvalue
96 decomposition denoted as $[C][G] = \lambda[G]$, where C , G and λ are the correlation tensor, basis
97 of eigenvectors, and eigenvalues, respectively. The POD eigenvectors illustrate the spatial
98 structure of the turbulent flow and the eigenvalues measure the energy associated with
99 corresponding eigenvectors. The summation of the eigenvalues presents the total turbulent
100 kinetic energy (E) in the flow domain. The cumulative kinetic energy fraction η and the
101 normalized energy content of each mode ξ can be represented as $\eta_n = \sum_{j=1}^n \lambda_n / \sum_{j=1}^N \lambda_n$ and
102 $\xi_n = \lambda_n / \sum_{j=1}^N \lambda_n$. POD is particularly useful in rebuilding the Reynolds shear stress using
103 a limited set (N_{lm}) of eigenfunctions as follows,

$$\langle u_i u_j \rangle = \sum_{n=1}^{N_{lm}} \lambda_n \phi_i^n \phi_j^n. \quad (3)$$

104 B. Anisotropy Stress Tensor

105 Turbulence is often described through the Reynolds stress tensor. Rotta (1951) developed
106 the Reynolds stress anisotropy tensor, as $a_{ij} = \overline{u_i u_j} - \frac{2}{3} k \delta_{ij}$, where δ_{ij} is the Kronecker
107 delta and k represents the turbulent kinetic energy. The deviatoric tensor is obtained,
108 $b_{ij} = \overline{u_i u_j} / \overline{u_k u_k} - \frac{1}{3} \delta_{ij}$. The second and third scalar invariants are defined as $6\eta^2 = b_{ij} b_{ji}$
109 and $6\xi^3 = b_{ij} b_{jk} b_{ki}$, respectively (see Pope (2000), Lumley and Newman (1977) for more
110 details). The second invariant, η , measures the degree of the anisotropy and the third
111 invariant, ξ , specifies the state of turbulence. Alternatively, the eigenvalue decomposition
112 of the normalized Reynolds stress anisotropy tensor can be used to derive the the second
113 and third invariants as $\eta^2 = \frac{1}{3}(\lambda_1^2 + \lambda_1 \lambda_2 + \lambda_2^2)$ and $\xi^3 = -\frac{1}{2} \lambda_1 \lambda_2 (\lambda_1 + \lambda_2)$. In an attempt
114 to additionally promote the study of turbulence anisotropy, Banerjee et al. (2007) presented
115 a linearized anisotropy tensor invariants, termed barycentric map (BM) as follows,

$$\hat{b}_{ij} = C_{1c} \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} + C_{2c} \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} + C_{3c} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (4)$$

116 where C_{1c} , C_{2c} and C_{3c} are the coefficients that represent the boundary of the barycentric

TABLE I: Summary of the special turbulence cases described by the barycentric map.

Cases	Eigenvalues
Three-component	$\lambda_1 = \lambda_2 = \lambda_3 = 0$
Two-component	$\lambda_1 = \lambda_2 = \frac{1}{6}, \lambda_3 = -\frac{1}{3}$
One-component	$\lambda_1 = \frac{2}{3}, \lambda_2 = \lambda_3 = -\frac{1}{3}$

map. The BM coefficients are determined as $C_{1c} = \lambda_1 - \lambda_2$, $C_{2c} = 2(\lambda_2 - \lambda_3)$, and $C_{3c} = 3\lambda_3 + 1$. The three basis matrices in equation (4) represent the three vertices of the equilateral triangle, with the following coordinates (x_{1c}, y_{1c}) , (x_{2c}, y_{2c}) and (x_{3c}, y_{3c}) . Table I presents the three turbulence states corresponding with the vertices of the BM, which also correspond to either isotropic (three-component), one- or two-component turbulence. As a result, any realizable turbulence state can be represented as follows,

$$x_{new} = C_{1c}x_{1c} + C_{2c}x_{2c} + C_{3c}x_{3c}, \quad (5)$$

$$y_{new} = C_{1c}y_{1c} + C_{2c}y_{2c} + C_{3c}y_{3c}. \quad (6)$$

Emory and Iaccarino (2014) also introduced a color map based visualization technique that aids to interpret the spatial distribution of the normalized anisotropy tensor. In this case, they attributed to each vertex of the barycentric map an RGB (Red-Green-Blue) color, see figure 1 for more details. This color map technique combines the coefficients C_{1c} , C_{2c} and C_{3c} to generate an RGB map such that,

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = C_{1c}^* \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_{2c}^* \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_{3c}^* \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (7)$$

where C_{ic}^* are the modified coefficients that can be determined as $C_{ic}^* = (C_{ic} + 5)^{0.65}$. As a result, one-component turbulence is associated to the red color, two-component turbulence to green, and three-component (isotropic turbulence) to blue, see figure 1. The anisotropy has been examined in different types of flow, including pipe and duct flows (Antonia et al. 1991, Krogstad and Torbergsen 2000), atmospheric boundary layer (Klipp 2010, 2012) as

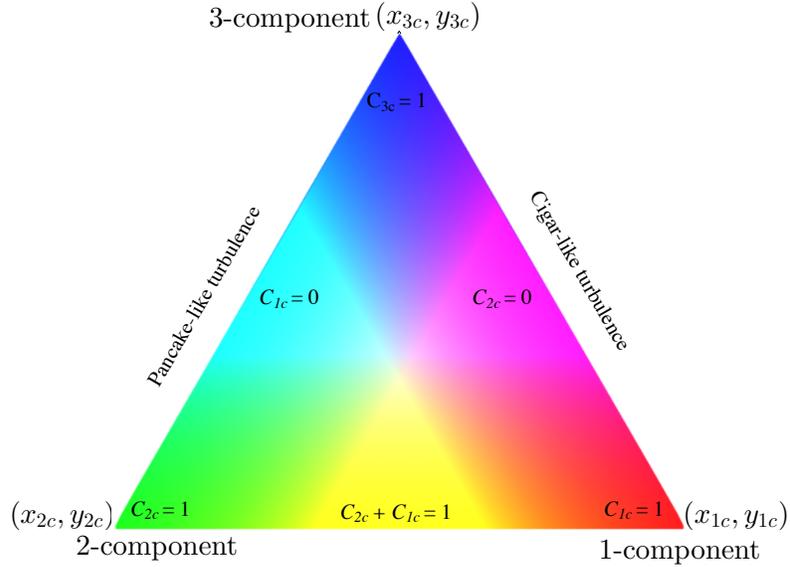


FIG. 1: Schematic representation of the Barycentric map (BM) with color map.

133 well as the wake of a wind turbine (Gómez-Elvira et al. 2005, Hamilton and Cal 2015, Ali
 134 et al. 2017b). Here we will use the anisotropy stress tensor to quantify the
 135 effect of the spacing on the turbulence states.

136 III. EXPERIMENTAL DESIGN

137 A 4×3 array of wind turbines was placed in the closed-circuit wind tunnel at Portland
 138 State University to study the effects due to variation in streamwise and spanwise spacing in
 139 a wind turbine array. The dimensions of the wind tunnel test section are 5 m (long), 1.2 m
 140 (wide) and 0.8 m (high). The blockage ratio is less than 5% in the current analysis. The
 141 entrance of the test section is conditioned by the passive grid, which consists of 7 horizontal
 142 and 6 vertical rods, to introduce large-scale turbulence. Nine vertical acrylic strakes, located
 143 at 0.25 m downstream of the passive grid and 2.15 m upstream of the first row of the wind
 144 turbine, were used to modify the inflow. The thickness of the strakes is 0.0125 m and are
 145 spaced every 0.136 m across the test section. Surface roughness was introduced to the wall
 146 as a series of chains with a diameter of 0.0075 m, spaced 0.11 m apart. Figure 2 shows the
 147 schematic of the experimental setup.

148 Sheet steel of 0.0005 m thick was used to construct the 3-bladed wind turbine rotors.

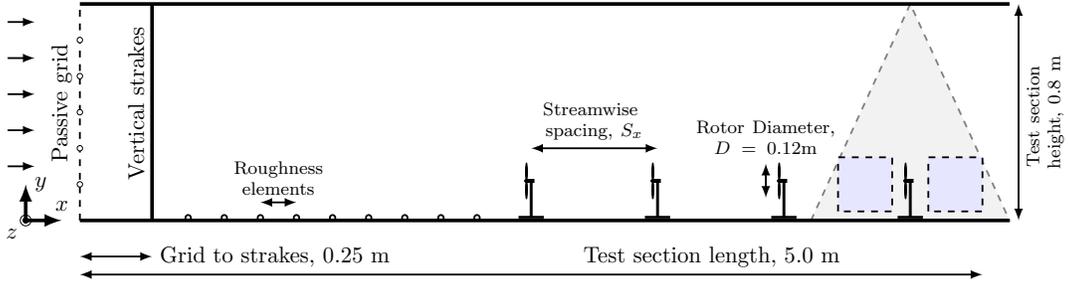


FIG. 2: Experimental Setup. Dashed gray lines indicate the placement of the laser sheet relative to the model wind turbine array. Filled gray boxes indicate measurement locations discussed below.

149 The diameter of the rotor was $D = 0.12$ m, equal to the height of the turbine tower. The
 150 scaled turbine models were manufactured in-house. Based on full scale turbines with a 100
 151 m rotor diameter and a 100 m hub height, the scaled models are at 1:830 scale. The rotor
 152 blades are steel sheets laser cut to shape and are 0.0005 m thick. The blades are shaped
 153 using a die press. The die press was designed in-house to produce a 15 degree pitch from the
 154 plane of the rotor and a 10 degree twist at the tip. Figure 3 presents the schematic of the
 155 wind turbine model. Operating conditions for the wind turbines are also scaled, namely the
 156 power coefficient, C_p and tip-speed ratio, λ , which are detailed in Hamilton et al. (2015b)
 157 The streamwise integral length scale is approximately 0.13 m, which is the same order of
 158 magnitude as the turbine rotor and representative of conditions seen by full-scale turbines in
 159 atmospheric flows. A DC electrical motor of 0.0013 m diameter and 0.0312 m long formed
 160 the nacelle of the turbine and was aligned with the flow direction. A torque-sensing system
 161 was connected to the DC motor shaft following the design outlined in Kang and Meneveau
 162 (2010). The torque sensor consists of a strain gauge, Wheatstone bridge and the Data
 163 Acquisition with measuring software to collect the data.

164 The flow field was sampled in four configurations of a model-scale wind turbine array,
 165 classified as $C_{S_x \times S_z}$, shown in Table II. Permutations of the streamwise spacing (S_x) of
 166 $6D$ and $3D$ and spanwise spacing (S_z) of $3D$ and $1.5D$ are examined. Thus, the four
 167 cases present aligned wind farm; the staggered wind farm does not consider in this study.
 168 Stereoscopic particle image velocimetry (SPIV) was used to measure streamwise, wall-normal
 169 and spanwise instantaneous velocity at the upstream and downstream of the wind turbine
 170 at the center line of the fourth row as shown in figure 4. At each measurement location,
 171 2000 images were taken, to ensure convergence of second-order statistics. SPIV equipment is

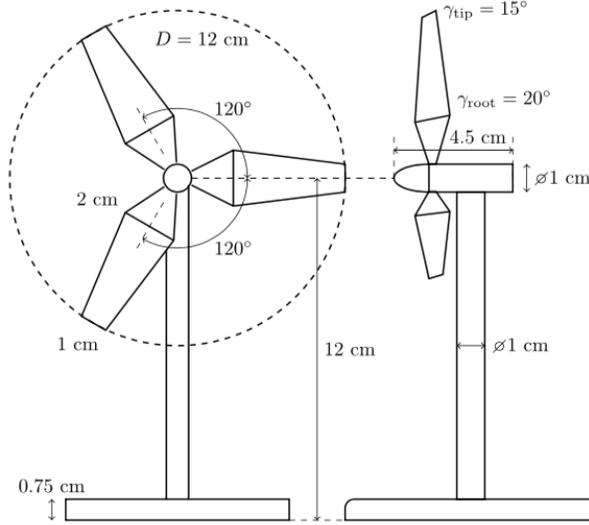


FIG. 3: Schematic representation of the wind turbine model (Hamilton and Cal 2015).

172 LaVision and consists of a Nd:Yag (532nm, 1200mJ, 4ns duration) double-pulsed laser and
 173 four 4 MP ImagerProX CCD cameras positioned for the upstream and downstream of the
 174 wind turbine. Neutrally buoyant fluid particles of diethyl hexyl sebacate were introduced to
 175 the flow and allowed to mix. Consistent seeding density was maintained in order to mitigate
 176 measurement errors. The laser sheet was approximately 0.001 m thick with less than 5 mrad
 177 divergence angle. Each measurement window was 0.2 m \times 0.2 m aligned with the center of
 178 each turbine, parallel to the bulk flow. A multi-pass fast Fourier transformation was used
 179 to process the raw data into vector fields. Erroneous measurement of the vector fields were
 180 replaced using Gaussian interpolation of neighboring vectors.

TABLE II: Streamwise and spanwise spacing of the experimental tests.

Cases	S_x	S_z	Occupied Area
$C_{6 \times 3}$	$6D$	$3D$	$18D^2$
$C_{3 \times 3}$	$3D$	$3D$	$9D^2$
$C_{3 \times 1.5}$	$3D$	$1.5D$	$4.5D^2$
$C_{6 \times 1.5}$	$6D$	$1.5D$	$9D^2$

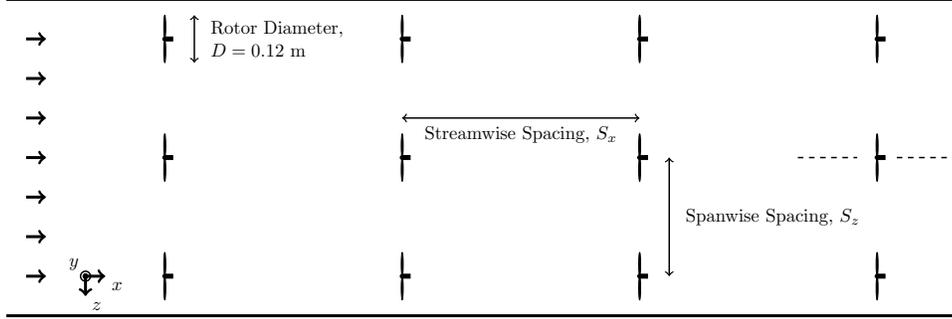


FIG. 4: Top view of 4 by 3 wind turbine array. The dash lines at the last row centerline turbine represent the measurement locations.

181 IV. RESULTS

182 A. Statistical Analysis.

183 Characterization of the wind turbine wake flow is presented by the streamwise mean
 184 velocity and Reynolds shear stress, with the aim to understand the influence of turbine-
 185 to-turbine spacing. Figure 5 presents the streamwise normalized mean velocity, U/U_∞ ,
 186 upstream and downstream of each wind turbine for the cases $C_{6 \times 3}$, $C_{3 \times 3}$, $C_{3 \times 1.5}$ and $C_{6 \times 1.5}$.
 187 The inflow mean velocity at the hub height U_∞ is used in the normalization, where $U_\infty =$
 188 5.5 m s^{-1} . The left and right contour plots of each case present the flow upstream and
 189 downstream of each turbine, respectively. At upstream measurement window, case $C_{6 \times 3}$
 190 exhibits the largest streamwise mean velocities due to greater recovery of the flow upstream
 191 of the turbine. Although the streamwise spacing of case $C_{6 \times 1.5}$ is similar that of case $C_{6 \times 3}$,
 192 the former shows reduced hub height velocity. The normalized mean velocity is about 0.567
 193 compared with 0.66 in case $C_{6 \times 3}$, confirming the influence of the spanwise spacing on wake
 194 evolution and flow recovery. Variations perceived between case $C_{3 \times 3}$ and $C_{3 \times 1.5}$ are small,
 195 where case $C_{3 \times 3}$ demonstrates higher velocities by approximately 2%. Downstream of the
 196 turbine, the four cases show more relevant differences especially above the top tip and below
 197 the bottom tip, where case $C_{6 \times 3}$, once again, shows the greatest velocities by approximately
 198 20%. Case $C_{3 \times 3}$ also shows higher velocities below the bottom tip compared with cases
 199 $C_{3 \times 1.5}$ and $C_{6 \times 1.5}$. The normalized mean streamwise velocity and the turbulence intensity in
 200 Nilsson et al. (2015) showed similar compound wakes from the upstream and downstream
 201 turbines and confirmed the current result of cases $C_{3 \times 3}$ and $C_{3 \times 1.5}$. In that study, there was

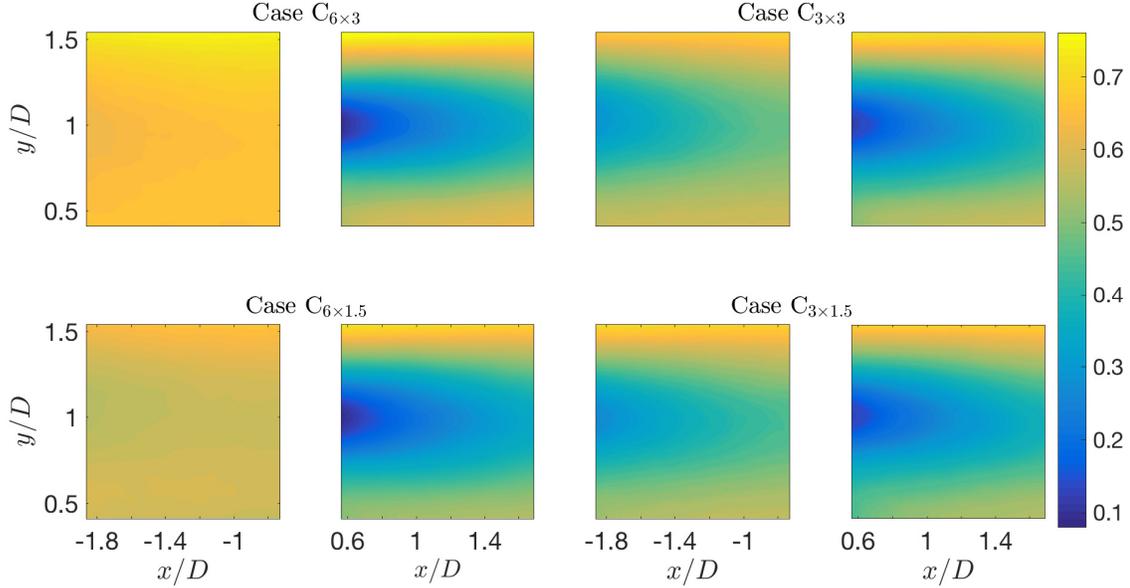


FIG. 5: Normalized streamwise velocity, U/U_∞ , at upstream and downstream of the cases $C_{6\times 3}$, $C_{3\times 3}$, $C_{3\times 1.5}$, and $C_{6\times 1.5}$.

202 one location with an absent turbine and the flow was given extra space for recovery. The
 203 recovered wake flow in Nilsson et al. (2015) is similar to the present cases $C_{6\times 3}$ and $C_{6\times 1.5}$.

204 Figure 6 compares the in-plane normalized Reynolds shear stress $-\overline{uv}/U_\infty^2$ for all test
 205 cases. The fluctuating velocities in streamwise and wall-normal direction are denoted as u
 206 and v , respectively. In the upstream window, cases $C_{3\times 3}$ and $C_{3\times 1.5}$ display higher stress
 207 compared with $C_{6\times 3}$ and $C_{6\times 1.5}$. Although the spanwise spacing of case $C_{3\times 1.5}$ is half of
 208 case $C_{3\times 3}$, no relevant differences are apparent. In the downstream window, comparison
 209 indicates that reducing streamwise spacing increases the Reynolds shear stress. The average
 210 value of the shear stress in the wake is 16% greater for $C_{3\times 3}$ than for $C_{6\times 3}$. A similar effect is
 211 observed in case $C_{3\times 1.5}$, where average value of the stress is 2% greater than that of $C_{6\times 1.5}$.
 212 The effect of spanwise spacing is more pronounced when the streamwise spacing is $3D$; the
 213 average shear stress is approximately 20% greater in $C_{3\times 1.5}$ than in $C_{3\times 3}$.

214 B. Averaged Profiles.

215 Spatial averaging of the flow statistics is undertaken by moving the upstream domain
 216 of each case beyond its corresponding downstream domain and performing streamwise av-

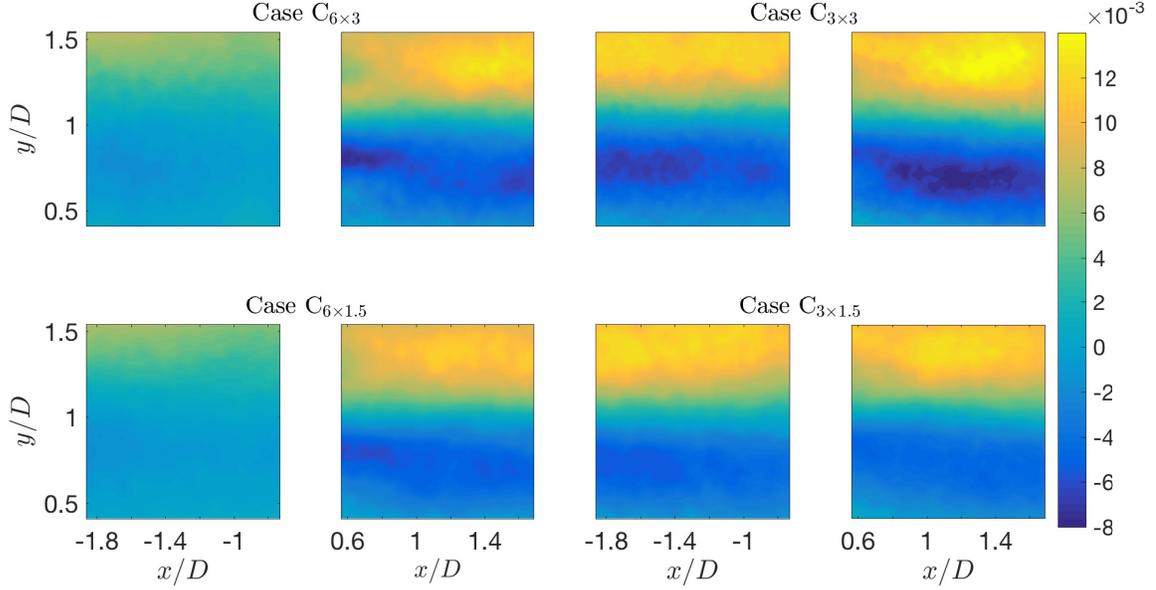


FIG. 6: Normalized Reynolds shear stress, $-\overline{uv}/U_\infty^2$, in upstream and downstream of the each measurement case.

217 eraging, following the procedure in Cal et al. (2010). Though the spatial averaging, it is
 218 possible to compare key data from different cases taking into account the different streamwise
 219 spacings. Streamwise averaging is denoted by $\langle \cdot \rangle_x$. Figure 7(a) shows profiles of streamwise-
 220 averaged mean velocity for all four cases. Cases $C_{6 \times 3}$ and $C_{3 \times 1.5}$ show the largest and
 221 smallest velocity deficits, respectively. At hub height, the velocity of the case $C_{6 \times 3}$ is ap-
 222 proximately 2.25 m s^{-1} whereas case $C_{3 \times 1.5}$ shows a velocity of approximately 1.6 m s^{-1} .
 223 Comparing to $C_{6 \times 3}$, the change seen in the spatially-averaged velocity is greater in $C_{3 \times 3}$
 224 than in $C_{6 \times 1.5}$, confirming that the impact of reducing streamwise spacing is greater than
 225 changing the spanwise spacing. A reduction in streamwise spacing shows less effect when
 226 the spanwise spacing $S_z = 1.5D$.

227 Figure 7(b) contains the streamwise-averaged Reynolds shear stress $\langle -\overline{uv}/U_\infty^2 \rangle_x$ for cases
 228 $C_{6 \times 3}$ through $C_{6 \times 1.5}$. Slightly decreased in $\langle -\overline{uv}/U_\infty^2 \rangle_x$ are attained in case $C_{6 \times 1.5}$, where the
 229 spanwise spacing is reduced. Reducing spanwise spacing shows an important influence when
 230 the streamwise spacing is $x/D = 3$. The streamwise spacing plays a larger role than the
 231 spanwise spacing, *i.e.* the maximum differences between the Reynolds shear stress profiles
 232 are detected between cases $C_{6 \times 3}$ and $C_{3 \times 3}$. Interestingly, the largest difference between the
 233 spatially-averaged Reynolds shear stress is found between cases $C_{6 \times 3}$ and $C_{3 \times 3}$, located at

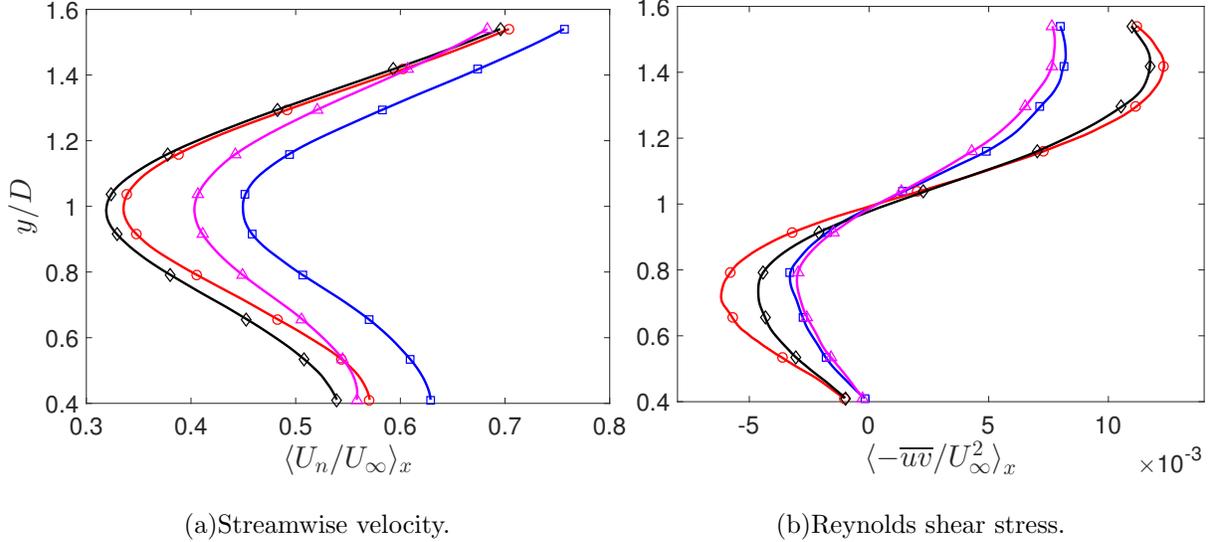


FIG. 7: Streamwise-averaged profiles of streamwise velocity, and Reynolds shear stress for four different cases $C_{6 \times 3}$ (\square), $C_{3 \times 3}$ (\circ), $C_{3 \times 1.5}$ (\diamond), and $C_{6 \times 1.5}$ (\triangle).

234 $y/D \approx 0.7$ and $y/D \approx 1.4$. Furthermore, the four cases have approximately zero Reynolds
 235 shear stress at the inflection point located at hub height. In addition, case $C_{3 \times 3}$ displays
 236 the maximum Reynolds stress and case $C_{6 \times 1.5}$ presents the minimum stress.

237 C. Proper Orthogonal Decomposition.

238 Based on the velocity field, the spatially integrated turbulent kinetic energy is expressed
 239 by the eigenvalue of each POD mode. The normalized cumulative energy fraction η_n for
 240 upstream and downstream measurement windows are presented in figure 8(a) and (b), re-
 241 spectively. Inset figures exhibit the normalized energy content per mode, ξ_n . Upstream of
 242 the turbine, cases $C_{6 \times 3}$ and $C_{6 \times 1.5}$ converge faster than cases $C_{3 \times 3}$ and $C_{3 \times 1.5}$, respectively.
 243 These results are attributed to the reduction the streamwise spacing. The convergence of
 244 case $C_{3 \times 3}$ is approximately coincident with case $C_{3 \times 1.5}$. For the downstream flow, case $C_{6 \times 1.5}$
 245 converges faster than the other cases, thereafter it is ordered as $C_{6 \times 3}$, $C_{3 \times 3}$ and $C_{3 \times 1.5}$ in
 246 succession. The comparison between the upstream and downstream windows reveals that
 247 energy accumulates in fewer modes upstream in each case, *e.g.*, case $C_{6 \times 3}$ requires 14 modes
 248 to obtain 50% of the total kinetic energy in the upstream window, whereas 26 modes are
 249 required to obtain the same percentage of energy downstream of the turbine. Cases $C_{6 \times 1.5}$
 250 and $C_{3 \times 1.5}$ show the maximum and minimum variations in λ_1 , respectively. This observation

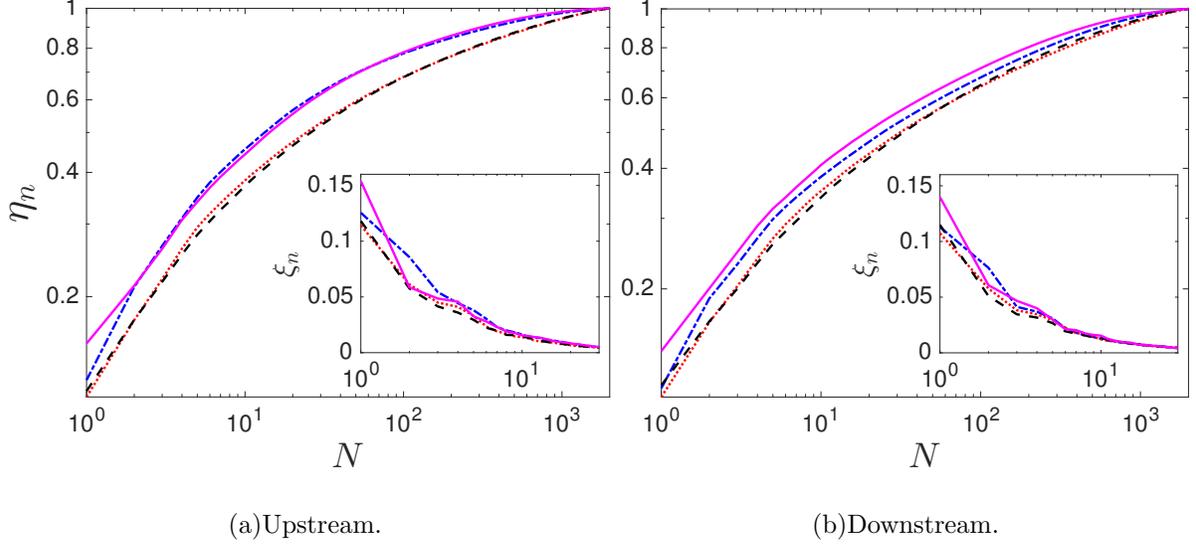


FIG. 8: Energy content of the POD modes for four different cases: $C_{6 \times 3}$ ($-\cdot-$), $C_{3 \times 3}$ ($\cdot\cdot\cdot$), $C_{3 \times 1.5}$ ($---$), and $C_{6 \times 1.5}$ ($-$).

251 can be attributed to the structure of the upstream flow of case $C_{6 \times 1.5}$, which is rather recov-
 252 ered, compared to the downstream flow, where the turbulence is high in energy content and
 253 more complex. However, the upstream and downstream windows of case $C_{3 \times 1.5}$ are more
 254 similar in terms of turbulence and organization. From mode 2 through 10, the starkest dif-
 255 ference between the upstream and downstream is found in case $C_{6 \times 3}$. Increasing the spacing
 256 area per turbine provides room for the flow to become more homogeneous in the upstream
 257 window and exhibit the most significant momentum deficit in the wake, accounting for the
 258 differences seen in η_n upstream and downstream.

259 The streamwise component of several POD modes is shown for all cases in figures 9
 260 through 11. These modes were selected because they provide a range of large and interme-
 261 diate scales, and highlight the discrepancies among the cases. Figure 9 presents the first
 262 POD mode at the upstream and downstream of the considered cases. The four cases show
 263 small gradients in the streamwise direction compared to a large gradient in the wall-normal
 264 direction. Although the four cases show a divergence between the eigenvalues of the first
 265 mode, the eigenfunctions display very similar structures. For case $C_{6 \times 3}$ energy of the first
 266 POD mode shows decreases by 1.25% comparing the upstream eigenvalue to the downstream
 267 one, see figure 8. Smaller variations of 0.68% and 0.32% are observed in the cases $C_{3 \times 3}$ and
 268 $C_{3 \times 1.5}$, respectively. Consequently, the structures of upstream and downstream of these
 269 cases are approximately equivalent. The similarity in the shape of the structure is observed

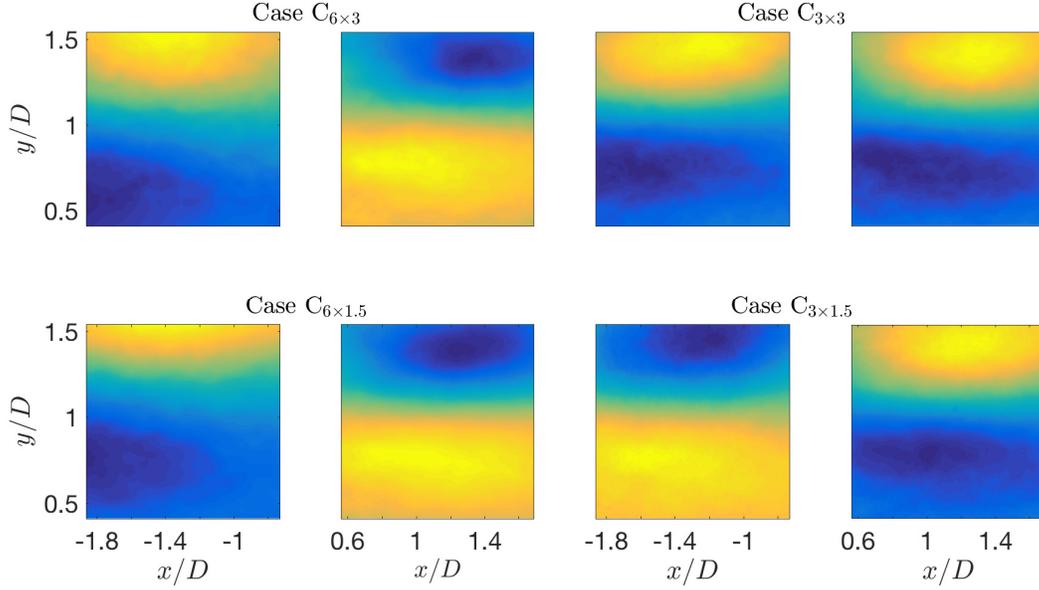


FIG. 9: The first mode upstream and downstream of the each case.

270 between cases $C_{6 \times 3}$ and $C_{6 \times 1.5}$ despite the turbulence kinetic energy difference between them
 271 being about 3%.

272 Figure 10 presents the fifth POD mode of the four cases that show a combination of POD
 273 and Fourier (homogenous) modes in the streamwise direction. Although the fifth mode of the
 274 four cases contains $\approx 74\%$ less energy of than the first mode, large scales are still pronounced.
 275 Smaller features also appear in the upstream and the downstream windows. The upstream
 276 window of cases $C_{6 \times 3}$, $C_{3 \times 3}$, and $C_{3 \times 1.5}$ is shifted horizontally in the downstream window.
 277 The upstream and downstream windows of case $C_{3 \times 1.5}$ look like the first mode, but at a
 278 reduced scale. The same trend is observed in the downstream window of the case $C_{6 \times 1.5}$.

279 Figure 11 presents the twentieth POD mode, where small structures become noticeable
 280 in both upstream and downstream windows. The upstream measurement window of cases
 281 $C_{6 \times 3}$ and $C_{6 \times 1.5}$ shows large scale structures compared with the other two cases. Although,
 282 after mode 10, there is no significant difference in the energy content from case to case, the
 283 structure of the modes shows a significant discrepancy between the cases confirming that
 284 the intermediate modes associate with the inflow characterizations.

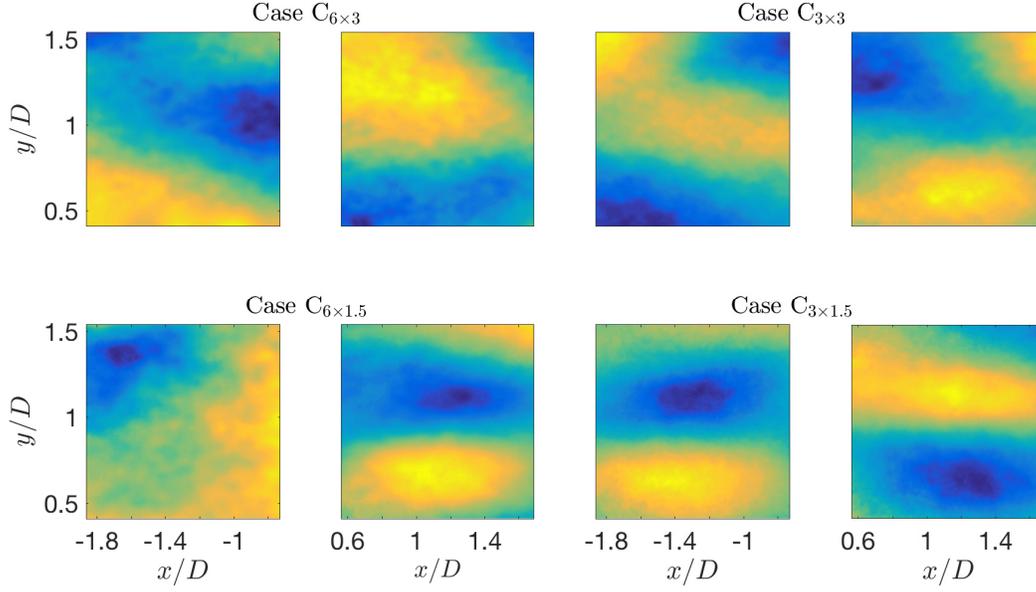


FIG. 10: The fifth mode upstream and downstream of the each case.

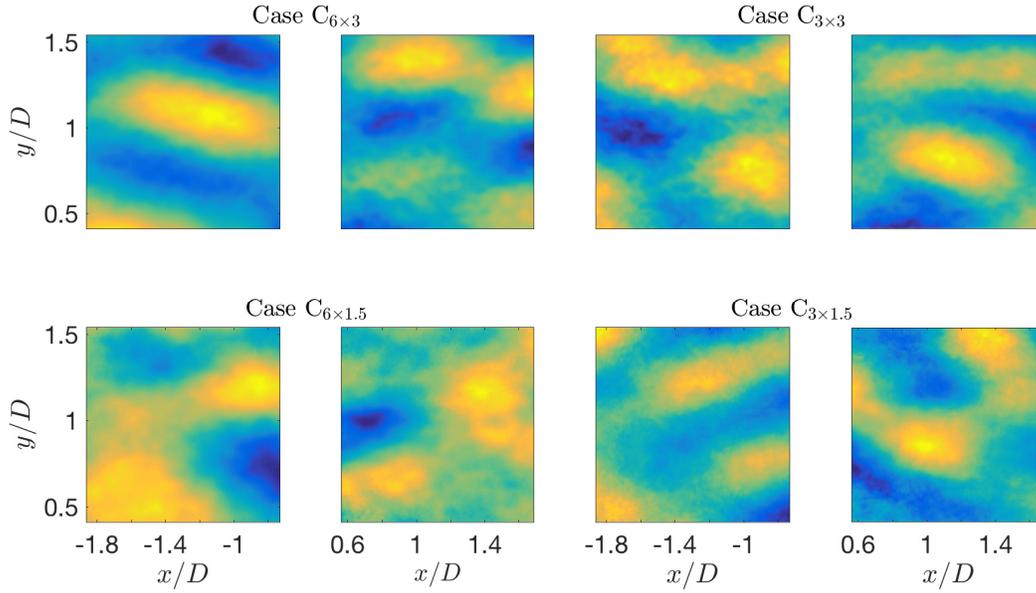


FIG. 11: The twentieth mode upstream and downstream of the each case.

285 **D. Reconstruction of Averaged Profile.**

286 Combining the POD modes with the corresponding time coefficient gives these modes the
 287 physical interpretation and shows the contribution of these modes in the flow perturbation.
 288 A reduced degree of the turbulence kinetic energy is considered using only a few modes

289 to reconstruct the streamwise-averaged profiles of Reynolds shear stress. Reconstructions
 290 are made using either the first mode, the first 5, 10, 25, or 50 modes to represent the
 291 stress as shown in figure 12. Inset figures present the Reynolds shear stress construction
 292 using the modes 5-10, 5-25, and 5-50, respectively, excluding the first four modes isolates
 293 contributions from intermediate modes. The black lines are the streamwise average of full
 294 data from figure 7(b). Using an equal number of modes, case $C_{6 \times 1.5}$ rebuilds the profiles
 295 of the Reynolds shear stress faster than the other cases. Case $C_{6 \times 3}$ also shows the fast
 296 reconstruction and the dissimilarity with case $C_{6 \times 1.5}$ is mainly in the profile of first mode
 297 (red line) and the first five modes (blue line). Cases $C_{3 \times 3}$ and $C_{3 \times 1.5}$ show approximately
 298 the same trends in reconstruction profiles. Below hub height, the four cases show the same
 299 trend of the first mode profiles, where the contribution in the reconstruction profiles is zero.
 300 The first five modes display exactly the form of the full data profile of individual case. The
 301 maximum difference between the successive reconstruction profiles occurs between the first
 302 mode and the first five modes. The cases $C_{6 \times 3}$, $C_{3 \times 3}$ and $C_{3 \times 1.5}$ show moderate variation
 303 between the profiles of the reconstructed stress resulting from first five and first ten modes
 304 (red and green lines, respectively). After mode 10 contributions by each additional mode
 305 are quite small, shown by pink and gray lines.

306 The maximum difference between the full data and the reconstructed profiles is located
 307 at $y/D \approx 0.75$ and $y/D \approx 1.4$, where the extrema in $\langle -\overline{uv} \rangle_x$ are located. Generally, faster
 308 reconstruction implies that the flow possesses coherent structures with a greater portion
 309 of the total kinetic energy. Consequently, the flow characterized with greater coherence in
 310 the cases $C_{6 \times 3}$ and $C_{6 \times 1.5}$; in cases $C_{3 \times 3}$ and $C_{3 \times 1.5}$, less energetic features are observed.
 311 Thus, streamwise spacing allows for the flow to recover and therefore produce larger struc-
 312 tures within the domain, which in comparison eclipses variations produced by the spanwise
 313 spacing.

314 To quantify the contribution of the moderate-scaled structures, Reynolds shear stress is
 315 reconstructed using the intermediate modes. As can be shown in the insets of figure 12,
 316 the full data profile (black line) is compared with profiles reconstructed from modes 5-10
 317 (red line), 5-25 (blue line), and 5-50 (green lines). The intermediate modes in each case
 318 approximately take the form of the full data profiles below the hub height, although the
 319 magnitudes of the reconstructions are smaller than those of the full data statistics. Recon-
 320 struction Reynolds shear stress in cases $C_{6 \times 3}$ and $C_{3 \times 1.5}$ show minute variations between

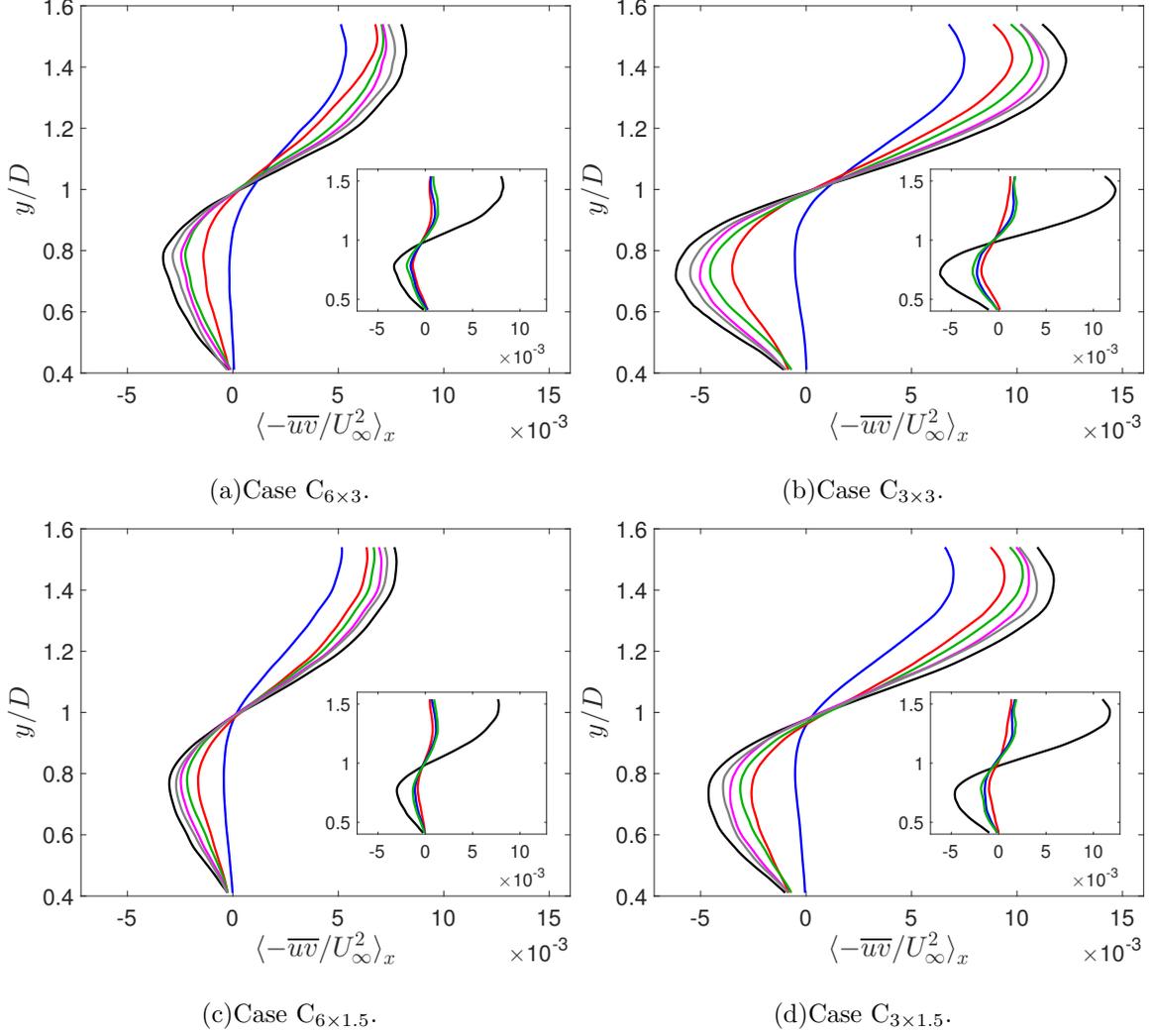


FIG. 12: Reconstruction Reynolds shear stress using: first mode (—), first 5 modes (—), first 10 modes (—), first 25 modes (—) and first 50 modes (—). Full data statistics (—). The insets show the reconstruction using modes 5-10 (—), 5-25 (—), and 5-50 (—).

321 the successive reconstruction profiles and are essentially vertical lines above the hub height.
 322 This trend is opposite to the trend that is shown in the first mode profile. Cases $C_{3 \times 3}$ and
 323 $C_{3 \times 1.5}$ show a difference between the successive profiles above the hub height. The maximum
 324 difference is observed between the reconstructed profiles from modes 5-10 and from 5-25.

325 E. Anisotropy Stress Tensor

326 To examine the dynamics and energy transfer in the wind turbine arrays with different
 327 streamwise and spanwise spacings, a description of the anisotropy in the upstream and

328 downstream of the wind turbines is presented in figure 13. A visualization of the turbulence
 329 state is obtained via the color map representing the barycentric map as described in section
 330 II B, where it efficiently distinguishes among the cases in terms of wake propagation and wake
 331 interaction. The variation in the spacings changes the background turbulence structure.
 332 The upstream of cases $C_{6 \times 3}$ and $C_{6 \times 1.5}$ shows the turbulence state close to the isotropy limit
 333 especially in hub height region as a result of the wake recovery occurring under a relatively
 334 long spacing distance. Below the bottom tip, these cases show pancake-like turbulence due to
 335 the surface effect that appear deeming the perturbation of the turbines virtually negligible.
 336 Near top tip, the flow shows a turbulence of axisymmetric state (between the pancake-like
 337 and cigar-like turbulence). With this representation, the spacing variation leads to a changed
 338 state of the turbulence and between the developed and developing flow conditions can be
 339 discernible. The upstream of case $C_{3 \times 3}$ shows a pancake-like turbulence state. However,
 340 the hub height and bottom tip regions shows an isotropic and axisymmetric turbulence,
 341 respectively. The upstream of case $C_{3 \times 1.5}$ exhibits axisymmetric and cigar-like turbulence
 342 in the most of the upstream domain, although the hub height region remains described by
 343 isotropic turbulence.

344 Past the turbine, the four cases exhibit the turbulence of isotropic state in the hub height
 345 region. The top tip region of the four cases shows axisymmetric turbulence although case
 346 $C_{3 \times 3}$ tends to be a cigar-like turbulence. Below the hub height, the turbulence is pancake-
 347 like and the difference amongst the cases is the covered area, where it is maximum at $C_{6 \times 3}$
 348 and minimum at $C_{3 \times 3}$. The longest extension is found in case $C_{6 \times 3}$ and the lowest in case
 349 $C_{3 \times 3}$ with. Comparing to $C_{6 \times 3}$, the change seen in the turbulence states is starker in $C_{3 \times 3}$
 350 than in $C_{6 \times 1.5}$, confirming that the impact of reducing streamwise spacing is greater than
 351 changing the spanwise spacing. However, the impact of the spanwise spacing is noticeable
 352 when S_x equals $3D$.

353 The ability to identify the turbulence structure allows for identification of its influence
 354 on subsequent turbines in terms of fatigue loads (Frandsen and Thøgersen 1999). Further,
 355 regions of the flow that are characterized by highly anisotropic turbulence are those in which
 356 one is likely to find large-scale, coherent turbulence structures. These structures impart
 357 the greatest axial and bending loads onto subsequent turbine rotors leading to accelerated
 358 fatigue and increased operational and maintenance costs for wind farms. In addition, regions
 359 of high anisotropy correlate with gradients in the mean flow and turbulence (Hamilton and

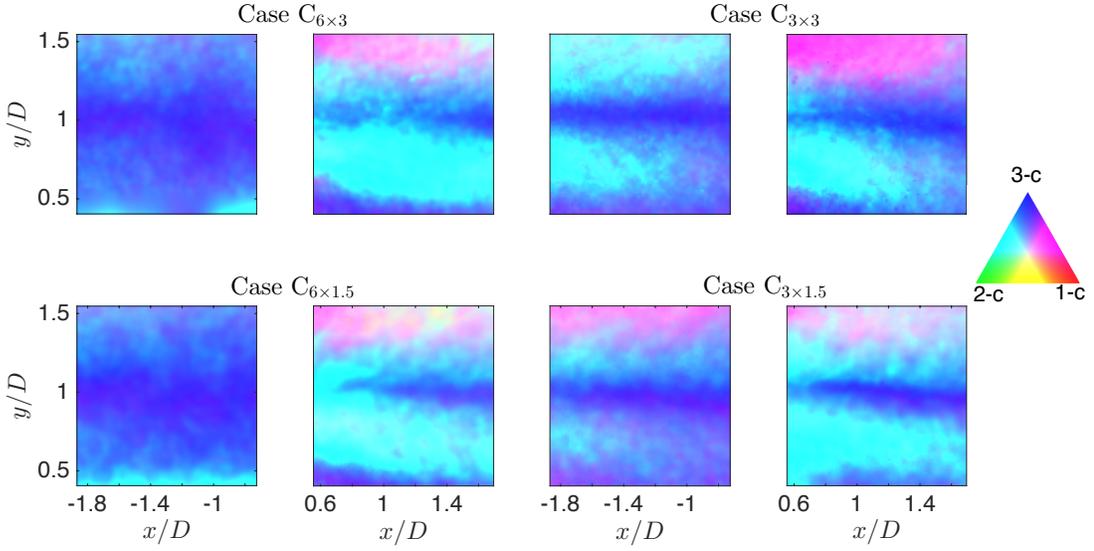


FIG. 13: Barycentric map map for the upstream and downstream of the considered cases. The small triangle is a color map key for ease of interpretation.

360 Cal 2015). These quantities are of particular interest in wind farm modeling and design.
 361 Accordingly, the accurate representation of gradients in wind farm design modeling is a
 362 necessary check to accurately representing production of and flux by turbulence kinetic
 363 energy, wake interaction, and structural loading on constituent turbines. Finally, the stress
 364 tensor invariants, by definition, do not depend on reflection or rotation of the coordinate
 365 system meaning that they are unbiased descriptive for the turbulent flow (Pope 2000).

366 V. POWER MEASUREMENTS.

367 Figure 14 demonstrates the power produced by each turbine, \mathcal{F}_x , obtained with the torque
 368 sensing system, versus the angular velocity, ω . The power measurements are normalized by
 369 the maximum theoretical power $\frac{1}{2}\rho A_c U_\infty^3$, where ρ is the air density, A_c is swept area of
 370 the turbine rotor $\pi D^2/4$. The angular velocity is normalized by the $2U_\infty/D$. It is apparent
 371 from the figure that the maximum power is extracted at the normalized angular velocity of
 372 15.8 ± 1 . The maximum normalized power of 0.062 is harvested at the largest spacing, *i.e.*,
 373 case $C_{6 \times 3}$. Fixing the spanwise spacing and decreasing the streamwise spacing reduces the
 374 normalized power produced by 33% for $S_x = 6D$ (from case $C_{6 \times 3}$ to case $C_{3 \times 3}$) and by 22
 375 % for $S_x = 3D$ (from case $C_{3 \times 1.5}$ to case $C_{6 \times 1.5}$). The complementary change in spacing
 376 holds the streamwise spacing constant while decreasing the spanwise spacing. In that case

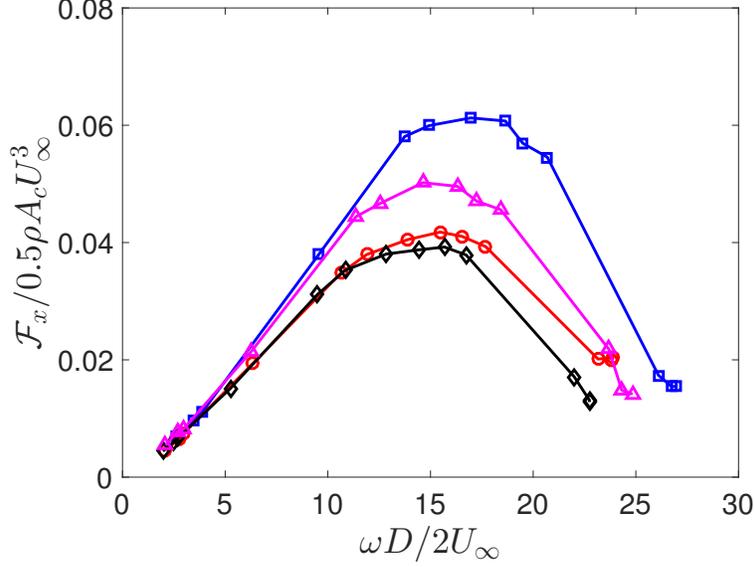


FIG. 14: Extracted power of the wind turbine at different angular velocities for four different cases $C_{6 \times 3}$ (\square), $C_{3 \times 3}$ (\circ), $C_{3 \times 1.5}$ (\diamond), and $C_{6 \times 1.5}$ (\triangle).

377 the normalized power produced is reduced by 20% for $S_z = 3D$ (from case $C_{6 \times 3}$ to case
 378 $C_{6 \times 1.5}$) and by 6% for $S_z = 1.5D$ (from case $C_{3 \times 3}$ to case $C_{3 \times 1.5}$). Nilsson et al. (2015) has
 379 complementary results to the ones present, where an increase in power produced is attained
 380 in the largest spacing and conversely, decreased in the limited spacing case. Furthermore,
 381 increasing the spanwise distance has a less notable effect in comparison to the streamwise
 382 spacing.

383 The trend of the power curves follows the one observed in the averaged profiles of the
 384 streamwise velocity, see figure 7 (a). Further, they verify the relationship between the power
 385 of the turbine with the deficit velocity. The maximum power and velocity are found in the
 386 case $C_{6 \times 3}$ and the minimum quantities are noticed in $C_{3 \times 1.5}$. The smallest variations in the
 387 power measurement and main velocity are observed between cases $C_{3 \times 3}$ and $C_{3 \times 1.5}$, whereas
 388 the largest difference is observed between cases $C_{6 \times 3}$ and $C_{3 \times 3}$. Increased longitudinal
 389 spacing produces larger energy content in the first few modes as to provide the imprint
 390 of the flow; thus, this is reflected in an increase in power as directly measured via a torque
 391 sensing device.

392 VI. CONCLUSIONS

393 Insight into the behavior of the flow in a wind turbine array is useful in determining how
394 to highlight the overall power extraction with the variation in spacing between the turbines.
395 The work above quantifies effects of tightly spaced wind turbine configurations on the flow
396 behavior. The findings of this study have a number of important implications, especially
397 regarding the cost of a wind farm or when large areas are not available. Stereographic
398 PIV data are used to assess characteristic quantities of the flow field in a wind turbine
399 array with varied streamwise and spanwise spacing. Four cases of different streamwise and
400 spanwise spacings are examined; the streamwise spacing being $6D$ and $3D$, and spanwise
401 spacing being $3D$ and $1.5D$. The flow fields are analyzed and compared statistically and by
402 snapshot proper orthogonal decomposition.

403 The streamwise mean velocity, and Reynolds shear stress are quantified upstream and
404 downstream of the wind turbine in the considered cases. In the inflow measurement window,
405 higher velocities are observed in cases $C_{6 \times 3}$ and $C_{6 \times 1.5}$ comparing to the other two cases
406 whose inflows are unrecovered wakes from preceding rows. In contrast, case $C_{3 \times 3}$ and $C_{3 \times 1.5}$
407 show higher Reynolds shear stress. The notable differences between the cases are found
408 above the top tip and below the bottom tip downstream the turbines, whereas the core
409 of the wakes shows fewer discrepancies. The streamwise and spanwise spacings have a
410 concerted effect on the flow, where the degree of the impact of one change highly depends
411 on the other. This relationship is shown in all statistical quantities discussed here, such as
412 reducing of the streamwise spacing by 50% leads to increases in the averaged Reynolds shear
413 stress by 16% when $S_z = 3D$. According to current statistical quantities, one can infer that
414 the higher influence of streamwise spacing is shown when the spanwise spacing is $S_z = 3D$,
415 and the significant effect of the spanwise spacing is observed when the streamwise spacing is
416 $S_x = 3D$. To make comparisons independent of the effects streamwise spacing, streamwise
417 average profiles of the statistical quantities are computed. Averaged profiles of the velocity
418 follow the order of higher velocity seen in the contour plots in case $C_{6 \times 3}$ and lowest velocity
419 in case $C_{3 \times 1.5}$. The maximum and minimum difference are observed between cases $C_{6 \times 3}$ with
420 case $C_{3 \times 1.5}$ and $C_{3 \times 3}$ with case $C_{3 \times 1.5}$. The result also reveals that the streamwise spacing
421 is more impactful than the spanwise spacing. Spatially-averaged profile of Reynolds shear
422 stress shows the maximum and minimum values occur in cases $C_{3 \times 3}$ and $C_{6 \times 1.5}$, respectively.

423 Based on the POD analysis, the upstream measurement plane of the four cases converges
424 faster than the downstream window. Case $C_{6\times 3}$ and $C_{6\times 1.5}$ show the rapid convergence in
425 cumulative energy content upstream of the turbine, but $C_{6\times 3}$ remains behind case $C_{6\times 1.5}$ in
426 the wake. The first mode of the case $C_{6\times 1.5}$ carries the maximum turbulent kinetic energy
427 content compared to the first mode of the other cases. No significant difference in energy
428 content is observed after mode 10 between the four cases. The streamwise-averaged profiles
429 of the Reynolds shear stress are reconstructed by back-projecting coefficients onto the set of
430 eigenfunctions. Low modes are used individually to demonstrate their contributions to the
431 overall flow. Cases $C_{6\times 1.5}$ and $C_{6\times 3}$ converge to the total spatially-averaged profile faster
432 than other two cases and the discrepancy in reconstruction is mainly observed in profiles
433 using only the first five modes. The same trend in reconstruction is observed in cases $C_{3\times 3}$
434 and $C_{3\times 1.5}$. Reconstructed profiles display the effects of the spacing, where the array of large
435 streamwise spacing exceeds and reconstruct faster than the other cases due to carrying more
436 coherent structure within the flow.

437 Based on the anisotropy stress tensor and color map visualization, the spacing modifies
438 the turbulence structure and the longest spacing attenuates the perturbation of the turbu-
439 lence, inducing the flow towards a more isotropic state. The hub height region shows an
440 isotropic turbulence state regardless the spacing. The differences of the color map visualiza-
441 tion between the downstream locations of the four cases show some structural dependency
442 on the spacing between turbine rotors.

443 Power production by the turbines is measured directly using torque sensing system. The
444 power curves follow the same trend as the velocity profiles. The maximum power extracted
445 at the normalized angular velocity of 15.8 ± 1 and it is harvested in case $C_{6\times 3}$. The small
446 difference in harvested power is observed between cases $C_{3\times 3}$ and $C_{3\times 1.5}$. The current work
447 demonstrates that wake statistics and power produced by a wind turbine depend more on
448 streamwise spacing than spanwise spacing. However, results above pertain only to a fixed
449 inflow direction. In the case where the bulk flow orientation changes, spacing in both the
450 streamwise and spanwise directions will be important to the optimal power production in
451 a wind turbine array. Continued efforts are required to understand the impact of stream-
452 wise and spanwise spacing in infinite array flow with Coriolis forcing and under different
453 stratification conditions.

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