The Authors response to the reviewers' comments

The authors would like to thank the referees for their new comments. In the following, the comments will be addressed:

- **Anonymous Referee #2**

1. Second part of (7): Is the LHS missing a transpose?

   Yes, a transpose sign was missing, but for the RHS term. Corrected.

2. Equations (9) and (10): $R_1 - R_4$ are scalars. Remove boldface italic. Also, mention here that they are the components of $\boldsymbol{R}$ in (2)

   Modified accordingly.

3. Equation (9): It is $U \cdot grad(U)$, not $U \cdot grad \cdot U$

   Corrected and written as $(\boldsymbol{U} \cdot \nabla)\boldsymbol{U}$.

4. Equation (9): p is really a modified pressure.

   The word "modified" was added to the explanation of the variable p.

5. Line 25: Do you mean $\bar{p}$, not q?

   Corrected. The adjoint variables are represented by variables with overbar line.

- **Anonymous Referee #4**

1. The abstract does not sufficiently describe the contents and findings of this manuscript. It is a very general description of what a reader can expect.

   More information is added to both the abstract and the conclusion to better represent the findings of the study.

2. Pg. 2, line 25: it still mainly $\rightarrow$ it is still mainly

   Corrected.

3. Pg. 6, line 1: $q \rightarrow p$

   Corrected.

4. Eq. (21): The first term $(-\nabla\bar{\boldsymbol{U}} \cdot \boldsymbol{U})$ may not be correct. Please reconfirm it. In literature I could only find $-\nabla\bar{\boldsymbol{U}}^T \cdot \boldsymbol{U}$ and one another form of the cross-convective term.

   The term $-\nabla\bar{\boldsymbol{U}} \cdot \boldsymbol{U}$ in the manuscript is correct and similar to what has been derived in the original study of Othmer. Moreover, as mentioned now in the manuscript, a more detailed derivation of the adjoint system of equation for topology optimisation, which includes this term, can be found in the following study:
   Hinterberger, C. and Olesen, M.: Industrial application of continuous adjoint flow solvers for the optimization of automotive exhaust systems, CFD & Optimization, Antalya, Turkey, 2011.

Please also be aware that,

$$\boldsymbol{R} = (R_1, R_2, R_3, R_4)^T$$

$$\frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} + \overline{\psi}^T \frac{\partial \boldsymbol{R}}{\partial \alpha}$$

$$\overline{\psi}^T = (\overline{\boldsymbol{U}}, \overline{p})$$

5. Figure 10: Can you please also add the evolution of gradient as a function of adjoint iteration?

   The plot of gradient history is now added to the Figure 10.

6. Conclusions: You should conclude the results and findings from your manuscript. Your current conclusions are too general.

   More information is added to both the abstract and the conclusion to better represent the findings of the study.