https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

Beam-like models for the analyses of curved, twisted and tapered HAWT blades in large displacements

Giovanni Migliaccio[1], Giuseppe Ruta[2], Stefano Bennati[1], Riccardo Barsotti[1]

[1]Civil and Industrial Engineering, University of Pisa, Pisa, 56122, Italy
[2]Structural and Geotechnical Engineering, University "La Sapienza" of Roma, Roma, I-00184, Italy

Correspondence to: Giovanni Migliaccio (giovanni.migliaccio.it@gmail.com)

Abstract. The continuous effort to better predict the mechanical behavior of wind turbine blades is related to lowering the cost of energy. But new design strategies and the continuous increase in the size and flexibility of modern blades make their aero-elastic modeling ever more challenging. For the structural part, the best compromise between computational efficiency and accuracy can be obtained by schematizing the blades as suitable beam-like elements. This paper addresses the modeling of the mechanical behavior of complex beam-like structures, which are curved, twisted and tapered in their reference state, undergo large displacements, 3D cross-sectional warping and small strains. A suitable model for the problem at hand is proposed. It can be used to analyze large deflections under prescribed loads and allows the 3D strain and stress fields in the structure to be determined. Analytical results obtained by applying the proposed modeling approach are illustrated.

1 Introduction

In the process of improving horizontal axis wind turbines (HAWT) performance new methods are continuously being sought for capturing more energy and developing more reliable structures, all with the ultimate goal of lowering the cost of energy (Wiser, 2016). As demonstrated by several researches, such goal can be achieved through the use of advanced materials, the optimization of the aerodynamic and structural behavior of the blades, and the exploitation of load control techniques. By way of example, one promising load control approach is based on the bend-twist coupling (BTC) of the blades, which can be obtained by sweeping the shape of the blades or by changing the orientation of their composite fibers (Ashwill 2010, Bottasso 2012, Stablein 2017). But new design strategies and the continuous increase in the size and flexibility of modern blades make the modeling of their aero-elastic behavior ever more challenging. For the structural part, schematizing the blades as suitable beam-like elements can be the best compromise between computational efficiency and accuracy. Modern blades can be considered complex beam-like structures, which are curved, twisted and tapered in their reference unstressed state. Even not considering the complexities related to the materials properties and the actual loading conditions, their shape alone is sufficient to make the mathematical description of their mechanical behavior a challenging task.

The present work addresses the mechanical modeling of modern blades considered as suitable curved, twisted and tapered beam-like structures. Beam models have historically found application in many fields, from the helicopter rotor blades in

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

aerospace engineering, to bridges components in civil engineering, and surgical tools in medicine. This contributed to the development of sophisticated theories over the years (see, for example, Love 1944, Antmann 1966, Rubin 1997). The need for geometrically non-linear models for complex beam-like structures has led to further researches also in recent years. One of the main drivers for the continuous research in this field is the need for rigorous and application-oriented models. In this paper the attention is focused on the effects of important geometrical design features, such as the curvatures of the reference center-line and the twist and taper of the cross-sections. After an introduction to modeling approaches for structures of this kind (section 2), a suitable model for the problem at hand is proposed (section 3). Finally, some analytical results obtained by applying the proposed modeling approach are illustrated (section 4).

2 Overview of modeling approaches

Aero-elastic modeling of modern blades can be addressed by means of different approaches (Wang 2016). Those ones based on 3D FEM and beam-like models (BLM) are two main choices for the structural part of this modeling. Although 3D FEM can be very accurate and flexible, they can be computationally expensive for the analyses of complex systems, especially if CFD aerodynamic analyses are executed in parallel. The overall computational cost can be reduced by using faster aerodynamic models, such as the blade element momentum (BEM) model, but even this solution may not be efficient enough for aero-elastic analyses and multi-objective optimization tasks. The coupling of BLM and BEM models can provide the best compromise between computational efficiency and accuracy. In this work we focus the attention on BLM for complex beam-like structures, such as modern blades, which can be curved, twisted and tapered in their unstressed state, be subjected to large deflections and 3D cross-sectional warpings. Suitable models are needed to simulate their mechanical behavior. In general, classical beam models (see Love 1944), which include extension, twist and bending, as well as the formulation of Reissner (1981), also accounting for transverse shear deformation, may not be sufficient. Geometrically exact models are a better choice, but a way to put them into a suitable form for engineering applications is usually needed (Antman 1966). In general, suitable models should be both rigorous and application-oriented, two important requirements pursued over the years by many investigators (see, for example, Simo 1985, Ibrahimbegovic 1995, Pai 2011, and Yu 2012).

For over a century researchers have sought to represent beam-like structures by means of 1D models. Several theories have been developed, from the elementary Euler-Bernoulli theory, to the classical theory which includes Saint-Venant torsion, up to more refined theories, such as the Timoshenko theory for transverse shear deformations, the Vlasov theory for torsional warping restraint, and 3D beam theories which include 3D warping fields. Broadly speaking, beam theories can be grouped into engineering and mathematical theories. Among the engineering theories, some formulations are based on ad-hoc corrections to simpler theories (Rosen 1978), while others are based on geometrically exact approaches (Hodges 2018). Among the mathematical theories, some approaches are based on the directed continuum (Rubin 1997), some others exploit asymptotic methods (Yu 2012). The reason for the extensive and continuous research efforts on beam theory is that it has always found applications in many fields. For example, many approaches have been developed for helicopter rotor blades

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

eawe european academy of wind energy
Open Access WIND ENERGY SCIENCE DISCUSSIONS

with an initial twist. Pre-twisted rods have attracted the interest of researchers in different fields. A wide-ranging review on this subject is due to Rosen (1991). In the 1990's, Kunz (1994) provided an overview also on modeling methods for rotating beams, illustrating how engineering theories for rotor blades evolved over the years. In those same years, Hodges (1990) reviewed the modeling approaches for composite rotor blades, discussing the importance of 3D warping and deformation coupling. More recently, Rafiee (2017) discussed vibrations control issues for rotating beams, summarizing beam theories and complicating effects, such as non-uniform cross-sections, initial curvatures, twist and sweep. It seems that, unlike the case of the pre-twisted rods, the published results for curved rotating beams with initial taper and sweep are quite scarce.

Up to now much has been done to develop powerful beam theories. However, there is still a gap between existing theories and those that could be suitable for complex beam-like structures. In general, the geometry of the reference and current states must be appropriately described. The curvature, twist and taper are important design features and should be explicitly included in the model. The analysis should not be restricted to small displacements and should consider deformation couplings. The model should provide the strain and stress fields, be rigorous and usable by engineers, and provide classical results when applied to prismatic isotropic homogeneous beams. Following these guidelines, a mathematical model to simulate the mechanical behavior of complex beam-like structures is proposed hereafter.

3 Mechanical model for complex beam-like structures

Here we are concerned with developing a mathematical model to describe the mechanical behavior of beam-like structures which are curved, twisted and tapered in their reference state and undergo large displacements. One of the main issues with such a task is how to describe the motion of the structure (see, for example, Simo 1985, Ruta 2006, and Pai 2014). The approach considered in this work is to imagine a beam-like structure as a collection of plane figures (i.e. the cross-sections) along a regular and simple three-dimensional curve (i.e. the center-line). We assume that each point of each cross-section in the reference state moves to a position in the current state through a global rigid motion on which a local general motion is superimposed. In this manner, the cross-sectional deformation can be examined independently of the global motion of the center-line. So, it is possible to consider the global motion to be large, while the local motion and the strain may be small.

3.1 Kinematics and strain measures

We begin by introducing two local triads of orthogonal unit vectors. The first one is the local triad, b_i, in the reference state, with b_1 aligned to the tangent vector of the reference center-line. This frame is a function of the reference arch-length parameter only, that is $b_i=b_i(s)$. The second local triad, a_i, is a suitable image of the local triad b_i in the current state. This frame is a function of the reference arch-length parameter and the time, that is $a_i=a_i(s,t)$. In general, a_1 is not required to be aligned to the tangent vector of the current center-line. See Figure 1.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

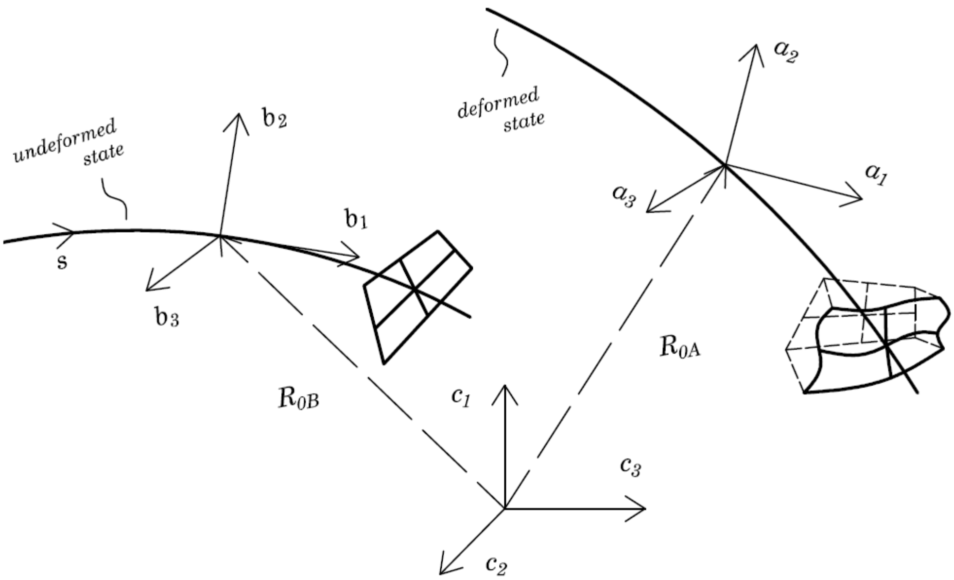

Figure 1: Schematic of the reference and current states, center-lines, cross-sections and local frames

We continue by introducing the kinematical variables we use to describe the motion of the considered structure. To this aim, the orientation of the frame a_i and b_i relative to a fixed rectangular frame, c_i, are defined as follows

95 $$a_i = Ac_i, \quad b_i = Bc_i \tag{1}$$

where A and B are two proper orthogonal tensor fields. We introduce an orthogonal tensor field T, which defines the relative orientation between the frames a_i and b_i and can be used to identify the deformed configuration of the structure, as

$$a_i = Tb_i = AB^T b_i \tag{2}$$

We also define two skew tensor fields, K_A and K_B, and their axial vectors, k_A and k_B, which are related to the curvatures of
100 the center-line of the structure, respectively in the current and reference states, as follows

$$\begin{aligned} K_A &= A' A^T, \quad a_i' = K_A a_i = k_A \wedge a_i \\ K_B &= B' B^T, \quad b_i' = K_B b_i = k_B \wedge b_i \end{aligned} \tag{3}$$

The prime denotes derivative with respect to the arch-length parameter s. Then, we introduce the skew tensor field Ω, and its axial vector field ω, associated with the variation of the vectors a_i over the time, t, as follows

$$\Omega = \dot{A} A^T, \quad \dot{a}_i = \Omega a_i = \omega \wedge a_i \tag{4}$$

105 The dot denotes derivative over the time t. At this point, it is easy to obtain the following identities

$$\begin{aligned} T' T^T &= K_A - T K_B T^T, \quad \dot{T} T^T = \Omega \\ \phi\left[T' T^T\right] &= k_A - T k_B, \quad \phi\left[\dot{T} T^T\right] = \omega \end{aligned} \tag{5}$$

where the operator ϕ[] provides the axial vector of the skew tensor between brackets.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

The function R_{0B}, which maps the points of the center-line in the reference state, does not depend on time, while R_{0A} can change over the time t. Its variation is the time rate of change of the position of the points of the current center-line

$$R_{0A}^{\bullet} = v_0 \tag{6}$$

We are now in a position to introduce two important kinematic identities

$$\begin{aligned} v_0^{'} - \omega \wedge R_{0A}^{'} &= T\gamma^{\bullet} \\ \omega^{'} &= Tk^{\bullet} \end{aligned} \tag{7}$$

where γ and k are well-defined measures of deformation for beam-like structures. They vanish for pure rigid motions and transform in the proper manner when a rigid motion is superposed to a not rigid motion. They are defined as

$$\begin{aligned} \gamma &= T^T R_{0A}^{'} - R_{0B}^{'} \\ k &= T^T k_A - k_B \end{aligned} \tag{8}$$

Now we start modeling the motion of the points of the cross-sections. In particular, we introduce two mapping functions, R_A and R_B, to identify the positions of the points of the 3D beam-like structure in its current and reference states. For what the reference state is concerned, we define the (reference) mapping function

$$R_B(z_i) = R_{0B}(z_1) + x_\alpha(z_i) b_\alpha(z_1) \tag{9}$$

where R_{0B} is the position of the points of the reference center-line relative to the frame c_i, b_α are the vectors of such local frame in the plane of the reference cross-section, x_α identify the position of the points in the reference cross-section relative to the reference center-line, and, finally, z_i are suitable coordinates which do not depend on time, with z_1=s.

Throughout this paper, Greek indices assume values 2 and 3, Latin indices assumes values 1, 2 and 3, and repeated indices are summed over their range.

It is worth noting that x_k may or may not be equal to z_k, with the first choice leading to the most common modeling approaches available in the literature (see, for example, Simo 1985, Pai 2011, and Yu 2012). In this work we use a different approach, by choosing suitable relations between x_k and z_k to simulate the shape of the considered beam-like structure, which is curved, twisted and tapered in its reference unstressed state. In particular, the span-wise variation of the shape of the cross-sections is analytically modeled by means of a mapping of this kind

$$x_i = \Lambda_{Bij} z_j \tag{10}$$

where the coefficients Λ_{Bij} are suitable functions of z_1. In the following we will consider the interesting class of the curved and twisted beam-like structures with bi-tapered cross-sections, in which case the map of Eq. (10) reduces to

$$x_1 = z_1, \quad x_2 = z_2 \lambda_{x2}(z_1), \quad x_3 = z_3 \lambda_{x3}(z_1) \tag{11}$$

where the coefficients $\lambda_{x\alpha}$ are suitable functions of z_1. It is worth noting that a suitable definition of such functions gives the possibility to reproduce interesting shapes, such as, for example, the one reported in Figure 2.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

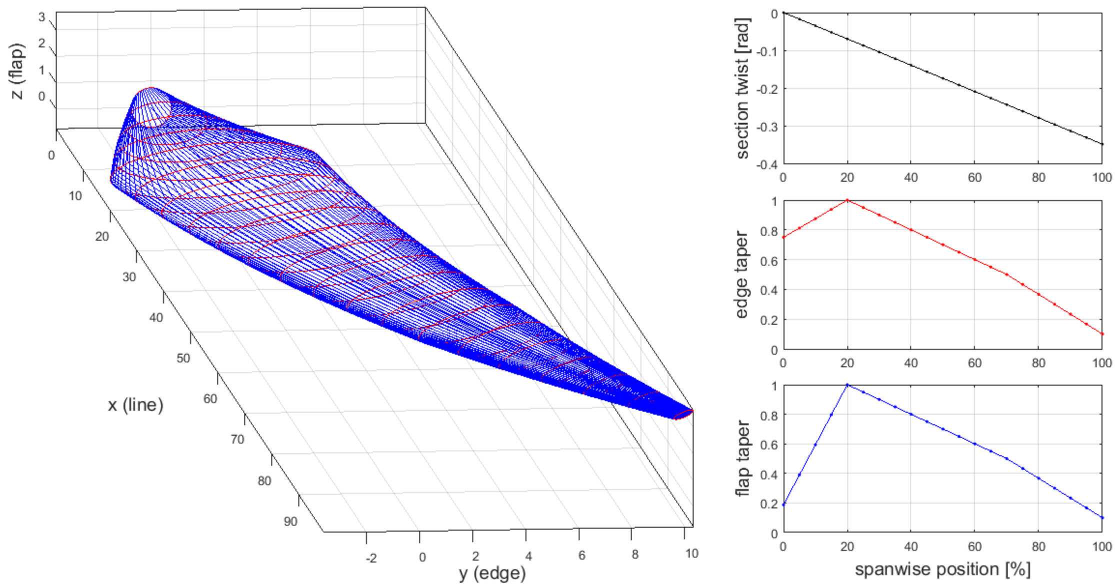

Figure 2: Schematic of a curved, twisted, and tapered beam-like structure

The position of the points in the current state are defined in a similar manner by means of the (current) mapping function

$$R_A(z_i,t) = R_{0A}(z_1,t) + y_\alpha(z_i)a_\alpha(z_1,t) + w_k(z_i,t)a_k(z_1,t) \tag{12}$$

where R_{0A} is a function mapping the position of the points of the center-line in the current state, while w_k are the components of the 3D warping displacements in the local frame a_k. Again, y_k is not equal to z_k. In general, we reserve the possibility to choose relations similar to Eq. (10), even with coefficients Λ_{Aij} (and $\lambda_{y\alpha}$) different from Λ_{Bij} (and $\lambda_{x\alpha}$). The main formal difference between the reference and current maps is due to the warping, w, introduced to describe the geometry of the deformed state without a-priori approximation.

The 3D deformation gradient, H, between the reference and current configurations, can now be calculated as follows

$$H = G_k \otimes g^k \tag{13}$$

where G_k and g_k are the covariant and controvariant base vectors in the current and reference states, respectively. They can be determined by using standard means. When the deformation gradient is given, the Green-Lagrange strain tensor, E, can be calculated. In particular, we write the Green-Lagrange strain tensor in a form based on simplifying assumptions applicable to the considered beam-like structure. To this end, we introduce the characteristic dimension of the cross-sections, h, the longitudinal dimension of the reference line, L, and assume h to be much smaller than L. Considering a thin structure, we assume its curvatures are much smaller than 1/h. The warping displacements w_k are also assumed to be small. More precisely, by introducing a non-dimensional parameter ε much smaller than one, they are considered of the order of hε, while their derivative with respect to z_1 is of the order of εh/L. In addition, all deformations are assumed to be small. In particular, γ is much smaller than one and kh is of the order of ε.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

For the considered structure, in the case of small strains and small local rotations, the following relation holds

$$E \simeq \frac{T^T H + H^T T}{2} - I \tag{14}$$

By way of example, in the case of uniform initial taper (Λ_{B22}=Λ_{B33}=Λ_0), y_k=x_k, and neglecting some higher order terms, the components of the Green-Lagrange strain tensor may be written in the form

$$\begin{aligned}
E_{11} &= \gamma_1 + \Lambda_0 \left(k_2 z_3 - k_3 z_2 \right) + k_{B1} \left(w_{1,2} z_3 - w_{1,3} z_2 \right) \\
2E_{21} &= \gamma_2 + \frac{w_{1,2}}{\Lambda_0} - k_1 \Lambda_0 z_3 + k_{B1} \left(w_{2,2} z_3 - w_{2,3} z_2 - w_3 \right) \\
2E_{31} &= \gamma_3 + \frac{w_{1,3}}{\Lambda_0} + k_1 \Lambda_0 z_2 + k_{B1} \left(w_{3,2} z_3 - w_{3,3} z_2 + w_2 \right) \\
E_{22} &= \frac{w_{2,2}}{\Lambda_0}, \quad E_{33} = \frac{w_{3,3}}{\Lambda_0}, \quad 2E_{23} = \frac{w_{2,3} + w_{3,2}}{\Lambda_0}
\end{aligned} \tag{15}$$

where

$$E_{ij} = E \cdot b_i \otimes b_j \tag{16}$$

3.2 Stress fields and constitutive models

Given the strain tensor, the stress fields in the structure can be calculated when a constitutive model is chosen. Limiting our attention to elastic bodies, in a pure mechanical theory, in the case of small strain, we use a linear relation between the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor (Gurtin 2003), as follows

$$S = 2\mu\, E + \lambda tr E\, I \tag{17}$$

In the case of small strain and small local rotations, we can also write

$$P \simeq TS, \quad C \simeq TST^T \tag{18}$$

where P is the first Piola-Kirchhoff stress tensor and C is the Cauchy stress tensor. In the considered case the tensor field T is sufficient to perform the pull back and push forward operations between the stress tensor fields S, P and C.

Now, we define the stress resultants, namely the force F and moment M, on each cross-section of the structure. Using the first Piola-Kirchhoff stress tensor, in the case of small warpings, small strains and small local rotations, we write

$$F = T \int_\Sigma P_{i1} b_i, \quad M = T \int_\Sigma y_\alpha P_{i1}\, b_\alpha \wedge b_i \tag{19}$$

where Σ is the domain corresponding to the cross-section on which the integration is performed and

$$P_{ij} = P \cdot a_i \otimes b_j \tag{20}$$

By combining Eqs. (15)-(19), the force and moment stress resultants can be related to the geometrical parameters of the structure and the 1D strain measures (8). However, such relations are actually known if we know the warping fields w_k. An approach to obtain suitable warping fields is illustrated in section 3.4.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

3.3 Expended power and balance equations

To complete the formulation, we conclude with considerations on the principle of expended power and the balance equations for the considered structure. For hyper-elastic bodies (Gurtin 2003), we write the principle of expended power in the form

$$\int_A p\cdot v+\int_V b\cdot v=\frac{d}{dt}\int_V \Phi \tag{21}$$

where p are surface loads per unit reference surface (A), b are body loads per unit reference volume (V), Φ is the 3D energy density function of the body, and v is the time rate of change of the current position of its points, which is given by

$$v=v_0+\omega\wedge y_\alpha a_\alpha+w^{\bullet} \tag{22}$$

where w• is the time rate of change of the warping displacement.
For small warpings, small strains, and small local rotations, it can be shown that the power expended by the surface and body loads on the warping velocities can be neglected. By using this simplification, the external power, Π_e, reduces to

$$\Pi_e=\Delta\left(F\cdot v_0+M\cdot\omega\right)+\int_s F_s\cdot v_0+M_s\cdot\omega \tag{23}$$

where the vector field v_0 is the time rate of change of the position of the points of the current center-line, the vector field ω is the time rate of change of the orientation of the vectors a_i, the terms F_s and M_s are suitable resultants of inertial actions and prescribed loads per unit length in the reference state, while the symbol Δ simply means that the function between brackets is evaluated at both the ends of the beam and the difference between those values is taken.
The 3D cross-sectional warpings may be important in calculating the 3D energy function, so they cannot be neglected in the internal power, Π_i. However, the internal power may be reduced to a useful form for beam-like structures by introducing a suitable 1D strain energy function, U. For example, if U can be expressed in terms of the strain measures, γ and k, we obtain

$$\Pi_i=\frac{d}{dt}\int_s U(\gamma,k,s)=\int_s f\cdot\gamma^{\bullet}+m\cdot k^{\bullet} \tag{24}$$

where f and m are the pulled back vector fields of the force and moment stress resultants, F and M, and are defined as

$$f=T^T F,\quad m=T^T M \tag{25}$$

By using the principle of expended power, we also obtain balance equations for the vector fields F and M in the form

$$\begin{aligned} &F'+F_s=0\\ &M'+R'_{0A}\wedge F+M_s=0 \end{aligned} \tag{26}$$

At this point, we have kinematic equations, (6), strain measures, (8) and (14), force and moment balance equations, (26), and the principle of expended power, $\Pi_e=\Pi_i$, in a suitable form for beam-like structures, (23)-(24). To complete the formulation of the model we need relations providing the 1D stress resultants in terms of the 1D strain measures. To this end, we need to know the warping fields. An approach to obtain suitable warping functions is discussed hereafter.

https://doi.org/10.5194/wes-2019-59
Preprint. Discussion started: 27 August 2019

3.4 Warping displacements

In general, a 3D nonlinear elasticity problem can be formulated as a variational problem. In any case, if we try to solve the variational problem directly, the difficulties encountered in solving the elasticity problem remain. For beam-like structures whose transversal dimensions are much smaller than the longitudinal one, assumptions based on the shape of the structure can lead to important simplifications. In particular, the 3D nonlinear problem can be split into a 1D nonlinear problem, governing the global deformation of the center-line and cross-sectional frames, and a 2D problem, governing the local distortion of the cross-sections. The warping displacements can be obtained by solving the 2D problem. Using such an approach, for small warpings, strain and local rotations, the following variational statement can be used

$$\delta \int_V \Phi = 0 \tag{27}$$

where δ is the variation of the functional for a corresponding variation of the warping fields.

The warping fields satisfying (27) can be obtained by the corresponding Euler-Lagrange equations. When the 2D problem is solved, the stiffness properties of the cross-sections are known. From the 1D problem we obtain the 1D strain measures (8). Finally, given the warping fields and the 1D strain measures, the 3D strain and stress fields can be determined (14)-(18).

4 An example with analytical results

An example illustrates the results the proposed modeling approach can provide. In particular, we consider a curved, twisted and tapered beam-like structure with elliptical cross-sections. The structure is clamped at one end and it is loaded by given forces at the other end. We use the assumptions introduced in the foregoing about the smallness of the warpings, strains and local rotations. In addition, here, we assume the initial twist is negligible and the initial taper is uniform ($\Lambda_{B22}=\Lambda_{B33}=\Lambda_0$).
The 3D nonlinear problem is split, as discussed, into a 2D linear problem and a 1D nonlinear problem. The 1D problem can be solved numerically when the stiffness properties of the cross-sections have been obtained from the 2D problem. Here we focus the attention on this latter problem, leaving to a successive paper the discussion on the numerical procedure we have implemented in MatLab to solve the 1D nonlinear problem. For the 2D problem, in the considered case, the Euler-Lagrange equations corresponding to (27), for the effects of extension, γ_1, and curvatures, k_i, are satisfied by the warping fields

$$\begin{aligned}
w_1 &= k_1 \frac{d_3^2 - d_2^2}{d_3^2 + d_2^2} \Lambda_0^2 z_2 z_3 \\
w_2 &= -\nu\gamma_1 \Lambda_0 z_2 - \nu k_2 \Lambda_0^2 z_2 z_3 + \nu k_3 \Lambda_0^2 (z_2^2 - z_3^2)/2 \\
w_3 &= -\nu\gamma_1 \Lambda_0 z_3 + \nu k_3 \Lambda_0^2 z_2 z_3 - \nu k_2 \Lambda_0^2 (z_3^2 - z_2^2)/2
\end{aligned} \tag{28}$$

where d_2 and d_3 are the semi-major axes of the reference elliptical cross-section. Using this result, we can calculate the strain and stress fields, (14)-(18), the force and moment stress resultants, (19), and the strain energy function U. For example, if we consider a local frame in the reference cross-section with its origin at the cross-section's center of mass and its axes aligned with the cross-section's principal axes of inertia, we can write the 1D strain energy function, U, in the form

$$U = \frac{1}{2} EA\Lambda_0^2 \gamma_1^2 + \frac{1}{2} GJ_1\Lambda_0^4 k_1^2 + \frac{1}{2} EJ_2\Lambda_0^4 k_2^2 + \frac{1}{2} EJ_3\Lambda_0^4 k_3^2 \tag{29}$$

In Eq. (29), E is the Young modulus, G is the shear modulus, while A, J_1, J_2 and J_3 are the following geometrical constants

$$A = \pi d_2 d_3, \quad J_1 = A d_2^2 d_3^2 / (d_2^2 + d_3^2), \quad J_2 = A d_3^2 / 4, \quad J_3 = A d_2^2 / 4 \tag{30}$$

An interesting result is that the initial taper, Λ_0, appears explicitly in all the previous relations, allowing us to analytically evaluate its effect. The effects of the not uniform initial taper, initial twist and other terms, on the strain energy function, on the strain and stress tensor fields, and on the force and moment stress resultants, will be discussed in a successive work.

5 Conclusions

Modern blades can be considered complex beam-like structures, with one dimension much larger than the other two and a shape that is curved, twisted and tapered already in the unstressed state. Their mechanical behavior can be simulated through suitable 3D beam models, which explicitly consider their main geometrical characteristics, possible large displacements of their center-line and 3D local warping of their cross-sections. In this work, curved, twisted and tapered beam-like structures have been modeled analytically. The main geometrical design features, such as the curvatures of the center-line and the twist and taper of the cross-sections, have been explicitly included in the model. The warping displacement has been thought of as additional small motion superimposed to the global generic motion of the cross-sectional frames. The resulting model is suitable to simulate large deflections of the center-line, large rotations of the cross-sectional frames and small deformation of the cross-sections. The strain tensor has been calculated analytically in terms of the geometrical parameters of the structure, the 1D strain measures and the 3D warping fields. The same has been done for the 3D and 1D energy functions. An approach based on energy functionals and the slenderness of the structure have been used to obtain suitable warping fields. The principle of expended power for curved, twisted and tapered beam-like structure has been discussed, as well as the balance equations for the force and moment stress resultants. Finally, an application example, which includes analytical results, has been presented to show the information the proposed modeling approach can provide.

References

Antman S.S., Warner W.H., Dynamical theory of hyper-elastic rods, Arch. Rational Mech. Anal., 23, 135-162, 1966.

Ashwill T.D., Kanaby G., et al, Development of the swept twist adaptive rotor (STAR) blade, 48th AIAA Aerospace sciences meeting, Orlando, FL, Jan. 4-7, 2010.

Bottasso C.L., Campagnolo F., et al, Optimization-based study of bend-twist coupled rotor blades for passive and integrated passive/active load alleviation, Wind Energy, 16, 1149-1166, 2012.

Gurtin M.E., An introduction to continuum mechanics, Mathematics in science and engineering, 158, 2003.

Hodges D.H., Review of composite rotor blades modeling, AIAA Journal, 28, 561-565, 1990.

Hodges D.H., Geometrically exact equations for beams, Encyclopedia of Continuum Mechanics, Springer Verlag, 2018.

Ibrahimbegovic A., On finite element implementation of geometrically nonlinear Reissner's beam theory: three-dimensional curved beam elements, Computer methods in applied mechanics and engineering, 122, 11-26, 1995.

Kunz D.L., Survey and comparison of engineering beam theories for helicopter rotor blades, Journal of Aircraft, 31, 473-479, 1994.

Love A.E.H., A treatise on the mathematical theory of elasticity, 4th Edition, Dover Publications, NY, 1944.

Pai P.F., Three kinematic representations for modeling of high flexible beams and their applications, Int. Journal of solids and structures, 48, 2764-2777, 2011.

Pai P.F., Problem in geometrically exact modeling of highly flexible beams, Thin-walled structures, 76, 65-76, 2014.

Rafiee M., Nitzsche F., Labrosse M., Dynamics, vibration and control of rotating composite beams and blades: a critical review, Thin-walled structures, 119, 795-819, 2017.

Reissner E., On finite deformation of space curved beams, Journal of applied mathematics and physics, 32, 734-744, 1981.

Rosen A., Friedmann P.P., Non linear equations of equilibrium for elastic helicopter or wind turbine blades undergoing moderate deformation, NASA, CR-159478, 1978.

Rosen A., Structural and dynamic behavior of pre-twisted rods and beams, American Society of Mechanical Engineers, 44, 483-515, 1991.

Rubin M.B., An intrinsic formulation for nonlinear elastic rods, Int. Journal of solids and structures, 34, 4191-4212, 1997.

Ruta G., Pignataro M., Rizzi N., A direct one-dimensional beam model for the flexural-torsional buckling of thin-walled beams, Journal of Mechanics of materials and structures, 1, 1479-1496, 2006.

Simo J.C., A finite strain beam formulation, the three-dimensional dynamic problem, part I, Computer methods in applied mechanics and engineering, 49, 55-70, 1985.

Stablein A.R., Hansen M.H., and Verelst D.R.: Modal properties and stability of bend-twist coupled wind turbine blades, Wind Energy Science, 2, 343-360, 2017.

Wang L., Liu X., Kolios A., State of the art in the aeroelasticity of wind turbine blades: aeroelastic modelling, Renewable and sustainable energy review, 64, 195-210, 2016.

Wiser R., Jenni K., et al, Forecast wind energy costs and cost drivers: the views of the world's leading experts, LBNL 1005717, 2016.

Yu W., Hodges D.H., Ho J.C., Variational asymptotic beam-sectional analysis - an updated version, Int. Journal of engineering science, 59, 40-64, 2012.