

An engineering model for 3D turbulent wind inflow based on a limited set of random variables

Fluck and Crawford

Response to comments from anonymous reviewer #1

Moderate comments:

- P8 & 9. Confirming that the temporal variability is more important than the spatial variability.*
This is an original thought derived from Veers' model. We have not come across any literature studying this fact.
Here is another way to look at it. Hopefully this clarifies our thoughts:
Assuming Veer's spectral model is adequate, we note that phase angles are independent over frequencies $\text{Cov}(\Theta_{mk}, \Theta_{nk})=0$, but somehow dependent between points $|\text{Cov}(\Theta_{mk}, \Theta_{ml})|>0$.
Without further discussion of the actual structure of this dependence we infer that the phase angle matrix Θ contains more randomness in the columns (time) than in the rows (space). Thus we decide to include randomness in time, but neglect randomness in space. The results presented in section 3 confirm the validity of this assumption.
We have revised the manuscript (see pp. 8, 9) to better reflect these thoughts.
- P9 & 10. Regarding Eq. (3), choosing $\Delta\Theta_{mk}$ to be deterministic.*
This indeed boils down the comment 1:
In order to arrive at a reduced order model something needs to be simplified. We choose to set $\Delta\Theta_{m(kl)} = \Theta_{ml} - \Theta_{mk}$ as deterministic constant. Thus we generate a realization for one part of the stochastic process, i.e. we collapse a part of the random domain of the problem into one sample, but we retain randomness in the other part. Comment 1 provides the reason for this choice, and although this means we neglect the randomness in the rows of Θ , and with it some part of the randomness in the wind field, the good agreement of our results with Veers original wind model justify this choice.
We have revised the manuscript (see p. 10) to better reflect these thoughts.
- P9 and later in the text. The concept of $\Delta\Theta_{mk}$.*
We have adopted this nice way to put paraphrase our concept. See changes under Eq. (6), p. 10.
We hope this will explain our concept more graphically.
- P10. L9. Spatial and temporal variability.*
See comment 1. This is a new thought, and we have not come across any studies in this direction.
- Figure 7. Difference between for the times lags less than 10 s?*
The cross correlation curves do not perfectly agree. The difference is most prominent in the oscillations of the (P1, P5) curve, and in the (P1, P6) curves around 10s and 50s. We do not think that this is related to high or low frequencies, but due to the fact that our model uses significantly less frequencies than *TurbSim*. Hence the turbulent energy is discretized over fewer modes in our model, and the curves are not as smooth.

6. *More emphasis on Figure 10.*

We fully agree. However, representing the underlying BEM model and the modifications made to transform it into a stochastic BEM model would exceed the scope of a single paper. These results will be presented in a separate paper (Fluck and Crawford: "A fast stochastic solution method for the Blade Element Momentum equations for long-term load assessment", Wind Energy, submitted). We included a reference at the end of Section 3.4.

7. *Validity of our method would be for the case of highly unsteady winds.*

Our method (like Veers' method) is a spectral representation of a random process. The wind field is generated through inverse Fourier transform with random phase increments. Thus the underlying process is (per definition) assumed to be stationary.

Consequently, fronts, downbursts, and other singular transient events can naturally not be modelled directly in any spectral model.

It might be possible to model these event through superposition of a discrete deterministic event with our random model, similar to Chay et al. (2006) who superposes a deterministic downburst profile over an ARMA model for the turbulent fluctuations. However, it is our goal to derive a formulation for wind as input to stochastic models. These models will have difficulties dealing with singular deterministic events themselves. We doubt if moving this way is beneficial. Nonetheless, this could obviously be the subject of a future study.

We have included this in Section 3.5.

Minor comments:

1. *P4, L24. Correction of "Moreover the stochastic wind..."*

This should be: "Moreover this model is driven by the decomposition (bi-orthogonal and Karhunen-Loève) of a specific set of wind."

We have changed this accordingly.

2. *P5, L7-9 difficult to understand.*

We have changed this to: "However, none of the previous models had an application in stochastic aerodynamic models in mind. Since random numbers can be generated very quickly, existing models rely on a large set of random variables to be used as a seed for a wind field realization. However, this random seed usually contains too many random variables to be applicable to a direct stochastic modeling of the aerodynamic wind turbine equations (path C in Fig. 1)."

3. *P5, L12. It should be Veers (1988).*

We have reviewed our references and believe they are correct now.

4. *P6. L2-6.*

We believe it is important to state why Veers model is briefly revisited. To avoid repetition, we rephrase this paragraph:

"Veers' method represents the established method for synthesizing turbulent wind (Nielsen et al., 2007; Lavelly et al., 2012) and at the same time is the baseline for our contribution. Hence

the method is briefly summarized here to lay out the basics for the following work. For a complete introduction the reader is referred to Veers' original paper (Veers, 1988) and successive work, e.g. Kelley (1992); Nielsen et al. (2004); Burton et al. (2011)."

5. *P6, L13.*
We have reworded this accordingly.
6. *Figure 2.*
We have clarified this in the caption.
7. *P8, L15, "block of wind".*
We have discarded the footnote and reword the sentence: "This is indeed the streamwise and thus the temporal variability of the wind field."
8. *P11, L21. "; the results" and **Veers**_{red}.*
This is a typo, which was removed. We also removed the bold typeface for the labels.
9. *9. P12, L1.*
We have also removed the bold typeface for this label.
10. *Figure 5 caption.*
We have changed this accordingly.
11. *Figure 6 caption.*
We have changed this accordingly.
12. *Eq. 9.*
We have changed this accordingly.
13. *Figure 9.*
We have changed this accordingly.
14. *Figure 10 caption.*
We have changed this accordingly.
15. *References.*
We have changed this accordingly.

Grammatical and stylistic comments:

We have revised the manuscript and correct the errors pointed out.

Response to comments from anonymous reviewer #2

1. *Page 5, line 15-17: main advantage of Veers' model.*

We consider three main advantages of Veers model and write:

“Due to its comparatively high independence of site specific parameters, ease of use, and low resource requirements, Veers' model is the preferred model for many applications (Lavelly et al., 2012).”

2. *Page 7, line 10-15: reduction of numbers of frequencies.*

In a frequency model the frequency bins as chosen arbitrarily, but such that the energy in the frequency spectrum can be represented adequately. As discussed on P5, L7-9, using a large set of frequencies (i.e. a large set of random numbers) was not a problem for deterministic models. However, moving to stochastic models requires a significant reduction of random variables, and hence frequencies. We show, that using 10 to 20 random variables for 10 to 20 frequencies is sufficient. This choice is not driven by physical arguments, but by the limitations of the stochastic models where such a reduced order wind field formulation is to be used.

We have amended the first paragraph of Section 2.2. to clarify this.

3. *Page 8, line 1: experimental of theoretical justification for grid resolution.*

The choice of grid resolution is a tradeoff between accuracy and computational effort. For a typical $D = 90$ m diameter rotor 15 x 15 points is a common choice. We change the bracket to “(for a $D = 90$ m diameter rotor somewhere in the order of 15 x15 points over the rotor disk is are typically used)”.

4. *Page 11, line 19: The number of frequencies.*

The number of frequencies does affect the quality of the resultant wind field. Too few frequencies result either in periodic wind speeds or distinct wind speed oscillations.

Eight to ten frequencies for a 333 s sample was found to be the minimum.

We hope the additions to the first paragraph of Section 2.2. clarify this, too.

5. *Location of Figure 4.*

We leave this to the final setting in the journal print.

6. *Size of Figure 5.*

We have increased the size.

7. *Page 12, line 16: grid resolution.*

In this study we are not concerned with wind turbine loads. Hence it is not necessary to model the wind field over the full wind turbine rotor disc. To assess the reduced order

wind model it is important to show that it adequately represents wind field properties both for close and distant points. In order to do this without unnecessary computational effort, we use a study grid, which contains only a few close and a few distant points. To make this clearer we changed p13, ll 6ff to:

“Since the goal here is not to calculate wind turbine loads, but to merely asses the quality of the reduced order wind model, we used a dummy wind field generated on $N_{py} \times N_{pz} = 5 \times 3 = 15$ points located on a regular grid as depicted in Fig. 4. This is fewer points than the usual grid for the analysis of a modern $D = 90$ m rotor diameter wind turbine. However, the reduced number of grid points enabled us to solve the equations quickly with all models and more clearly illustrate the method. At the same time, the configuration of Fig. 4 still allowed us to study both the wind speed time series of points in close proximity (e.g. P1 and P6), as well as at more distant points (e.g. P1 and P5). The origin [...]”

8. *Page 15, line 2-10: model names*

Thank you for catching this! We erroneously swapped model names in line 3. We changed this paragraph (p. 13, ll 2ff):

“From Fig. 8 it can be seen that the covariance from all three model agrees fairly well. Our implementations of Veers’ model, $Veers_{red}$ and $Veers_{red, \Delta\theta}$, which both use a limited set of frequencies, agree almost perfectly. The TurbSim version with the full set of roughly 3,000 frequencies, on the other hand, yields slightly different covariance. A more detailed investigation reveals the reason for this: the covariance depends on the cross-spectrum and thus the spectrum at each individual point. Consequently, the discrepancy between the covariance functions is connected to the fact that Veers’ model distorts the spectrum at each individual point, such that with Eq. 3 $|U_{mk}| = \sqrt{\tilde{S}_{mk}} \neq \sqrt{S_{mk}}$ (see discussion in section 2.1). When we replace S in our implementation by the distorted spectrum \tilde{S} at each particular point P_k in Eq. 8 all three curves do match. However, \tilde{S} does not in fact represent the prescribed Kaimal spectrum. Thus we conclude that our phase increment model actually represents the desired covariance better than Veers’ original model and *TurbSim*.”

9. *Page 16, line 7-8: logarithmically spaced bins.*

With a logarithmic spacing more bins are located at low frequencies and fewer at high frequencies. This enables us to represent the wind and its spectrum more efficiently. For clarity we changed on p. 12, ll. 13-14 to:

“[...] with $N_f = 10$ logarithmically spaced frequencies, which allowed a more efficient representation of the wind and its spectrum. We set $f = []$ ”

10. Page 18, line 15: auto spectrum

The auto-spectrum is included in Fig. 2. We realize that this might get lost towards the end of the paper. We have changed the heading of section 3.3. to “Power spectra” and changed the beginning of the section:

“Wind speed spectra are again obtained as average from 100 realizations (from 100 different random seeds). However, this time 6,000 s were sampled to obtain sufficiently long data sets for a proper resolution of the low frequency components. Note that the same set of 20 frequencies $[f_k] \in [1 = 600; 5]$ Hz are used for both the 20 frequency (Veers_{red}) and the phase increment (Veers_{red, Δθ}) implementations. Hence the $T = 6,000$ s signal repeats after 600 s. The spectrum is binned into discrete bins of frequencies f_m equal to the logarithmically spaced frequencies initially used to generate the wind speed time series.

The wind speed auto-spectrum is included in Fig. 2. By definition (Eq. 8) the reduced order model produces the prescribed (auto-) spectrum exactly. Fig. 9 shows a comparison of the cross-spectra estimates for different [...]

An engineering model for 3D turbulent wind inflow based on a limited set of random variables

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Abstract. Emerging stochastic analysis methods are of potentially great benefit for wind turbine power output and loads analysis. Instead of requiring multiple (e.g. ten-minute) deterministic simulations, a stochastic approach can enable quick assessment of a turbine’s long term performance (e.g. 20 year fatigue and extreme loads) from a single stochastic simulation. However, even though the wind inflow is often described as a stochastic process, the common spectral formulation requires a large number of random variables to be considered. This is a major issue for stochastic methods, which suffer from the ‘curse of dimensionality’ leading to a steep performance drop with an increasing number of random variables contained in the governing equations. In this paper a novel engineering wind model is developed which reduces the number of random variables by 4–5 orders of magnitude compared to typical models while retaining proper spatial correlation of wind speed sample points across a wind turbine rotor. The new model can then be used as input to direct stochastic simulations models under development. A comparison of the new method to results from the commercial code *TurbSim* and a custom implementation of the standard spectral model shows that for a 3D wind field the most important properties (cross-correlation, covariance, auto- and cross-spectrum) are conserved adequately by the proposed [reduced order](#) method.

Nomenclature

Latin Letters:

e	Euler’s number
$\mathbf{f} = [f_m]$	frequency
$i = \sqrt{-1}$	imaginary unit
N_F	number of frequencies
N_P	number of wind speed points
N_R	number of random variables
P	a point in Euclidean space
S_{kk}	(auto) power spectrum
S_{kj}	cross power spectrum
t	time

U wind speed Fourier coefficient

u wind speed

Greek Letters:

θ phase angle

$\Delta\theta$ phase angle increment

ξ random number

$\omega = [\omega_m]$ angular frequency

Indices:

j, k points in space

m frequencies

1 Introduction

Engineering design tasks frequently face uncertain or random model parameters (e.g. imprecise component geometries), system properties (e.g. tolerances on manufacturing quality), and/or boundary conditions (e.g. varying wind conditions). In a deterministic modeling framework the analysis of such uncertain systems produces one specific solution for each realization of the random quantity. A ‘realization’ (also referred to as one ‘sample’) is one specific observation of the random quantity, for example a specific solution for one specific geometry, or one specific set of inflow conditions. In a numerical experiment, a realization is usually obtained ~~based on the generation of from~~ one specific random seed. However, through this process the stochastic dimension of the problem at hand is either ignored entirely, by analyzing the most likely case only (the purely deterministic approach), or it requires multiple parallel solutions to assess the statistics of the results a posteriori, for example via extreme value, sensitivity analysis, or Monte Carlo simulation. Often the first two options are insufficient, and the latter is computationally too expensive. To solve this dilemma the focus of recent research has lately moved towards stochastic analyses and uncertainty quantification (Sudret, 2007; Najm, 2009; Le Maître and Knio, 2010; Sullivan, 2015). Rather than generating one specific solution for each realization of a random input or model quantity, a stochastic analysis can help assess uncertainties quicker and even include uncertain quantities directly into the system analysis. ~~This is because it not only provides~~ A stochastic analysis may not only provide one specific solution, but ~~it solves also solve~~ the problem for the whole ensemble of all possible realizations at once, see Fig. 1. ~~This is made possible via stochastic methods that~~ Thus it becomes possible to transform the problem from multiple deterministic realizations with random seeds to a stochastic formulation of the ~~governing~~ governing equations that directly ~~describe handles~~ the stochastic variables in the system. The stochastic solution then directly describes the statistics (e.g. the probability distributions) of the outputs, based on the properties given for the input variables in the forcing terms of the governing system equations.

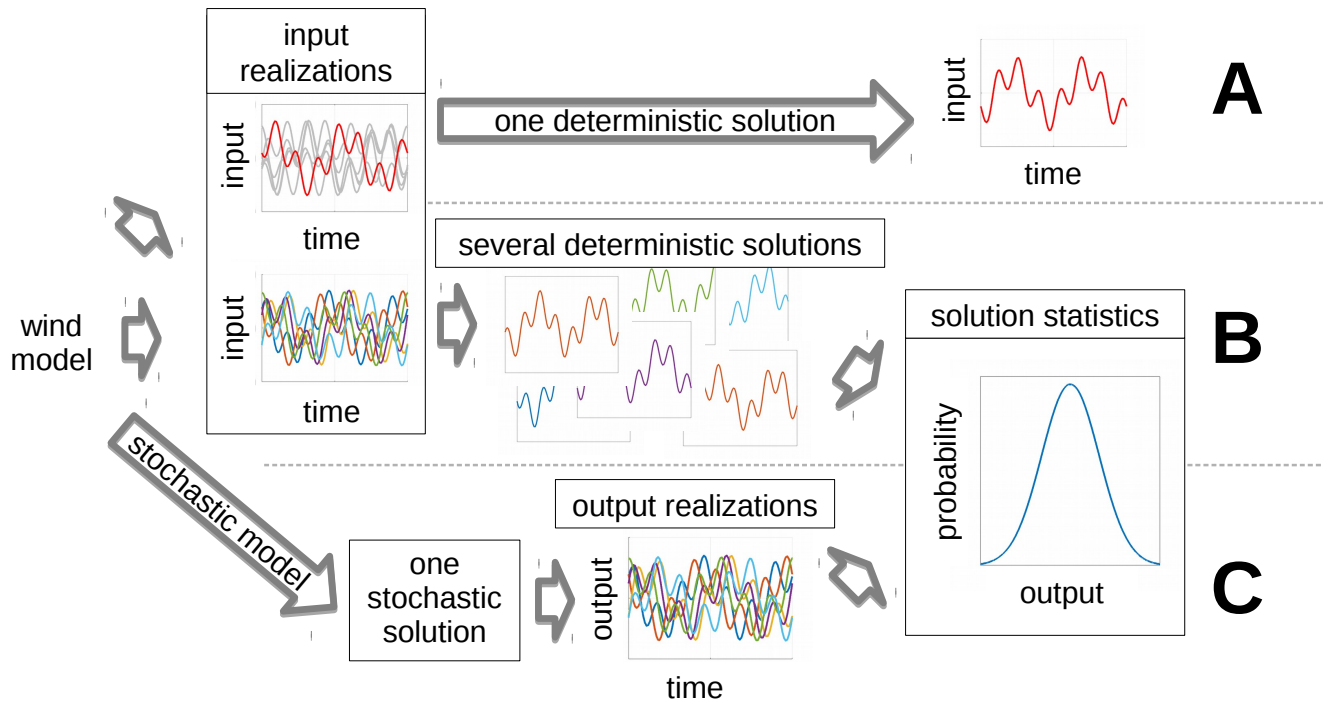


Figure 1. Comparison of the solution processes in a pure deterministic (A), a deterministic-statistic (B), and a stochastic framework (C).

In wind turbine engineering, the driving force is the turbulent atmospheric wind which is commonly described as a stochastic field derived from turbulent wind models developed around stochastic ten-minute mean wind speed distributions. This naturally invites the use of stochastic methods to assess extreme and fatigue loads, annual power production, power fluctuations, etc. in a stochastic sense and thus exploit the advantages of stochastic methods. However, wind turbine design and analysis is usually carried out in a deterministic fashion, or at best as a Monte-Carlo-like set of several subsequent deterministic solutions (path A and B in Fig. 1 respectively). The wind turbine design standard IEC 61400-1, Ed. 3 (2005) is indicative of this deterministic framework. It bases the turbine load analysis on multiple deterministic simulations, carried out at many different mean wind speeds, for about 20 different load cases, each simulated for ten minutes and repeated several times with different realizations of the turbulent inflow, each generated from a different numerical random seed. For a land based turbine this quickly amounts to evaluating several hundred ten-minute samples. For offshore turbines, where various wind conditions (wind speed and direction) additionally have to be combined with various sea states (combinations of wave height and direction) this number increases to several thousand ten-minute evaluations. However, even with a large number of deterministic simulations, extrapolation to extreme loads is a delicate exercise and results can vary greatly (Moriarty, 2008; Burton et al., 2011). Moreover, For a single load case and one mean wind speed bin Zwick and Muskulus (2015) show that basing a wind turbine analysis on six ten-minute wind speed simulations, generated from six different random seeds, results in a difference of up to 34% in the ultimate loads for the most extreme 1% of seed combinations. Tibaldi et al. (2014) present a study, which

indicates that turbine loads extracted even from 20 different ten-minute wind fields, generated from 20 different random seeds, vary greatly. This shows that in a deterministic framework load variations from different random seeds can dominate effects from design parameter changes even with a fairly large number of realizations analyzed. Obviously this constitutes a severe problem, particularly when concerned with gradient-based optimization where not only relatively fast solutions times, but also reliable design variable gradients are vital.

A direct stochastic treatment of the wind loading (path C in Fig. 1), on the other hand, considers the wind as a stochastic process throughout the turbine simulation procedure. It postpones the generation of realizations until after the calculation of a solution for the system equations, which thus become stochastic equations. Hence it can be a means to efficiently include stochastic parameters, directly obtain a stochastic solution, and arrive at the statistics of the resulting loads much quicker. Fluck and Crawford (2017a) present an example of a stochastic analysis for wing loads in turbulent inflow, and show that such a stochastic approach does not rely on the repeated analysis of multiple (e.g. 600 s) realizations of the wind field. Instead, one (possibly short, e.g. 10 s) stochastic result yields all possible realizations and hence contains the full spectrum of uncertainties. Thus it will enable the analyst to obtain a more complete description of the resulting load ensemble at large, calculate its statistics, and eventually arrive at more precise estimates of e.g. the probability of exceedance of some load threshold more quickly.

Recently, progress has been made towards stochastic analysis of wind turbines. For example, results have been shown for an aeroelastic analysis with one uncertain system parameter, stiffness or damping (Desai and Sarkar, 2010), and for a stochastic formulation of airfoil lift, drag, and pitching moment in stall conditions (Bertagnolio et al., 2010). Moreover, stochastic models have been used for wake modeling, treating wake center and shape as random processes (Doubrawa et al., 2017). However, only very early steps have been completed to include the biggest source of uncertainty: the uncertain inflow from turbulent atmospheric wind. On a wind farm scale, Padrón et al. (2016) recently presented a layout optimization based on a polynomial chaos formulation for the freestream wind speed and direction. Finally, Guo (2013) offers a stochastic wind model used for a stochastic analysis of wind turbine loads. However, he still bases the stochastic analysis on deterministic sampling (i.e. path B in Fig. 1). Moreover this model is driven by the decomposition (bi-orthogonal and Karhunen-Loève) of a specific set of wind field data. Hence, it is not generally applicable, but relies on the availability of sufficient data.

As turbulent wind is already represented as a stochastic field in many common wind models, a transition from a deterministic aerodynamic model for specific wind realizations to a stochastic model yielding the whole stochastic load ensemble at once seems an obvious step. However, this step comes with a simple, yet fundamental challenge: current wind models, even simple spectral models, rely on a large number of random variables to set the wind sample's phase angles. Since realizations of large sets of random variables can be generated very quickly, this is not a problem for deterministic load analyses. However, the computational cost of stochastic analysis methods increases dramatically with the number of random variables included, a fact commonly known as the 'curse of dimensionality' (Majda and Branicki, 2012). This renders current wind models inaccessible to stochastic methods, and thus poses a major barrier to the further development of stochastic models for the analysis of wind

turbine loads based on a stochastic description of the turbulent wind input.

To address this problem we reformulate an industry standard wind model into a reduced order engineering model. The aim of our work is to develop a wind model that can generate a realistic wind field with appropriate (long term) dynamic properties from considerably less random variables than the current models. In the last three decades numerous turbulent wind models have been proposed. ~~Kleinhans et al. (2008) summarizes~~ Kleinhans et al. (2008) summarize a few. However, none of the previous models had an application in stochastic aerodynamic models in mind. ~~Hence, as generating~~ Since random numbers can be generated very quickly, existing models rely on a large set of random ~~numbers-variables to be used~~ as a seed for a wind field realization ~~is usually no problem, the existing models we are aware of rely on a large number of random variables — too large.~~ However, this random seed usually contains too many random variables to be applicable to a direct stochastic modeling of the aerodynamic wind turbine equations (path C in Fig. 1).

In the ~~following present study,~~ we focus on a formulation for the IEC standard spectral wind description (IEC 61400-1, Ed. 3, 2005), so that it may be directly ~~useful-in-industry~~ used for stochastic aerodynamic models. Veers' model (Veers, 1988) was chosen as baseline and starting point. This model is widely used, for example in the stochastic wind simulator *TurbSim* (described by Jonkman and Kilcher (2012)), which synthesizes a sample of turbulent atmospheric wind from Veers' spectral formulation. Although it is well known that Veers' model does not capture all physical details of 'real' atmospheric wind (e.g. Mücke et al. (2011); Morales et al. (2012); Lavelly et al. (2012); Park et al. (2015)), it is for many cases an appropriate engineering model (Nielsen et al., 2007). Due to its comparatively high independence of site specific parameters, ease of use, and low resource requirements, Veers' model is the preferred model for many applications (Lavelly et al., 2012). Moreover, it is endorsed by the governing wind turbine design standard IEC 61400-1, Ed. 3 (2005), and thus is widely used in the wind energy industry. This underlines that its fidelity is accepted as a reasonable compromise in engineering practice for wind energy. As such, Veers' model ~~provided~~ provides a well accepted foundation to base further development on. Note that our goal is not improving on known deficiencies of Veers' model, but to arrive at a model that can generate a wind samples of comparable (and accepted) fidelity with significantly less random variables, geared towards eventual inclusion in a stochastic wind turbine simulation.

The following sections will first briefly review Veers' model to set the stage for the proposed modifications. Subsequently, the new reduced order wind model is introduced, and finally results are presented, which confirm that key statistical properties (cross-correlation, covariance, auto- and cross-spectrum) are conserved by the new model. The paper concludes by giving direction for continued work on integrating the wind model into a turbine simulation and on refinements with other turbulent wind descriptions. To not overload this paper, the focus is solely on the details and validation of the stochastic wind inflow model itself. Interested readers should refer to Fluck and Crawford (2017a) for the basic stochastic aerodynamic model, or ~~Fluck and Crawford (2016b)~~ Fluck and Crawford (2016b) for an example how the reduced order wind model is used to calculate stochastic loads on a stationary wing.

2 Method

~~In this section we first briefly summarize Veers' method, as it Veers' method represents the established method for synthesizing turbulent wind (Nielsen et al., 2007; Lively et al., 2012), and at the same time is the baseline for our contribution. Subsequently, we will introduce our new reduced-order method. Note that section 2.1 is only meant as a summary. Hence the method is briefly~~
5 summarized here to lay out the basics for the following work. For a complete introduction, the reader is referred to Veers' original paper (Veers, 1988) and successive work, e.g. Kelley (1992); Nielsen et al. (2004); Burton et al. (2011).

2.1 Veers' method

In a Veers' spectral method, the wind speed time series $u_k(t)$ at each point P_k , $k = 1 \dots N_P$, in the sampled wind field is obtained through the inverse discrete Fourier transform of a set of discrete frequencies components from the double-sided
10 (symmetric) spectrum U_{mk} at $\omega_m = 2\pi f_m$, $m = -N_F \dots N_F$

$$u_k(t) = \sum_m U_{mk} e^{i\omega_m t} \quad (1)$$

Here m is used to index the frequency bins, and k is used to index the points in space where wind speed data is recorded. Usually, the terms U_{mk} are binned Fourier amplitudes centered at the frequency ω_m , prescribed by the wind speed spectrum $S(\omega_m)$ at each point P_k . Often a Kaimal spectrum is used (IEC 61400-1, Ed. 3, 2005) (Kaimal et al., 1972).
15

Following Veers' method (Veers, 1988), $U_{mk} \in \mathbb{C}$ contains not only the amplitude but also the random phase angles at point P_k for each frequency ω_m . To obtain the desired coherence for all frequencies and between any two points in the wind field, all phase angles

$$\theta_{mk} = \arctan\left(\frac{\text{Im}(U_{mk})}{\text{Re}(U_{mk})}\right) \quad (2)$$

20 need to be correlated correctly. To achieve this, Veers multiplies the starts with a set of $N_R = N_F \cdot N_P$ independent, uniformly distributed random variables $\xi_{jm} \sim U(0, 1)$ with the and multiplies these with a weighting tensor H_{jkm} , obtained from the discrete cross spectrum $S_{jk}(\omega_m)$ (given by the relevant design standard or physics model), to obtain the complex Fourier coefficients U_{mk} at for each frequency band ω_m :

$$U_{mk} = \sum_{j=1}^k H_{jkm} e^{i2\pi\xi_{jm}} \quad (3)$$

25 where $S_{jk}(\omega_m)$ is given by the relevant design standard or physics model. Note that through Through Eq. 3 the phase angles at each point P_k are related to the phases at all previously computed points $P_{j < k}$. Thus, correctly correlated Fourier coefficients are obtained, which can now be inserted into Eq. 1 to obtain a correlated wind field.

This method works well to generate multiple (deterministic) wind speed data sets at many points. However, as already
30 noted by Veers in his original publication (Veers, 1988), Eq. 3 changes the amplitude of each Fourier coefficient, such that

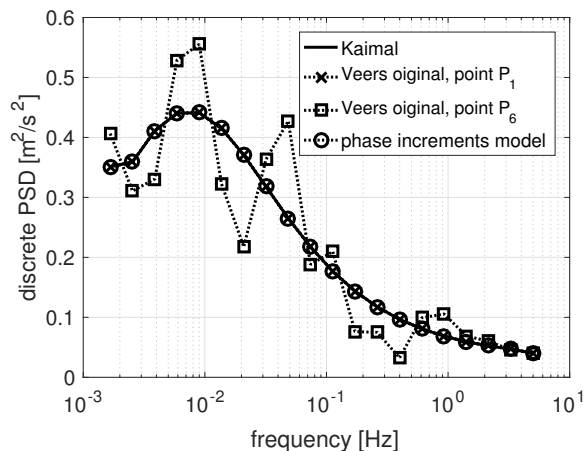


Figure 2. Raw wind spectra from a single wind speed sample, no averaging. Kaimal: the analytic spectrum; Veers: sample of the spectrum resulting from Eq. 3 at two different points P_1 and P_6 ; Phase increments: the spectrum from the reduced order phase increment model Eq. 8 (identical for all points). The values from Kaimal, Veers original (P_1), and the phase increment models all collapse to the same line

$|U_{mk}| \neq \sqrt{S_{kk}(\omega_m)}$ for all but the point computed first. Thus, the prescribed (e.g. Kaimal) spectrum $S_{kk}(\omega_m)$ is not conserved anymore at each point for any single realization, see Fig. 2. However, if spectra are averaged over either several points or several realizations, the wind field's average spectrum converges to the prescribed spectrum as $\lim_{N \rightarrow \infty} 1/N \sum_{k=1}^N |U_{mk}(\omega_m)| = \sqrt{S_{kk}(\omega_m)}$, with N the number of samples or realizations. This means the field still is stochastically homogeneous, as expected.

5 However, for a stochastic analysis where only a limited number of samples might be used, this may pose a challenge. ~~In~~ the following we We introduce a reduced order model based on phase angle increments. This model not only yields a significant reduction in random variables required to synthesize a stochastic wind field, but it also analytically preserves the prescribed spectrum at any single point for each realization (see Fig. 2 ‘phase increments’).

10 2.2 The reduced order model with phase increments

To arrive at a reduced order model we follow a two step process. First is a reduction in the number of frequencies necessary for the spectral composition of the wind speed time series at a single point in space, ~~and with it which yields~~ a reduction in the number of random phase angles associated with each frequency. This frequency reduction has been done before. For example Fluck and Crawford (2017a) showed that with ten frequencies from the IEC Kaimal spectrum, logarithmically spaced in [0.003, 5] Hz (a $T = 333$ s sample, resolved at 10 Hz, a reasonable time step for wind turbine simulations, cf. Bergami and Gaunaa (2014)), a realistic wind speed time series can be produced, ~~with probability distribution (and thus turbulence intensity), as well as the wind speed auto-correlation.~~ The choice of 10 frequencies is not driven by physical arguments, but by the fact that it is sufficient to obtain a wind speed time series similar to results from a full *TurbSim* simulation at 10 Hz

for 10,000 s (roughly $5 \cdot 10^4$ frequency bins). ~~Ten~~, including similar probability distribution (and thus turbulence intensity), as well as similar wind speed auto-correlation. ON the other hand, ~~ten~~ frequencies, and thus ten random variables for the phase angles, is manageable as input to a stochastic model. However, when dealing with a wind field big enough to be used for wind turbine calculations, many points (~~typically a grid for a D = 90 m diameter rotor somewhere~~ in the order of 15×15 points over the rotor disk ~~is are typically used~~) of correlated wind speed are necessary. The challenge is to extend this limited frequency wind description from a single point to a spatially varying wind field without excessively increasing the number of random variables required. ~~Fung et al. (1992)~~ Fung et al. (1992) introduced a wind model which ~~models~~ describes both the spatial and the temporal dimension through Fourier modes. They reduced the number of modes down to as little as 38, however, the model then relied on several random numbers associated with each mode. ~~Fung et al. (1992)~~ Fung et al. (1992) did not report in detail how many random variables they used for their model, but the equations indicated that this number was still considerably larger than manageable by stochastic methods. The following paragraphs will introduce a new approach, which will allow to create a stochastic wind field from a significantly reduced number of random variables, independently of the (~~spatial~~) spatial size of the wind field, i.e. independently of ~~both~~, the number of data points over the rotor disc ~~as well as the lateral extent~~.

15 In Veers' model the phase angle matrix $\Theta = [\theta_{mk}]$ is populated with random numbers. We note that random phase angles in the rows and columns of Θ carry out two distinctly different functions. At each individual point P_i the different phase angles in the column vector $[\theta_m]_i = \Theta_i$ generate constructive/destructive interference of the ensemble of base sinusoids. Thus, different realizations of Θ_i generate the "gusty" nature of the wind speed time series at that point. This is indeed the streamwise and thus the temporal variability of the wind ¹ field. On the other hand, the wind speed structure in space, for example the fact that 20 strong winds at one point correlate with strong winds at a nearby point, is captured through the relation of phase angles for the one particular frequency ω_l at different points P_i and P_j – that is in each row of Θ , $[\theta_k]_l = \bar{\Theta}_l$. This is the *spatial* variability of the wind.

While the phase angles at each point (the columns Θ_i) are uncorrelated, the phase angles between two points (the rows $\bar{\Theta}_l$) have to be ~~correlated somehow dependent on each other~~ to reproduce the spatial structure correctly. For two column vectors $[\theta_m]_i$ and $[\theta_m]_j$ this means while the entries within each column vector are uncorrelated ~~the two vectors themselves are element-wise correlated~~, ~~($cov(\theta_{mk}, \theta_{nk}) = 0$) the elements within each vectors are not independent, i.e. correlated ($|cov(\theta_{mk}, \theta_{mj})| > 0$),~~ cf. Fig. 3. For wind, this correlation decreases with both, increasing frequency and increasing distance.

~~Schematic of random phase angle vectors and deterministic phase increments.~~

30 ~~For~~ Based on these observation, we note the following for our use-case of turbulent wind as input to dynamic wind turbine analysis, ~~we observe the following:~~

¹ ~~Note that one dimension of the block of wind, in the average wind direction, corresponds to a temporal correlation as the wind moves downwind.~~

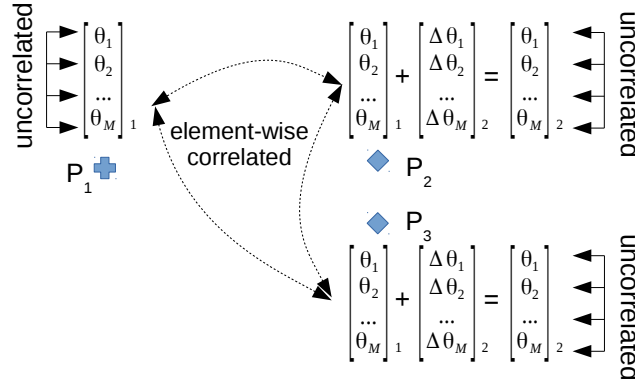


Figure 3. Schematic of random phase angle vectors and deterministic phase increments.

1. The temporal variability (in the columns of Θ) is of primary importance, since it drives the dynamic excitation of the system under investigation. This is the duration-structure of gusts and lulls, captured by the energy distribution in the frequency spectrum of the wind sample.
2. ~~The spatial-variability~~ Also the spatial variability (in the rows of Θ) needs to be represented correctly to yield representative wind loads, ~~since the length-scale of spatial wind structures needs to be correct to~~ which eventually result in the correct integral loads. For example, ~~for at~~ any instance when a sensor *A* somewhere on the blade experience an increased load, another sensor *B* a certain distance away from *A* needs to experience a load correctly correlated to the load at *A*. However, since there will necessarily be some averaging of the loads across the blades this is of secondary importance.
3. For each point, all elements in each column vector Θ_i are independent (Fig. 3). However, the column vectors Θ_i and Θ_j at two points P_i and P_j are element-wise correlated. This means the phases in each row vector $\bar{\Theta}_i$ are *not* independent. Following Veers' method, only the elements in Θ_1 are independent, while the phases at all other points are mapped from i.i.d. random variables ξ_{mi} such that they are correlated to the phases at the base point P_1 (and thus to each other), Eq. 3. This means there is more "randomness" in the columns of Θ than in the rows – an important fact, which we will soon exploit.

15

To obtain a reduced order model which requires fewer random variables we propose splitting the complex Fourier coefficients U_{mk} , into a temporal and a spatial part. The temporal part will contain the amplitude of each Fourier mode as well as the random phase angles. It therefore will determine the structure of the wind speed sample in time. The spatial part will contain the phase correlation between different points across the wind field. It will thus set the wind field structure in space. To reflect this

approach we can write:

$$U_{mk} = \underbrace{U_{m1}}_{\text{temporal}} \cdot \underbrace{e^{i\Delta\theta_{mk}}}_{\text{spatial}} \quad (4)$$

The temporal part contains the amplitude according to the prescribed power spectrum $S(\omega_m)$ and a vector of random phase angles $\theta_{m1} = 2\pi\xi_m$ at an arbitrary base point P_1 within the wind field:

$$U_{m1} = \sqrt{S(\omega_m)} e^{i\theta_{m1}} \quad (5)$$

with independent and identically distributed $\xi_m \sim U(0,1)$ as before. Similar to the wind speed increments used for wind interpolation by Fluck and Crawford (2016a), the spatial part is based on the idea of phase increments $\Delta\theta_{mk}$, which are specific to each point and each frequency relative to the base point P_1 :

$$\Delta\theta_{mk} = \theta_{mk} - \theta_{m1} \quad (6)$$

The increment $\Delta\theta_{mk}$ holds the correlated phase information to generate the correct spatial structures. Since θ_{mk} and θ_{m1} are random numbers, the increments $\Delta\theta_{mk}$ ~~are~~ should be random, too. ~~In~~ However, in contrast to Veers' approach of employing the cross spectrum to map a set of uncorrelated random variables to a set of correlated phases for each point in the wind field, we neglect the random nature of $\Delta\theta_{mk}$ and consider the phase increments deterministic constants ~~to move between points as~~ illustrated in Fig. This means that for different wind field realizations the correlation coefficient between two points in space is fixed for each frequency (n.b. this is the stochastic correlation; It does *not* establish a deterministic one-to-one dependence of the wind speeds at two points!). For example, for a frequency bin at $f_1 = 10$ Hz and point P_4 , $\Delta\theta_{14}$ has the same value for each realization. Similarly, for a frequency bin $f_5 = 0.1$ 3-Note-Hz and the same point, $\Delta\theta_{54}$ has once again always the same value (but but different from $\Delta\theta_{14}$).

Assuming $\Delta\theta_{mk}$ to be constant is the core assumption of the presented reduced order model. It clearly is a simplification, but essential to arriving at a model reliant on a reduced number of random variables. The results presented in Section 3 will confirm the validity of this assumption. Note moreover that $\Delta\theta_{mk}$ only contains the spatial structure, but not the temporal part. That means 'gusty' features of the wind (lulls and gusts at different points) are still generated from random numbers; only the wind field's ~~structure in space~~ spatial correlation is fixed with each specific set of phase increments. Based on the three observations above (1-3) this seems justified for two reasons. Firstly, the phases in each row vector $\bar{\Theta}_l$ are correlated, while the phases in each column vector Θ_i are uncorrelated (3). This means there is more 'randomness' in the temporal dimension than in the spatial dimension. Secondly, for the dynamic analysis of a wind energy device, the temporal part is of primary importance. While the spatial structures have to be represented correctly, their exact variability can be considered secondary (1,2).

It is important to note that focusing on the temporal part does not mean that each realization of the reduced order wind field will exhibit the same spatial structure of gusts and lulls, i.e. that a gust at point P_i would e.g. necessarily come with a

lull at another point P_j . ~~IN- In~~ contrary, the proposed method does not alter the original correlation between wind speeds at two ~~distinct~~ points which is generally smaller than unity. Graphically speaking gusts and lulls result from the interference of different frequency component sinusoids and phase offsets. Based on the specific realization θ_{m1} the phase angles at each point $\theta_{mk} = \Delta\theta_{mk} + \theta_{m1}$ will be different each time. Thus, the interference between the frequency components and consequently the structure of the gusts and lulls will be different with each different realization of phases at the base point θ_{m1} . Figure 6, which will be discussed later, demonstrates this fact. Nonetheless, the proposed reduction of random variables necessarily causes a certain increase in dependence of wind speeds across the wind field. An investigation into how this dependence actually looks like in detail ~~is left for~~ will be the subject of future work.

10 Inserting Eqs. 5 and 6 into Eq. 4 yields the Fourier coefficients based on only one vector $[\theta_{m1}]$ of random phases $[\theta_{m1}]$ and the (auto-) spectrum:

$$U_{mk} = \sqrt{S(\omega_m)} e^{i(\theta_{m1} + \Delta\theta_{mk})} \quad (7)$$

Substituting $\theta_{m1} = 2\pi\xi_m$ ~~with $\xi_m \sim U(0,1)$ as before~~, Eq. 1 can be turned into our reduced order model (with $\xi_m \sim U(0,1)$ as before):

$$15 \quad u_k(t) = \sum_m \sqrt{S(\omega_m)} e^{i(\omega_m t + 2\pi\xi_m + \Delta\theta_{mk})} \quad (8)$$

Note that while Eq. 3 changes the amplitude of each Fourier coefficient and thus distorts the spectrum at each point, Eq. 8 fully conserves the spectrum.

In contrast to Veers' original model, where $N_R = N_F \cdot N_P$, in the reformulated model $N_R = N_F$. This means the number of random variables N_R only depends on the number of frequencies N_F used for the wind speed Fourier series, not on the number of wind speed measurement points N_P in the 3D wind field. With the available strategies to reduce the number of frequencies required in a spectral wind model Eq. 8 now allows expression of a turbulent wind field consistent with Veers' model, but with significantly fewer frequencies.

25 What remains is to obtain the phase angle increments $\Delta\theta_{mk}$. Since these determine the cross-correlation between any two points in the wind field, and since the cross-correlation and the cross-spectrum are linked as a Fourier transform pair (correlation theorem, see e.g. ~~(Kauppinen and Partanen, 2011)~~ Kauppinen and Partanen (2011)), it should be possible to analytically generate one set (one realization) of phase increments directly from the cross-spectrum. For now, however, we extract one phase angle increment set from one realization of Veers' Eq. 3, with an analytic solution left to future work.

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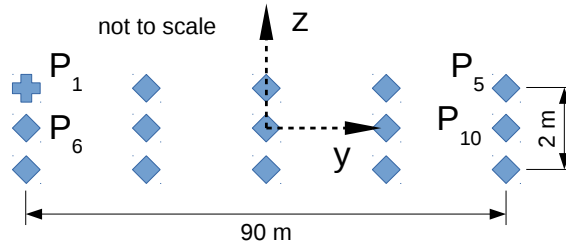


Figure 4. Schematic of grid points of wind speed data (minimal test case). We arbitrarily chose the right hand top point (P_1) to be the base point.

3 Results and discussion

In the following, we will take a closer look at statistical metrics of the synthetic reduced order wind field. As mentioned earlier our goal is not to develop a more physically faithful wind model, but rather to reduce the number of random variables required while retaining similar fidelity as the methods currently in use. *TurbSim* (Jonkman and Kilcher, 2012) is widely used in industry and the de facto standard to generate synthetic wind fields for wind turbine analysis. Hence we use *TurbSim* wind speed data sets as the benchmark. In the following, we compare results obtained from *TurbSim* to two different reduced order models.

The first is our implementation of Veers' model, which allowed us to freely choose the number of frequencies at each data point and the frequency binning. ~~As suggested by Veers equations~~ Following Veers this implementation relies on ~~the conventional a~~ inverse discrete Fourier transform with random phase angles at each frequency bin. This model was validated directly against *TurbSim*. If many frequencies are used and identical phase angles are enforced perfect agreement of the resulting data set was found as expected. As shown by Fluck and Crawford (2017a) the wind speed time series at a single point for a 333 s sample can be well represented with $N_f = 10$ logarithmically spaced frequencies, which allowed a more efficient representation of the wind and its spectrum. We set $\mathbf{f} = \omega / (2\pi) = [f_m] \in [f_1, f_{N_F}] = [0.003, 5]$ Hz with $f_m = 10^{a_m}$ and $a_m = \log_{10} \left(\frac{f_{N_F}}{f_1} \right) \frac{m-1}{N_F-1}$ for $m = 1, \dots, N_F$. For better comparison (equal sample length $T = 600$ s), we use $N_F = 20$ frequencies in $\mathbf{f} = [f_m] \in [1/600, 5]$ Hz. ~~I; the The~~ results of this model are labeled '~~Veers_{red}~~ Veers_{red}' in the following discussion. This model does not include new theory, yet it is a critical step between *TurbSim* (and thus Veers' original model) and our reduced order model. The second model presented is our reduced order model as described above (Eq. 8). The newly introduced theory of deterministic phase increments $\Delta\theta$ is employed here, together with a limited number frequencies $N_F = 20$ and thus a reduced number of stochastic variables $N_R = 20$. These results are labeled '~~Veers_{red, \Delta\theta}~~ Veers_{red, \Delta\theta}'.

Tab. 1 gives a comparison of the three models. Note particularly the total number of random variables required by each model, assuming a typical grid resolution in the order of 15×15 points over a rotor disk of $D = 90$ m diameter. While the use of a limited set of frequencies (Veers_{red}) yields a noticeable reduction in random numbers, for a turbulent wind field with several wind speed data points in x- and y-direction, this alone is not enough to arrive at a wind model with few enough random

Table 1. Comparison of random numbers used in different wind models for a common grid size.

	<i>TurbSim</i>	Veers _{red}	Veers _{red, Δθ}
sample length	10 min	10 min	10 min
grid size $N_{Py} \times N_{Pz}$	15×15	15×15	15×15
frequencies N_F	$\sim 3,000$	20	20
total number of random variables N_R	$6.75 \cdot 10^5$	4,500	20

numbers to be applicable in a stochastic method (several dozen random variables to be tractable). Only the additional introduction of deterministic phase increments (Veers_{red, Δθ}) to decouple the number of random variables from the number of wind speed data points reduces the number of random variables drastically enough to obtain a wind model which can be reasonably handled by a stochastic method.

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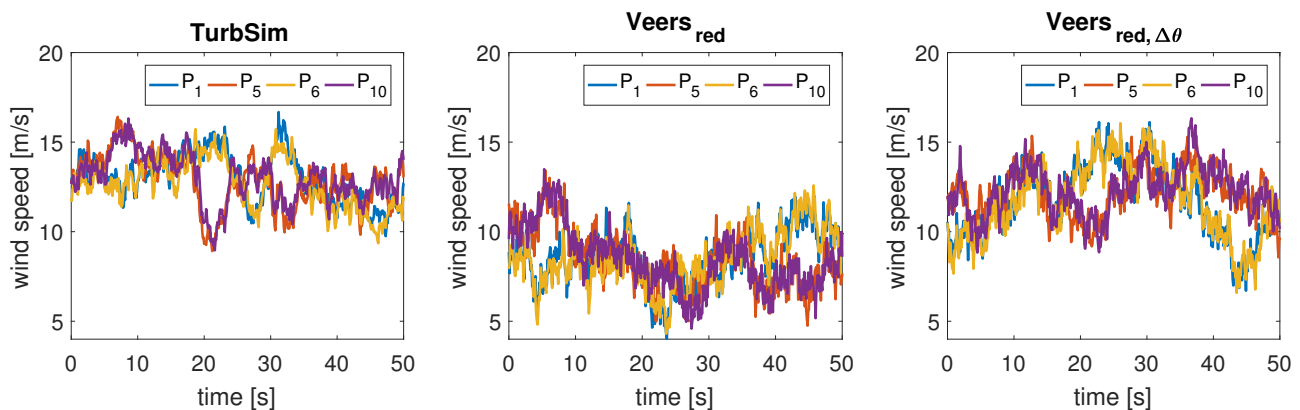


Figure 5. Three 50 s excerpts of a wind speed time series [sample samples](#) at four points generated from different models and different random seeds. TurbSim: NREL’s original *TurbSim* model; Veers_{red}: Veers model with a limited number of frequencies ($N_F = 20$); Veers_{red, Δθ}: Veers model with a limited number of frequencies and [deterministic](#) phase increments [to model spatial structures at different points, see](#). [See](#) Tab. 1 [for additional information](#).

~~As a test case we study a~~ [Since the goal here is not to calculate wind turbine loads, but to merely asses the quality of the reduced order wind model, we used a dummy](#) wind field generated on $N_{Py} \times N_{Pz} = 5 \times 3 = 15$ points located on a regular grid as depicted in Fig. 4. ~~The origin of the wind field was located 100 above ground with mean wind speed $\bar{u} = 10$ and no wind shear. The IEC class A normal turbulence model with a Kaimal spectrum (IEC 61400-1, Ed. 3, 2005) and homogeneous turbulence was used. Data was sampled at 10. We arbitrarily chose the top-left hand point (P_1) as the base point. Note that the grid used here contains~~ [This is](#) fewer points than the usual grid for the analysis of a modern $D = 90$ [D = 90 m](#) rotor diameter wind turbine ~~(where $N_{Py} = N_{Pz} = 15$ is more likely).~~ However, the reduced number of grid points enabled us to solve the

equations quickly with all models and more clearly illustrate the method. At the same time, the configuration of Fig. 4 still allowed us to study both the wind speed time series of points in close proximity (e.g. P_1 and P_6), as well as at more distant points (e.g. P_1 and P_5). The origin of the wind field was located 100 m above ground with mean wind speed $\bar{u} = 10$ m/s and no wind shear. The IEC class A normal turbulence model with a Kaimal spectrum and homogeneous turbulence was used
 5 (IEC 61400-1, Ed. 3, 2005). Data was sampled at 10 Hz. We arbitrarily chose the top left hand point (P_1) as the base point.

Fig. 5 shows realizations of the wind speed time series sampled at four points (P_1 , P_5 , P_6 , and P_{10} in Fig. 4) from the three different models. For each model the samples are generated from different random seeds. Thus the time series are not identical. Still, it can be seen that the fundamental structures are conserved through both reduced order models. In particular, even if
 10 wind samples are synthesized with only 20 random numbers and deterministic phase increments ($\text{Veers}_{\text{red}, \Delta\theta}$) the wind speeds at two points in close proximity (P_1 and P_6 , or P_5 and P_{10}) are highly correlated, while at more distant points (e.g. P_1 and P_5) the correlation is weaker. It is important to note that this holds not only for points in relation to the base point, but for all point pairs. For example, points P_5 and P_{10} are both far away from the base point, but close to each other. As expected, the wind speeds at these two points are well correlated.

15

Fig. 6 shows three realizations of wind speed time series plots at three points obtained from the new phase increment model ($\text{Veers}_{\text{red}, \Delta\theta}$), Eq. 8. The phase increments are considered deterministic, and $\Delta\theta_{mk}$ is fixed for all realizations. The randomness enters the time series only via random phase angles at the base point P_1 with $\theta_{m1} = 2\pi\xi_m$. As can be seen from the figure, this does *not* result in a complete determination of the spatial relation between wind speeds at different points, since the
 20 samples still contain different gusts and lulls at different instances in time.

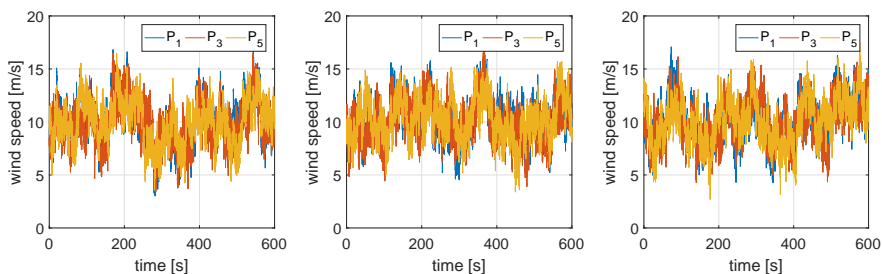


Figure 6. Three realizations of wind speed time series at three points generated from the the new reduced order model with fixed phase increments ($\text{Veers}_{\text{red}, \Delta\theta}$ model).

Beyond this qualitative visual comparison of the wind speed time series the remainder of this section will show that the phase increment model produces the same statistics as Veers' original model (with only 20 frequencies) as well as the full *TurbSim* model (with the full set of frequencies) for the most important ~~statistical~~ metrics.

3.1 Cross-correlation

Fig. 7 compares the cross-correlation for two different point pairs, P_1-P_5 (90 m apart) and P_1-P_6 (1 m apart) as obtained from six 99 s windows from a 600 s sample from our reduced model with fixed phase increments ($\text{Veers}_{\text{red}, \Delta\theta}$), from Veers' model with 20 frequencies ($\text{Veers}_{\text{red}}$), and from the full *TurbSim* simulation. To reduce noise and compare meaningful (rather than possibly extreme) values the results are presented as averages of 100 realizations from different random seeds for both phase angles and phase increments.

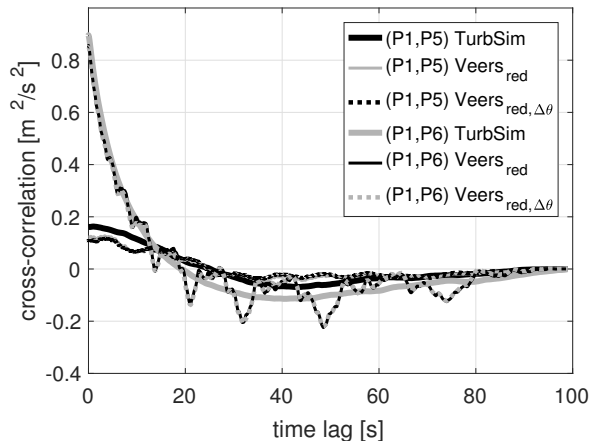


Figure 7. Wind speed cross-correlation for two point pairs generated from different models; a close pair (P_1, P_6) and a distant one (P_1, P_5).

As can be seen from the figure the Fig. 7 the cross-correlation in general agrees very well for both the close points and the distant point pair. The results from our implementation of Veers' model and from the phase increment model are almost identical and hence difficult to distinguish in Fig. 7. Note that the *TurbSim* data is smoother, presumably due to the significantly higher number of frequencies contained in the *TurbSim* data set.

Further investigation with the pair P_5 and P_{10} , two points close to each other but far away from the base point P_1 (not included in Fig. 7), shows that for all three models the cross-correlation is almost identical to the curve for P_1-P_6 . This confirms that with our phase increment model the cross-correlation of the homogeneous turbulence field, and with it the length scale of spatial structures, is indeed only dependent on the distance between two points, but not on the two specific points themselves.

3.2 Covariance

Now we look at the covariance as a function of the distance between two points and compare data from *TurbSim* to the 20 frequency of Veers' model ($\text{Veers}_{\text{red}}$) and to our reduced model with phase increments ($\text{Veers}_{\text{red}, \Delta\theta}$). As above we use averages

from 99 s windows out of 100 realizations of 600 s samples.

From Fig. 8 it can be seen that ~~our implementation of Veers' model agrees well with the results from TurbSim. The phase increment model, however, yields slightly, but consistently less~~ the covariance from all three model agrees fairly well. Our implementations of Veers' model, Veers_{red} and Veers_{red, Δθ}, which both use a limited set of frequencies, agree almost perfectly. The TurbSim version with the full set of roughly 3,000 frequencies, on the other hand, yields slightly different covariance. A more detailed investigation reveals the reason for this: the covariance depends on the cross-spectrum and thus the spectrum at each individual point. Consequently, the discrepancy between the covariance functions is connected to the fact that Veers' model distorts the spectrum at each individual point, such that with Eq. 3 $|U_{mk}| = \sqrt{\tilde{S}_{mk}} \neq \sqrt{S_{mk}}$ (see discussion in section 2.1). When we replace S in our implementation by the distorted spectrum \tilde{S} at each particular point P_k in Eq. 8 all three curves do match. However, \tilde{S} does not in fact represent the prescribed Kaimal spectrum. Thus we conclude that our phase increment model actually represents the desired covariance better than Veers' original model and ~~TurbSim.~~ TurbSim.

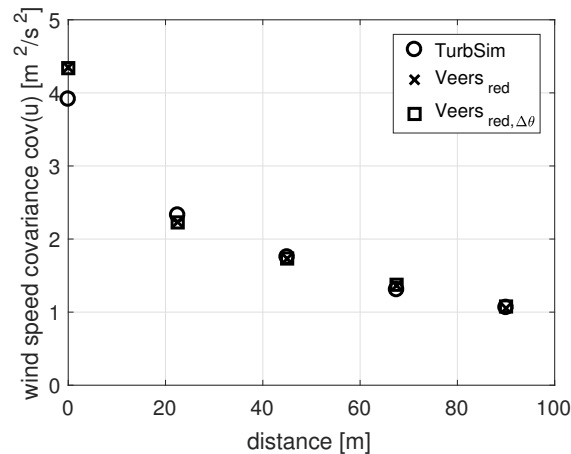


Figure 8. Wind speed covariance for points different distances apart.

3.3 Cross-spectrum Power spectra

15 ~~Next we compare the cross-spectrum, which is~~ Wind speed power spectra are again obtained as ~~the average spectrum average~~ from 100 realizations (from 100 different random seeds). However, this time 6,000 s were sampled to obtain sufficiently long data sets for a proper resolution of the low frequency components. Note that the same set of 20 frequencies $[f_k] \in [1/600, 5]$ Hz are used for both the 20 frequency (Veers_{red}) and the phase increment (Veers_{red, Δθ}) implementations. Hence the ~~$T = 6,000$~~ $T = 6,000$ s signal repeats after 600 s. The spectrum is binned into discrete bins of frequencies f_m equal to the

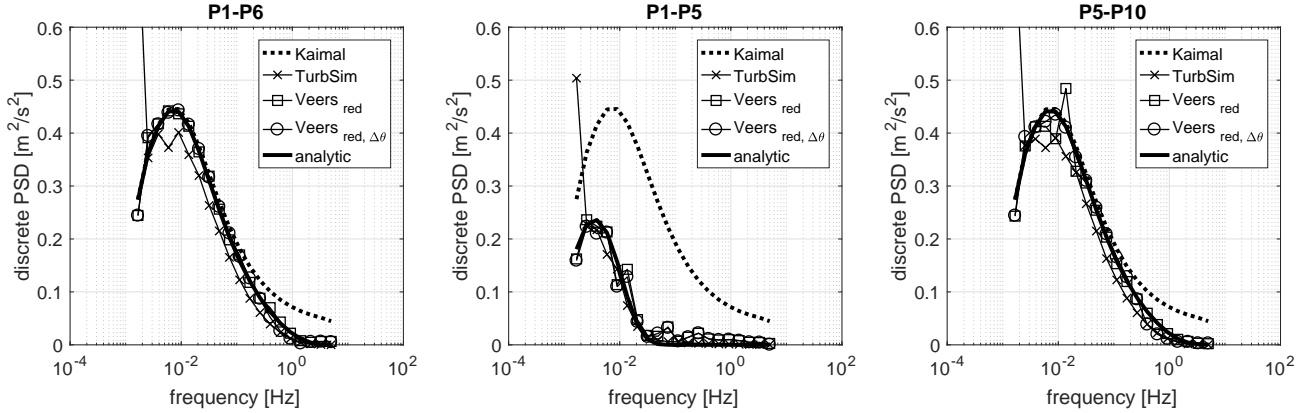


Figure 9. Wind speed cross power spectral density for three point pairs from different models, together with the analytic results (Eq. 9) and the prescribed Kaimal auto-spectrum. Left: the base point, and its closest neighbor. Middle: the base point and a point far away. And right: a point pair close together, but far away from base point.

logarithmically spaced frequencies initially used to generate the wind speed time series.

The wind speed auto-spectrum is included in Fig. 2. By definition (Eq. 8), the reduced order model produces the prescribed (auto-) spectrum exactly. Fig. 9 shows a comparison of the cross-spectra estimates for different point pairs obtained through Welch’s periodogram method (Welch, 1967) employed on the full 6,000 s samples with no extra windowing (Welch, 1967). We study the base point, and its closest neighbor (P_1 - P_6); the base point and a point far away (P_1 - P_5); and a point pair close together, but far away from base point (P_5 - P_{10}). For reference, the prescribed Kaimal spectrum S is included, as well as the analytic cross-spectrum obtained by:

$$S_{ij} = \text{Coh-coh} \sqrt{S_{ii}S_{jj}} \quad (9)$$

from-with the (auto-) spectra $S_{ii} = S$ and the coherence function Coh-coh as defined by the standard IEC 61400-1, Ed. 3 (2005).

Again, the phase increment model ($\text{Veers}_{\text{red}, \Delta\theta}$) in all cases reproduces the analytic spectrum well with only 20 random variables. This time, however, the TurbSim TurbSim results do not match as well. The reason is that TurbSim TurbSim chooses the lowest frequency f_1 and the frequency bin width Δf such that $\Delta f = f_1 = 1/T$, and thus uses a wider frequency band for the first bin compared to our logarithmically spaced bins. When re-binning to the logarithmic range this results in excess power (and an artificial peak) in the first bin and hence less power in higher frequency bins. Note, however, that this is an artifact of the discrete spectrum and the frequency binning, and not a discrepancy in the underlying data.

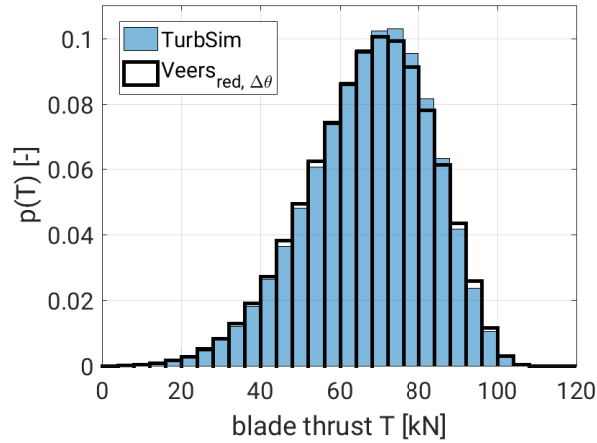


Figure 10. Blade thrust load probability distribution from [Blade Element Momentum \(BEM\)](#) model based on wind fields generated with either [TurbSim](#) or from the reduced order Veers model with constant phase increments.

3.4 Outlook: wind turbine rotor blade loads

To further assess the validity of the reduced order wind model, loads were calculated for one single blade on a three bladed $R = 35$ m diameter wind turbine rotor spinning at a tip speed ratio $\lambda = 6.1$. Loads were obtained at $\Delta t = 0.1$ s time steps through a simple blade element momentum model supplied with wind generated either from [TurbSim](#), or from our reduced model with fixed phase increments ($\text{Veers}_{\text{red}, \Delta\theta}$) on a 15×15 grid of data points over the rotor disc. The hub height is set to $h_{\text{hub}} = 90$ m, with the hub height mean wind speed $\bar{u} = 12$ m/s, power law wind shear with power law exponent $\alpha = 0.2$ (according to Jonkman and Kilcher (2012)), and IEC normal turbulence model, class A (IEC 61400-1, Ed. 3, 2005). Fig. 10 shows the probability distribution $p(T)$ of thrust loads T on one blade calculated from 100 realizations of a 600 s wind field. [TurbSim](#) used the full set of roughly 3,000 frequencies at each of the 15×15 grid points. The reduced order model, on the other hand, relied on only 20 frequencies with all 100 realizations generated one set of fixed phase increments. It can be seen, that the reduced order model, although relying on significantly fewer random variables ($N_R = 6.75 \cdot 10^5$ versus $N_R = 20$ for each realization, see Tab. 1), produces almost the same load probability distribution. [In a forthcoming publication \(Fluck and Crawford, 2017b\) we will use the reduced order wind model presented here to develop a stochastic wind turbine blade load model \(BEM\). This publication will also discuss these blade load PDFs in greater detail.](#)

3.5 Discussion

As shown ~~by the results presented~~ in this section the phase increments wind model presented in section 2.2 can reproduce important statistics (both of a wind field, as well as for resulting wind loads) with the ~~same accuracy as~~ [accuracy comparable to](#) the full model. At the same time, the phase increments model requires significantly less random variables. As indicated by Fig. 6 the phase increments model does *not* produce identical spatial structures with each realization, even though a large

part of the spatial randomness is neglected ~~is in~~ Eq. 8. This further illustrates the method's ability to retain important stochastic information. Like Veer's model, our method relies on a spectral representation of a random process. The wind field is generated through an inverse Fourier transform with random phase increments. Thus the underlying process is (per definition) assumed to be stationary. As with any spectral model, singular transient (and as such deterministic) events like fronts or

5 downbursts, can consequently not be modeled directly. It might be possible to model these events through superposition of a discrete deterministic event with our random model. This could be similar to Chay et al. (2006), who superposes a deterministic downburst profile over an ARMA model for the turbulent fluctuations. However, it is our goal to derive a formulation for wind as input to stochastic models. These models will have difficulties dealing with singular deterministic events themselves. Hence we doubt if progressing along this way would be beneficial; Nonetheless, this could obviously be the subject of a future study.

10

The results from sections 3.1-3.3 are generated from a set of 100 different phase increments generated from 100 different random seeds. This was necessary because, due to the random equations, it was not possible to compare the results from a single realization. This might have resulted in uncharacteristically bad (or good) agreement only by the chance of comparing 'bad' (or 'good') realizations. Instead only the averages over multiple realizations could be compared. For a stochastic analysis

15 as outlined in the introduction, however, only a very limited set of phase increment realizations would be used. Hence, some part of the randomness of the wind field will be lost. This is the price to be paid for using a reduced order model. In section 2.2 we justify this choice. The results, particularly Figs. 6 and 10, support the notion that a very limited set of phase increment realizations, or even a single one, can be sufficient. It is still to be determined, however, how many sets will actually be necessary for adequate results, and how the associated reduction in randomness influences the relevant output quantities,

20 e.g. for a wind turbine analysis the resulting loads, especially the probability of extreme loads. Preliminary results for wind turbine blade loads calculated from a Blade Element Momentum model indicate that only one single set of phase increments is sufficient to obtain almost the same statistical load distribution as from the conventional analysis based on standard *TurbSim* TurbSim wind fields.

4 Conclusions

25 Stochastic analysis and uncertainty quantification are ~~generally~~ very active fields of research in engineering with the developed methods increasingly adopted by industry. To enable practitioners to apply these methods to wind turbine aerodynamics and more generally wind loading analysis on various structures, we presented a new method, which significantly reduces the number of random variables used in the wind model. This reduction is critical, because the computational effort of the common stochastic solutions is very sensitive to the number of random variables involved.

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The model introduced here employs a separation of the temporal (correlation in time) and spatial (coherence in space) part of the random dimension of turbulent wind. While the temporal part is still determined from random variables, the spatial part is collapsed into deterministic phase increments. Thus the number of random variables is reduced by several orders of

magnitude compared to the commonly used model developed by Veers and implemented in *TurbSim*, currently the (de facto) standard tool for synthetic wind generation. A comparison of the most important stochastic metrics (cross-correlation, covariance, auto- and cross-spectrum) showed that the reduced order model based on phase increments still reproduces these metrics as accurately as Veers' equations or *TurbSim*. Moreover, preliminary results were presented, which indicate

5 that the reduced order wind model based on phase increments also preserves wind turbine blade loads well. A detailed study quantifying the impact of using deterministic phase increments on the overall statistics of wind turbine loads is yet to be carried out. Subsequent to the implementation of this reduced order wind model in a full wind turbine simulator, which is the focus of ongoing work, these ultimate questions can be addressed.

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