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Contact person Giovanni Migliaccio
E-mail giovanni.migliaccio.it@gmail.com

Subject **Author's response**

University of Pisa
Department of Civil and Industrial Engineering

Address
Largo Lucio Lazzarino, 2, 56122, Pisa
Italy

Dear Reviewers,

The authors would like to express their gratitude for the constructive feedbacks which have helped us to further improve the quality of the paper. In our attempt to accounts for the comments, we have revised different parts of the paper. The objective of this document is to respond to the points raised by the Reviewers and to provide an overview of the corresponding changes that will be included in the revised paper. In the following sections, we respond to the review report provided by each Reviewer.

Your sincerely,

Giovanni Migliaccio

Sections: Response to comments of Anonymous Referee #1
Response to comments of Anonymous Referee #2
Response to comments of Anonymous Referee #3

Note: Author's response to each Referee's comment follows the comment itself and is in blue.

Response to comments of Anonymous Referee #1

The authors are proposing a novel beam like model specifically developed for wind turbine blade structures. The authors motivate the need for development with computational efficiency required for design optimization in conjunction with aeroelastic analysis. The model is capable of considering lengthwise geometrical variations (LGVs) such as twist, curvature and pre-bend and is suitable for large deformation analysis.

General comments:
The research significance of the proposed model is high and the authors are addressing two of the renowned challenges in wind turbine blade simulations namely computational efficiency and accuracy. Regarding the latter, the implementation of LGVs into blade beam models bears indeed a considerable research demand.

- Concerning the introduction, the important contiguous contributions in the realm of this paper made by Giavotto and coworkers were not mentioned in the literature review.

  Giavotto and coworkers will be cited in the literature review.

- The model proposed in this paper is presented in a sole formal mathematical format. I am conceding the necessity of such a formal solution, albeit, the model can hardly be falsified in its current form. The authors mention that the model was indeed implemented and allude the intention to publish the procedure in a follow up paper. However, the complete absence of information concerning the implementation e.g. the pseudo code impedes reproducibility and judgement. With the information provided it is not possible to judge whether the model is a scientific breakthrough or not. In Section 4 an analytical example is presented in which no tangible results e.g. stress/strain fields are presented that would be vital for corroboration. It would especially be pertinent (and straightforward) to compare the model predictions with analytical solutions of a tapered beam the third author published previously. I recommend the paper for publication, provided that the solution is explicated in more detail with particular emphasis on the adopted numerical procedure. Moreover, the paper would gain credence by provision of concrete model predictions, which can be tried against analytical/other numerical solutions.

  A section with information on the current implementation of the model in Matlab code will be provided, along with numerical results (e.g. tip deflection, strain measures, stress resultants) that can be obtained by using such a model. Comparison with corresponding results obtained with a 3D FEM commercial software will also be provided.

- Specific comments/ questions:
  1. P.2 line 40: Please define ‘beam like models (BLM)’ or provide a reference to its stipulation

  BLM is used as shorthand of “beam-like model”. It will be better defined.

- 2. P.4 line 95: Please more clearly define the meaning of ‘proper orthogonal tensor fields’ by preferably using a physical interpretation. The same pertains to the meaning and purpose of the skew tensor fields KA and KB. Alternatively, please provide references.

Further details will be provided. In particular, the terms used will be better defined and more references to classical works of rational and continuum mechanics will be provided.

- 3. P.5 line 110: Please more clearly enunciate the meaning of ‘well-defined measures of deformation’.

  This will be done and useful references will be provided.

- 4. P.5 line 115: Please define ‘proper manner’.

  This will be done and useful references will be provided.

- 5. P.6 lines 150-155: The entire paragraph appears hard to follow. Can it be confate in a more comprehensible way?

  This paragraph will be revised and further details and references will be provided.

- 6. P.7 top: Please clearly state which higher order terms (from which order) are neglected.

  This will be better specified.

- 7. P.7 line 170: In contrast to mathematics, I presume the majority of readers affiliated with wind energy might not be familiar with the rather specific terms stemming from differential geometry such as ‘pull back’ and ‘push forward’. Auxiliary explanations and additional references to relevant literature would be very helpful to follow the derivation.

  Auxiliary explanations and additional references to classical works of rational and continuum mechanics will be provided.

- 8. The first author of one reference is misspelled: It should rather read ‘Stäblein’ with umlaut.

  This will be done.

- 9. P. 8 ff: Is it correct that the general beam problem is decoupled into what is stipulated as ‘1D’ solution and into a ‘2D’ solution? If this is indeed correctly understood, on what grounds can the decoupling be justified? What is the error estimation of such an assumption?

  For beam-like structures with transversal dimensions much smaller than the longitudinal one, in the case discussed in section 3 (small warping, etc), the resolution of the classical three-dimensional nonlinear elasticity problem can be reduced to the resolution of two main problems. One of them governs the local warping of the cross-sections. It will be referred to as the cross-sections problem. The other one governs the global deformation of the center-line. It will be referred to as the center-line problem. In the revised version of the paper, the mathematical models to determine the deformation of cross-sections and center-line will be discussed with further details. Additional references will be provided to help understanding how those problems can be solved. A final section with numerical simulations will be added to show the accuracy of the results obtainable with the proposed modeling approach. Comparison with corresponding results of a 3D FEM commercial software will also be provided.

- 10. P.9 line 210: If correctly understood, the 2D solution of the warping displacements must be obtained prior to the 1D solution. Yet, in equation 28 the analytical expressions for the cross sectional properties (moments of areas) of an isotropic, prismatic ellipsoid are used. It is not abundantly clear how exactly the general 6x6 cross section stiffness matrix is obtained in case of a wind turbine rotor blade.

  The analytical results proposed in section 4 are for the case of tapered (not prismatic) beam-like structures with elliptical cross-sections. For that case we can provide analytical results. For general cross-sections shapes the formulation of the problem of 'how to determine the deformation of the cross-sections' is the same (as discussed in section 3.4), but in such a case the solution has to be obtained by using a numerical method. However, this is not surprising, since even in the classical linear theory of prismatic beams the analytical solutions are available for a limited number of cases only. For what concerns the relations between the stress resultants and strain measures, they can be obtained by integration of the three-dimensional stress fields over the cross-sections of the beam-like structure. This will be better specified in the revised version of the paper.

- 11. A figure showing the cross section, CSYS and cross-section forces used in section 4 would help a lot to illustrate the matter.

  Some current figures will be modified and other figures will be added to better introduce and explain the problem. Some of them will show the cross-sections and also the local frames which are used to write the stress and strain fields in components notation.

Response to comments of Anonymous Referee #2

The motivation of this work is highly relevant to wind energy. It is common place for beam-like models to be used, due to their balance between computational efficiency and accuracy. One limitation to these theories is the assumption of prismatic geometry. The closest example of relaxing this constraint is that of Hodges and Yu with VABS, where the beam can be curved and twisted, yet, cannot taper. Ignoring taper has some consequences for wind energy, near the root region where the loads are highest. So, the taper region can be important for structural design, while contemporary models cannot properly model these complex stresses.

- Although the ambition of this work is important to wind energy, I cannot recommend that this article is published in it's current form. A critical weakness is that the solution to the warping field is not well developed. Only a simple analytical example is given, which makes this contribution only valid for special cases. Thus, it cannot be used for wind turbine blades in general.

  The paper addresses the modeling of the mechanical behavior of beam-like structures which are curved, twisted and tapered in their reference unstressed state, undergo large displacements, in- and out-of-plane cross-sections warping and small strain. The problem of 'how to determine the warping of the cross-sections' is formulated for generic cross-sections shapes (in section 3.4). For what concerns the resolution of such a problem, we can provide analytical results in some cases (like in section 4, for the case of tapered elliptical cross-sections) and numerical results in all the other cases. However, this is not surprising, since even in the classical linear theory of prismatic beams analytical results are available for a limited number of cases only. This will be better specified in the revised version of the paper. Moreover, an additional section with numerical results for different beam-like structures undergoing large displacements will be provided.

- Currently, the state of the art are the contributions of Hodges, Yu and Giavotto. They have already developed general purpose beam models and cross section solvers. So this is the ultimate level of ambition that is needed to make a contribution to wind energy in this area. However, the key aim of this work, to incorporate taper, will be an important improvement over these earlier contributions. So I would strongly encourage the author to continue this important work. I can recognize that getting to the level of these earlier contributions will be difficult. I think this particular manuscript can still maintain an analytical approach and be improved by expanding greatly on the example. There is still an open question on what effects a beam model with taper could capture. So, the author could demonstrate the stresses and strains that this solution gives, that are not present in a more conventional beam formulation. Furthermore, the author could also make comparisons to FEM models to highlight the effects that are not captured. This I think is possible at this level and results like this would greatly improve the manuscript. Furthermore, if you had an tapered elliptical blade, how does taper affect things like frequencies or tip deflection? Again, these results will shed light on what more we can expect from simple engineering models if this limitation was relaxed, yet although simple and analytic, it would have relevance to wind energy.

  In addition to what said in the answer to the previous comment, in the revised paper we will provide an additional section with numerical results (including tip deflection in cases of large deflections, corresponding strain measures and stress resultants) for different reference beam-like structures. Moreover, comparison with corresponding results of a 3D FEM commercial software will also be provided.

- The authors did a well at explaining the motivation of their work. It could be made more widely applicable by explaining current engineering design challenges that this would help overcome. I have highlighted some points at the beginning of this review.
  This is a very mathematical paper written in a concise manner, using a lot of terminology that is typically not familiar outside of the continuum-mechanics community. To make this article accessible to wind energy readers I recommend several points where the author expand on the terminology.

  Auxiliary explanations as well as references to classical works of rational and continuum mechanics will be provided.

- The authors should further develop their techniques for solving the warping solution so it can be applied to general cross section shapes that are typically found in wind turbine blades. The authors should aim to solve the structural dynamics of real wind turbine blades. Furthermore the explanation of this work should be expanded so it is more clear.

  The method is already applicable to generic cross-sections shapes (see the answer to the first comment above). This will be better specified in the revised version of the paper and a section with numerical results for different test cases will be provided.

- There are several minor points that can be improved:
  Equation 15 with sub-equations would be more clear

  The corresponding paragraph will be modified and further details and references will be added.

- A general comment as with a theoretical development, please elaborate on the assumptions taken and the limitations of this approach.

  Further details about assumptions and limitations (e.g. beam-like structure, transversal dimensions much smaller than the longitudinal one, small warping, small strain, etc) will be provided and other useful references to the literature will be added.

- Generally speaking the wind energy community is not familiar with continuum mechanics. The author should explain verbally what all the terms mean. I personally have read about all these terms from my text books, but it would be nice if I didn't have to dust off my old texts to understand this article.

  Auxiliary explanations and further details will be provided. In particular, the terms used will be better defined and more references to classical works of rational and continuum mechanics will be provided to make it easier to follow the mathematical aspects of the proposed modeling approach.

- In the equations, the time rate of change is indicated by a dot. Typically this is given by a dot over the variable, however in this work it appears to be a super-script. This can be a little confusing because they use the same dot for dot products. If you use latex, ndot{x} would be the command that you would use.

This will be done, that is, the time rate of change will be indicated by a dot over the variable.

- The 'ˆ' operator is used in the equation. It is not clear that the 'ˆ' operator is in many of the equations. The authors should elaborate more on the formal definitions of the mathematics.

  The operator '∧' will be better defined in the revised version of the paper.

Response to comments of Anonymous Referee #3

The proposed method in the manuscript is a novel model of beam-like structures with curved, twisted and tapered geometries. Since the wind turbine blade designs are curved, twisted and tapered beam-like structures and go through large displacements in their operational life, the proposed model is highly related to the wind turbine blade analysis. Today, beam models are generally preferred in load and aeroelastic stability analysis of the turbine blades due to their accuracy and computational speed compared to the 3D finite element models. Although, curved and twisted beam models already exist in the literature (Hodges, Dewey H. Nonlinear composite beam theory), counting the taper effects are the main novelty of the study.

- Although the motivation of the study is very interesting and notable for state of the art blade analysis, there are essential things to be done before it is published.
  The manuscript is written in mathematical format, however the equations are hard to follow and re-derive because authors skip intermediate steps and give no reference in the derivation of the equations. I strongly recommend to write the intermediate steps explicitly or give relevant references for these steps instead of the statements such as 'well defined measures', 'proper manner' or 'when the 2D problem is solved'. Figures depicting the cross-sectional warping effects, loads and 'suitable coordinates' (coordinate curves) would be helpful to the readers.

  Further details, explanations and references will be provided to make it easier to follow the mathematical aspects of the proposed modeling approach. Moreover, some of the current figures will be modified and other figures will be added.

- Another substantial point is the lack of reproducible results. The analytical example results given in the manuscript can't be reproduced by the explanations given in the manuscript, hence the solution needs to be explained clearly. If the authors come up with the analytical example by themselves, they should provide more information about it. If the analytical example is taken from another study, please give reference. They should also compare the their results with a higher fidelity analysis results such as 3D finite element results to show that the taper effects are captured correctly by their formulation. The authors mention that they already implemented the method in a MatLab code. However, there is no information about the implementation of the method. Example results of authors' code and comparison of them by higher fidelity models would increase the value of the study. A wind turbine blade example would also intensify the proposed methods' relevance to wind turbine applications.

  Section 4 provides analytical formulas we have obtained for beam-like structures with tapered elliptical cross-sections. This solution is also included in the code that we have implemented in Matlab. Further details on this analytical solution will be provided. Also, an additional section with numerical results (e.g. tip deflection, strain measures, stress resultants) for different beam-like structures will be added to show the effectiveness of the proposed modeling approach. Finally, comparison with corresponding results of a 3D FEM commercial software will be provided.

- Please below suggestions:
  1- Section 2 : 'BeamDyn' is very relevant to the application of the geometrically exact

beam models to wind turbine analysis. Consider citing it.

It will be cited.

- 2- Section 3.1 : Instead of Figure-2 with wind turbine blade, a figure with cross-section warpings and coordinate curves would be elucidating.

  Some figures will be modified and other figures will be added to better introduce and explain the problem.

- 3- Section 3.1 : Please explain 'y' clearly (in current position vector R).
  4- Section 3.1 : Please explain deformation gradient explicitly or give reference for it.
  5- Section 3.1 : Please explain 'some higher terms' after equation 14.
  6- Section 3.1 : Please write intermediate steps between equation 13 - 15.

  The paragraph containing 'y', 'deformation gradient', 'higher order terms' and 'equation 13-15' will be modified. More precisely, further details on the mathematical model will be provided, along with useful references to classical works of rational and continuum mechanics.

- 7- Section 3.2 : A figure with cross-section forces and moments would help the readers.

  Some figures will be modified and other figures will be added to better introduce and explain the problem. Some of them will show the cross-sections and also the local frames which are used to write the stress and strain fields in components notation.

- 8- Section 3.4 : Please elaborate the section by providing the solution of the warping fields.

  Section 3.4 addresses the problem of 'how to determine the warping fields', which are responsible of the cross-sections deformation. Section 4 provides analytical results for beam-like structures with tapered elliptical cross-sections. For that case we can provide analytical results. For generic cross-sections shapes the formulation of the problem is the same (as in section 3.4), but the solution has to be obtained by using numerical methods. But this is not surprising, since even in the classical linear theory of prismatic beams analytical solutions are available for a limited number of cases only. However, this will be better specified in the revised version of the paper.

- 9- Section 4 : Please give more information about the example and how you obtain the final results. Please, compare them with higher fidelity solution to show your model captures the taper effects correctly. Comparison can also show the results of a model which ignores the taper effects. So, reader can see the effect of taper term in final results.
  10- A section which explains the numerical implementation should be added.
  11- A section with results of your numerical model and higher fidelity model should be added.

  Further details on how to determine cross-sections warping and center-line deformation will be provided. An additional section will introduce the model we have implemented in Matlab and the results that it can provide. Comparison with corresponding results of a 3D FEM commercial software will also be provided.