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Interactive comment on "Analytical model for the power-yaw sensitivity of wind turbines operating in full wake" *by* Jaime Liew et al.

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Response to reviewers 1 and 2

We would like to thank both reviewers for the constructive and detailed comments on our article. The authors have considered the reviewer comments in detail, and we believe that the suggestions have helped strengthen the document before publication. Please find below our responses to your comments. In addition to changes suggested by the reviewers, we have added two additional citations, and minor grammatical changes to the article. Please find attached in the supplementary document a marked-up version showing all changes in the paper.

Yours Sincerely,

Jaime Liew, Albert M. Urbán, and Søren Juhl Andersen

Reviewer 1

Major Comments

1. p1, l19: 'Part of the discrepancy and uncertainty might stem from unintentional yaw misalignment'. This formulation is quite vague, and should be made more precise if possible. Could the authors make more clear how big they expect / know the effect of yaw misalignment to be? The subsequent citations to literature clearly indicate that yaw misalignments are common, but do not really measure their contribution to the aforementioned discrepancies and uncertainties.

The reviewer is correct about the vague formulation of the statement present in the paper. However, the authors do not have a precise answer yet regarding the uncertainties of the real implication of the yaw misalignment when wakes are present. This paper needs to be seen as a first step towards that quantification. The intention of this paper is to provide the relevant information and a model to capture such complex phenomena precisely. Next steps are to include measured yaw misalignment in various wind farms to quantify the power loss via our model and compare it with measurements extending the study case for other common inflow cases as partial wakes.

2. It is not completely clear to me why r_m is a random variable, I suppose this is due to uncertainty on γ. The probabilistic framework introduced in 2.1 hence came a bit as a surprise when reading the text. The source of uncertainty should be stressed

more.

r_m is chosen to be a random variable not because of the uncertainty in γ, but rather because of the definition of the blade segment effective wind speed, $\bar{U}(r_R)$ in Eq. (3). $\bar{U}(r_R)$ can be thought of as a *probability weighted average wind speed*. In order to determine the weighting, one must know the location of the blade segment in the wind field (determined by the elliptical orbit). It is assumed that the blade segment azimuth angle is uniformly distributed, which leads to the probability distribution of the variable r_m, outlined in Appendix A.2. The authors determined that following this line of reasoning is the most straight forward and insightful path to arriving at Eq. 4.

3. The yaw misalignment γ is introduced on p2, l5 as the yaw misalignment with the free wind direction. Given that the local wind direction can change throughout a wind farm (e.g. due to Coriolis effects), downstream turbines can have a non- zero γ whilst still being aligned with the local wind (as the authors also point out on p2. However, does this definition of γ then still uphold in the remainder of the text (e.g. Figure 1, and the analytical model), or should γ be interpreted as the misalignment to the local wind direction?

The reviewer raises an interesting concern regarding the definition of yaw misalignment. Indeed, the authors are referring to the misalignment between the rotor and the local wind direction. For this reason, the definition of γ after Equation 1 has been updated to reflect this.

4. The main novelty of the current work is being able to account for non-uniform flow conditions in the wake. In a uniform wake, $\bar{U}(r_R) = U(r_R)$, and I suppose from section 2.2 that α would simply equal α 0 . Is this true? If yes, please mention this. If not, please explain.

It is indeed true that $\alpha = \alpha_0$ if the wind turbine faces a uniform wind field. This fact has now been mentioned at the end of section 2.2.

5. p 7, l4: Please don't use D both as symbol for the downstream distance. In literature, D virtually always is the turbine diameter. Even in this manuscript, p7 l16 (ranges of 2D to 14D), I suspect D indicates the turbine diameter. Use a small d instead to represent the variable downstream distance. A similar thing can be said about the subscript R, in the rotor coordinate system r R ,ψ R . R is almost always used to indicate the rotor radius (like you also do in line 29, p8). However, I guess finding another meaningful symbol to represent this is not as simple (since r is already taken for the radial position).

The reviewer raises a valid point regarding nomenclature. We have changed the notation for downstream distance to x, and have removed all uses of the variable, R, so that there is no confusion with the subscript in r_R. Section 3.1.2 now refers to distances in terms of rotor diameters instead of rotor radii, and Appendix A.1 has been corrected to not contain any variables with the name R.

6. Section 3.2.2: The LES wake is post-processed: it is time-averaged, the shear is removed, and azimuthal variations are removed by averaging over the azimuthal direction to obtain a radial wind speed function, which the authors claim to be comparable to that generated by the DWM model. In this sense, I think the added value of including LES in this study should be justified more clearly and perhaps earlier in the manuscript. For instance in Section 5, the authors indicate that wake break down occurs earlier in LES than in DWM. This could probably also be seen from the data in Figure 4, if some cuts at different streamwise locations are taken. I think it would be useful to discuss the relevant observed differences between LES and DWM wakes in Section 3.

The reviewer raises a good point about providing additional justification of the use of LES. We have extended the first paragraph of the Method section, as well as provide insight into the early breakdown point in section 3.2.2. Please refer to the response of the following question for further remarks.

7. Added value of LES: Fig2 (why not LES), why even use LES if you make it so close to a DWM? Can the model account for a truly turbulent wake?

The purpose of Fig. 2 is to demonstrate a typical wake profile (and its derivative) in the form of a radial function. For this purpose, the choice of DWM or LES in this figure is arbitrary. The use of LES in the paper is not to process the LES to resemble the DWM wake, but rather to have a higher fidelity wake model in the aeroelastic simulations to make up for model limitations of the DWM model. The DWM model is inherently a simplified engineering model, and does not capture the behaviour of a wake in as much detail as LES. The use of both medium and fidelity wake models strengthens the validity of the investigation. Regarding the second question, the strength of the analytical model is that it provides comparable results for α when compared to α determined from turbulent aeroelastic simulations.

8. Figure 5. This is an interesting Figure. Some comments.

- Would it be insightful to include panel(s) with both the analytical and HAWC2 results on top of each other to highlight the differences?
- The lines have X markers. I'm assuming this is where the yaw angles have been sampled for the simulations. Please indicate this.

The authors found that such a figure is not insightful due to overlap of the lines resulting into a crowded figure. It was decided that Fig. 6, showing the curvature of the curves in Fig. 5, is the clearest way to compare the differences in power output due

to yaw in a wake situation. The markers are indeed indications of the sampled points. This has now been indicated in the figure caption.

9. Figure 6.

- Perhaps add markers where you sampled these lines (i.e. which simulations you actually ran)
- The differences between your model and HAWC2 seem to increase at lower turbine spacings. However, in practice, such low turbine spacings are rare. Mentioning this in the text would further justify the utility of your model.

We agree with the reviewer that although the discrepancy between HAWC2 and the model increases at low turbine spacings, these scenarios are rare in practice. We have included this point in paragraph 3 of the Results section. Additionally, we have added markers in Figure 6 indicating the simulations that were run.

10. In the context of wake steering with yaw misalignment, literature has shown that wakes tend to take curled shapes, hence axisymmetric wakes would be rare in farms with active wake steering. Could such wakes also be incorporated using the azimuthal-time averaging? I believe so, and explicitly mentioning this would further strengthen your case.

The reviewer raises an interesting point regarding the use of the analytical model for asymmetric wakes, such as a curled wake. Indeed the azimuthal averaging that we present in 3.2.2 can be applied to a curled wake. This is now mentioned in the last paragraph of Section 5.

11. The Lemma's in Appendix A would be better readable if they were self-contained (i.e. defining symbols etc.). Also, in both Lemma's you use captial symbol

R again, for a different meaning than simply a subscript indicating the rotor reference frame. Please consider revising this, as this comes off confusing.

The reviewer raises an valid concern regarding the Lemmas in Appendix A. The Lemmas have been extended to be self contained, and the use of the variable R has been removed.

12. You propose an analytical method which allows (p 13, l25) a quick and reliable method to calulate α. Can you compare the computational complexity of your model with HAWC2?

Thank you for the suggestion. The computational time to determine the power loss exponent using the analytical model is in the order of seconds, whereas stochastic aeroelastic simulations require a time in the order of hours to reliably achieve the same result. We have included this point in the final paragraph of the Introduction.

Minor comments

1. p2, eq (1): P_0 is not defined explicitly, I assume this is simply P_γ with $\gamma = 0$ (so a similarly waked turbine with misalignment 0), but in literature P_0 is sometimes referring to an unwaked turbine. Better to make this explicit.

We agree with the reviewers suggestion, and have defined P_0 explicitly after Eq. 1

2. p4, l10: Mention that r is radial position and ψ is azimuthal angle.

r and ψ have now been defined below equation 2.

3. p4, Figure 1. This is a nice figure which clearly presents the trajectory of a blade segment. However, in this general theory section, it would be more appropriate to normalize the length scales in the figure by the rotor radius or diameter, instead of visualizing the concept for a specific turbine with rotor radius 45m. Further- more, the windspeed deficit could also be normalized with respect to freestream velocity.

The reviewer raises a good point regarding the dimensions used in Figure 1. The figure has been adjusted accordingly, and is non-dimensionalized in terms of rotor radius and the free wind speed.

4. p5, l9: Indicate clearly that the Lemma is in the appendix. Also, consider changing the order of Lemma A.2 and A.1 (A.2 is referred to earlier in the text than A.1)

This is a good suggestion. We have now directly referred to Appendix A, and have switched the order of the Lemmas for added clarity.

5. p6, l4. ρ and a are not defined.

Changed accordingly. Thank you.

6. Figure 4. The yellow color in the LES flow field is not present in the color scale. Are all figures plotted using the same color scale as shown on the right?

The color bar has been updated accordingly and the figure layout has been updated.

Typographical

1. p1, l17: includes include

2. p1, l24: show shown, misalignment misalignments
3. p1, l25: I think it's better to say either 'The probability ... was more than 25%', or 'The waked turbines were yaw misaligned more than 25% of the time', not both.
4. p2, l8: overestimate overestimates
5. p2, l20 includes include
6. p2, l5: wind wind wind
7. p2, l30: increase increased
8. p2, l31: its their
9. p3, l18: benefit by benefit from
10. p3, l19: focus focuses
11. p3, l24: need of need for
12. p3, l26: optimization optimisation (be consistent in American vs British English)

Thank you for the suggestions. The paper has been changed accordingly.

Reviewer 2

Major Comments

1. p.1-2: Here, the terms 'power-yaw sensitivity', 'power-yaw loss function' and 'power-yaw loss coefficient' are introduced. While the authors are establishing a name for the already existing description, the terminology is only used on the first two pages. I think that if one introduces new terms, one should consistently use them.

The reviewer raises an important concern regarding the naming convention of

the variable, α. The authors have chosen to rename the variable the 'power-yaw loss exponent' to better reflect its behaviour. This terminology has now been included throughout the document.

2. p. 2/ l. 3: P_0 is not introduced, please clarify whether P_0 is the power with respect to the free inflow or the inflow of the respective situation where the formula is applied (e.g. wake inflow).

We agree with the reviewers suggestion, and have defined P_0 explicitly after Eq. 1.

3. p. 2/ ll. 14: First, it is written, that 'The wake recovery rate is highly dependent on turbine spacing and ambient turbulence intensity' and directly afterwards it is mentioned that 'It was found that α is maximum for a turbine located approximately 4 rotor diameters (4D) downstream of another turbine when in a full wake situation' - as the ambient conditions determine the wake evolution, it should be pointed out that α is maximum 4D downstream in the situation that was investigated but that this distance might vary depending on the inflow conditions of the upstream turbine.

The reviewer raises an excellent point regarding the relationship between inflow conditions and the location of the maximum value of α. Indeed, the location of the peak is dependent on the atmospheric conditions, especially the turbulence intensity. We have updated the paragraph to make this point clear.

4. p. 3/ ll. 11: I think that this paragraph needs some work. First, this paragraph is together with the following paragraph used to motivate the necessity of considering a wake inflow for the calculation of α. However, the idea is discussed within the framework of wake steering, which is discussed in the previous paragraph. An integration into the previous paragraph could probably help the readability as one

aim of an optimized α is a higher precision of the power optimization procedures used for wake steering. Second, the 'trade off of wake steering' is a bit vague and also, the aim of wake steering is power production optimization. Third, the term 'power gain loss' is not clear to me in this context. Do you mean the trade-off between power losses due to yawing the upstream turbine and the power gain of the downstream turbine?

We agree with the reviewer that this paragraph can be integrated with the previous paragraph. This has been done in the revised version. Thank you for bringing our attention to the term 'power gain loss'. This was a typo, and has been corrected to 'power loss'.

5. p. 4/ l. 4: r_R is mentioned the first time here but introduced on page 5

We have now defined r and ψ before Equation (2)

6. p. 4/ fig. 1: Was this wind field generated by the DWM model? It should be specified that the inflow conditions of the wake generating turbine are uniform.

Fig 1 was indeed generated using the DWM model. This has now been mentioned in the caption, as well as its inflow conditions.

7. p. 5/ ll. 20: 'Fig. 2 shows the waked wind field for various downstream distances.' Figure 2 shows the radial variation of the wind speed and its derivative for different downstream positions in a wake generated by the DWM model.

Thank you for the suggestion. The document has been changed accordingly.

8. p. 7: While D is used as variable for the rotor diameter on page 2, it is used

here as variable for the downstream distance. As D is usually used for the rotor diameter, a different variable for the downstream distance should be used.

The reviewer raises a valid point regarding nomenclature. We have changed the notation for downstream distance to x.

9. p. 7: Methods: In the introduction, the four different test cases are explained. As the aeroelastic simulations are used to validate the analytical modes, I would mention this again here.

Thank you for the suggestion. The first paragraph of the Method section has been updated accordingly.

10. p. 8/ l. 12: I would mention that the wake deficit profile generated by the DWM model is depending on the downstream distance.

We have specified this now in the text.

11. p. 10/ l. 11: '...where N is the number of time steps in the LES wind field, and N is the desired azimuthal discretization (in this case, N = 500)' I guess that M is the number of time steps.

Thank you for bringing up the typographic error. M is actually the azimuthal discretization, and N is the number of time steps. This sentence has been updated accordingly.

12. p. 11/ l. 4: '... is present at a low turbine spacing between 3D and 5D' the maximum values are between 3D and 4D.

Thank you. Changed accordingly.

Minor Comments

1. p. 1/ll. 19: ' Part of the discrepancy and uncertainty might stem from unintentional yaw misalignment (or yaw error) of turbines inside wind farms.' This sounds like an assumption of the authors to explain the uncertainty. Connecting this idea with the fact that yaw misalignment occurs regularly (following sentence) could improve the readability: 'As unintentional yaw misalignment (or yaw error) of turbines inside wind farms occurs frequently, this could explain the discrepancy and uncertainty partially. For example, Mikkelsen et al. (2010) reported yaw error on a turbine in freestream wind conditions of up to 20^o during a measurement campaign of approximately 3 hours.'

2. p. 2/ l. 12:: a new paragraph for the discussion of the power-yaw loss coefficient with respect to wake inflows would emphasize the new focus.

3. p. 3/ l. 3: '...to determine the trade off of directing a wake' It would be nice if the sentence was more precise, e.g. '... to determine the trade-off between power losses due to yawing the upstream turbine and the power gain of the downstream turbine...'

4. p. 3/ l. 5: '... is adjusted based on the blade pitch angle of the yawed turbine' - while it should be clear from the context that the blade pitch is meant, I would specify this since new works on floating turbines discuss the pitch, yaw and roll movements of the turbine.

5. p. 7/ l. 16: states that downstream positions between 2D and 14D were investigated, but figure 6 does only show results from 3D downstream.

6. p. 8/ll. 5: 'the first set uses...', 'the second set uses...': I would prefer 'simulation'

over 'set'.

7. p. 8/ll. 7: 'downstream distance' and 'turbine spacing' are used synonymously; here I would prefer the term 'turbine spacing' over 'downstream distance'.

9. p. 8, 3.1.2 LES wake: – several parameters are not introduced (α (this variable should be renamed), ϵ, L and Γ, and R) 10. p. 9, fig. 4: the color bar depicting the wind speed deficit is incomplete as it lacks the yellow colors occurring in the LES results.

11. p. 9 – 3.2 Analytical calculation: As in 3.1, it was mentioned that cases (1) and (2) are explained, it should be mentioned, that in the following, cases (3) and (4) will be discussed.

12. p. 11/ l. 14: '...is due to the strong positive curvature of $U(r)$...' it could be added here 'is due to the strong positive curvature of $U(r)$ at small turbine spacings (cf. Fig. 2)'

13. p. 12/ l. 1: "'To highlight the significance of the results in Fig. 6, a wind farm layout consisting of two turbines with a spacing of 6D is considered' I would probably use a different formulation, for example 'To give an example of deviations in the power estimation that result from using a constant α as compared to using the new, adapted α, a wind farm layout consisting of two turbines with a spacing of 6D is considered'

14. p. 13/ ll. 21: 'theoretical formulation' - before, 'analytical model' was used.

Thank you for the suggestions. We have taken the suggestions into account and have changed the document accordingly.

Typographical/Grammar

If you list several sources, it would be nice to add an "'and' instead of a "';' as the separation between the last two sources.
p. 1/ l. 5: (+ other positions in the text) '"waked" does not exist in this context, instead of '"waked wind field", you could use '"wake" or in the context of this paper '"wake inflow"
p. 1/l. 2/3: '"the unintentional yaw misalignment increases for turbines operating in the wakes . . ."'
p. 1/l.18: 'Often, the wake effects and therefore the power production are not accurately modeled when employing engineering wake models which includes substantial uncertainty'
p. 1/ll. 21: 'However, McKay et al. (2013) have shown yaw misalignments of up to 35^o for turbines operating in ËŽ the wakes of aligned upstream turbines based on field measurement for a 6 month period'
p. 2/l. 5: 'free wind wind'
p. 3/l. 21: 'DWM model'
p. 4/ l. 6: 'as the yaw angle...'
p. 6/ l. 2: 'based on'
p. 7/ l. 4: ' is the power output of a turbine for a yaw misalignment γ and a down stream distance D'
p. 13 / l. 11: 'high fidelity wake profiles'

Thank you for the suggestions. We have taken the suggestions into account and have changed the document accordingly.

Please also note the supplement to this comment:
https://www.wind-energ-sci-discuss.net/wes-2019-65/wes-2019-65-AC1-supplement.pdf

Interactive comment on Wind Energ. Sci. Discuss., https://doi.org/10.5194/wes-2019-65, 2019.

Analytical model for the power-yaw sensitivity of wind turbines operating in full wake

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Abstract. Wind turbines are designed to align themselves with the incoming wind direction. However, turbines often experience unintentional yaw misalignment, which can significantly reduce the power production. The unintentional yaw misalignment ~~increase~~ increases for turbines operating in the wake of upstream turbines. Here, the combined effects of wakes and yaw misalignment are investigated~~with~~ , with a focus on the resulting reduction in power production. A model is developed, which considers the trajectory of each turbine blade element as it passes through the ~~waked wind field~~ wake inflow in order to determine a power-yaw loss ~~coefficient~~exponent. The simple model is verified using the HAWC2 aeroelastic code, where wake flow fields have been generated using both medium and high-fidelity computational fluid dynamics simulations. It is demonstrated that the spatial variation of the incoming wind field, due to the presence of ~~wake(s)~~wakes, plays a significant role in the power loss due to yaw misalignment. Results show that disregarding these effects on the power-yaw loss ~~coefficient~~ exponent can yield a 3.5% overestimation in the power production of a turbine misaligned by 30°. The presented analysis and model is relevant to low-fidelity wind farm optimization tools, which aim to capture the ~~effects of wake effects~~ combined effects of wakes and yaw misalignment as well as the uncertainty on power output.

1 Introduction

As the global wind energy sector continues to grow, there is a strong demand for a decreased levelized cost of energy. With this demand comes an increasing need for accurate and efficient computational tools, which are able to improve the design of wind farms and optimize annual energy production. In the early phases of wind farm design, optimization tools provide estimates of energy production and the costs during construction, installation or operation. The wind farm planning tools must account for the interactions between nearby wind turbines using wake models. Often, the wake effects and~~therefore~~ , therefore, the power production are not accurately modelled when employing engineering wake models~~and includes~~ , and include substantial uncertainty, see *e.g.* Nygaard (2015) and Peña et al. (2018). ~~Part of the discrepancy and uncertainty might stem from unintentional~~ Unintentional yaw misalignment (or yaw error) of turbines inside wind farms ~~.~~ occurs frequently, which can partially explain the discrepancy and uncertainty. For example, Mikkelsen et al. (2010) reported yaw ~~error~~ errors on a turbine in freestream wind conditions of up to 20° during a ~~measurement campaign of approximately~~ 3 ~~hours. However, McKay et al. (2013) has shown yaw misalignments~~ hour measurement campaign. McKay et al. (2013) have shown yaw misalignment of up to 35° for turbines operating in ~~wake~~ the wakes of aligned upstream turbines based on field ~~measurement~~ measurements for a 6 month period. Furthermore, it was ~~show~~ shown that the yaw misalignment ~~were~~ was accentuated further downstream for turbines affected by ~~more~~ multiple wakes. The probability of a ~~wake~~ waked turbine to be yaw misaligned $\pm 25°$ was more than 25%~~of the time~~.

Wind turbines which experience yaw misalignment show a reduction in power production. ~~The~~ This power sensitivity to yaw misalignment can be quantified by the power-yaw loss ~~function is often expressed as~~exponent, α, which is found in the expression:

$$\frac{P_\gamma}{P_0} = \cos^\alpha \gamma \tag{1}$$

where γ is the yaw misalignment angle between the turbine rotor and the ~~free wind wind,~~ local wind direction, and P_γ is the power generated by a wind turbine with a yaw misalignment of γ~~, and α is the~~ . P_0 is the power generated when the turbine experiences no yaw misalignment. Numerous values of the power-yaw ~~loss coefficient. Numerous α values~~ loss exponent, α, have been proposed in ~~the~~ literature. Based on Blade Element Momentum (BEM) theory, it is commonly concluded that $\alpha = 3$. However, experimental results have often ~~showed~~ shown that this value ~~overestimate~~ overestimates the power loss due to yaw (Aagaard Madsen et al., 2003). Schepers (2001) found experimentally that $\alpha = 1.8$, and Dahlberg and Montgomerie (2005) found a range between $\alpha = 1.88$ and $\alpha = 5.14$. Gebraad et al. (2016) uses a constant $\alpha = 1.88$, determined by using the wind farm simulator, SOWFA (Fleming et al., 2013). Medici (2005) found a value of $\alpha = 2$ from wind tunnel data. However, these considerations are simplified and only valid for free-stream conditions.

The investigation performed by Urbán et al. (2019) shows that yaw misalignment of a turbine in the wake of another turbine ~~,~~ can exhibit significant variations ~~in the value of α~~of the power-yaw loss exponent. In particular, α depends on the shape of the wake deficit profile, which evolves as it propagates downstream. The wake recovery rate is highly dependent on turbine spacing and ambient turbulence intensity. It was found that α is maximum for a turbine located approximately 4 rotor diameters ($4D$) downstream of another turbine when in a full wake situation. Furthermore, α tends to decrease rapidly as turbine spacing decreases below $4D$, while α converges slowly to a fixed value as turbine spacing increases.

Considering the findings in McKay et al. (2013), the implication of unintentional yaw misalignment can be significant for the total power production of large wind farms. Efforts to reduce the yaw error ~~includes~~ include improved measuring techniques for individual turbine control ~~, see~~ (*e.g.* Kragh et al. (2013) and Schlipf et al. (2013)), as well as farm level control, where the information on wind direction is shared between turbines in close proximity to improve the overall alignment ~~, see Annoni et al. (2019)~~(see Annoni et al. (2019)). However, it should be mentioned that part of the yaw misalignment compared to the free stream wind direction should not necessarily be considered a yaw error. The ~~turbines~~ turbine attempts to align itself with the local inflow direction to optimize the power production, where the presence of wake effects may alter the local flow wind direction. Such behaviour was also described by McKay et al. (2013) and shown experimentally by Bartl et al. (2018) as well as through the use of surrogate models based on high fidelity simulations in Hulsman et al. (2019), where ~~a optimization based on surrogate models showed that~~ the second turbine should indeed should align itself with the local wind direction to optimize power production. Archer and Vasel-Be-Hagh (2019) used LES to also show how turbines deep inside the farm could be intentionally yawed for improved performance.

In recent years, there ~~is an increase~~ has been an increased focus on applying control strategies for both ~~standalone~~ stand-alone wind turbines and entire wind farms to increase ~~its~~ operational performance. The main focus is generally on power optimization, for example, Knudsen et al. (2015) and Gebraad et al. (2015). A common form of wind farm control for power optimization is wake steering, in which a wake can be redirected away from a downstream turbine by inducing a yaw misalignment in the upstream turbine. Numerous studies on wake redirection have been performed by ~~Fleming et al. (2016); Gebraad et al. (2016, 2017); Jiménez~~ ~~,~~ Fleming et al. (2016), Gebraad et al. (2016), Gebraad et al. (2017), Jiménez et al. (2010), Bossanyi (2018), and Munters and Meyers (201 , showing improved annual energy production in wind farms ranging between 2% and 8%. These investigations often assume a constant value of α to determine the ~~trade off of directing a wake, with the exception of Munters and Meyers (2018)~~trade-off between power losses due to yawing the upstream turbine and the power gain of the downstream turbine. An exception to this can be found in Munters and Meyers (2018), who modelled the turbines as actuator disks ~~,~~ which could yaw, and Bossanyi (2018), where α is adjusted based on the blade pitch angle of the yawed turbine. Overlooking the causes and effects of a varying ~~α~~ power-yaw loss exponent becomes a problem in the framework of low fidelity wind farm optimisation, where the layout of the wind farm itself can change the values of α for each turbine. For example, both ~~Gebraad et al. (2017); Howland et al. (2019)~~ Gebraad et al. (2017), and Howland et al. (2019) demonstrate potential power increases in a wind farm by performing wake steering, where the analysis relies on a constant α despite the fact that some turbines are yawed in wake situations. It is beneficial to further investigate the behaviour of α in order to better predict the ~~trade off~~ trade-off of yaw steering, especially in the event that a turbine is yawed when operating in the wake of another turbine. The estimates of these power ~~gain~~ losses could benefit from a more accurate estimation of α by taking into account the increased uncertainty of yaw alignment when a turbine is in a wake.

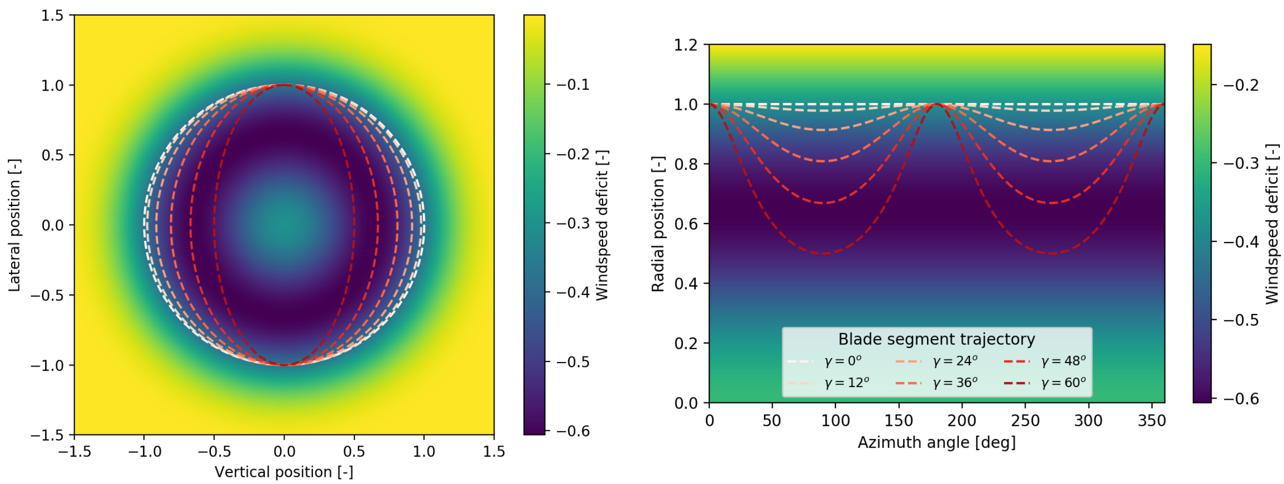
By overcoming the assumption of a constant ~~α~~power-yaw loss exponent, uncertainty in wind farm modelling tools can be decreased. Low fidelity wind farm optimisation frameworks such as TOPFARM (Réthoré et al., 2014), FLORIS (Fleming et al., 2017) or FarmFlow (Soleimanzadeh et al., 2012) could benefit ~~by~~ from including the presented model for estimating α, allowing for more accurate results. This paper ~~focus~~ focuses on the estimation of power loss of a wind turbine when yawed in wake, and aims to extend the work of Urbán et al. (2019), who used the Dynamic Wake ~~meandering~~ Meandering (DWM) model in conjunction with the aeroelastic tool, HAWC2, to study the effects of axisymmetric wake profiles on a misaligned wind turbine. In the presented work, the DWM generated wakes are validated against large eddy simulation (LES) generated wakes. Furthermore, an analytical formulation, based on concepts of blade element momentum (BEM) theory, is presented, which captures the behaviour of α in axisymmetric wake situations. The analytical formulation is able to estimate values of α rapidly, without the need ~~of aeroelastic simulations~~for aeroelastic simulations. The calculation of α for a range of turbine spacings can be performed in a seconds, whereas aeroelastic simulations performing the same task require a time frame in the order of hours. The analytical formulation is validated against simulations using the aeroelastic code , HAWC2, where the dynamic wake flow is generated using both DWM and LES. The formulation , an aeroelastic code which uses an unsteady BEM induction model for non-uniform inflow conditions (Madsen et al., 2019). The HAWC2 simulations are used in conjunction with both the DWM model and LES to generate the dynamic wake inflow. The presented analytical model can be used in existing wind farm optimization optimisation frameworks as a power correction for misaligned wind turbines in full wake scenarios.

2 Theory

When a downstream turbine in a full wake situation is perfectly aligned with the incoming wind, each blade segment follows a circular trajectory relative to the mean incoming wind direction. For misaligned cases, where the turbine is yawed, each blade segment follows an elliptical path, where the eccentricity of the ellipse increases with yaw angle (Fig. 1a). When these trajectories are plotted on an unfolded polar grid (Fig. 1b), it can be observed that the blade segment passes through different regions of the ~~waked wind field. As~~ wake inflow. As the yaw angle increases, all blade segments on a rotor experience flow near the wake center for an increasing period of time. This suggests that the spatial distribution of the wake profile could have an effect on how the power output of a turbine changes with yaw angle. It is therefore proposed that the power output contribution of each blade segment depends on the average wind speed experienced as a result of following a trajectory through a nonuniform wind field. It is convenient to define a transformation between rotor coordinates (r_R, ψ_R) and meteorological coordinates, (r_m, ψ_m) where r and ψ are the radial position and azimuth angle, respectively. The transformation is based on the definition of an ellipse~~as follows~~:

$$\begin{bmatrix} r_m \\ \psi_m \end{bmatrix} = \begin{bmatrix} r_R \frac{\cos\gamma}{\sqrt{1-\sin^2\gamma\cos^2\psi_R}} \\ \psi_R \end{bmatrix} \quad (2)$$

where the semi-major ~~axis~~ and semi-minor axes of the ellipse ~~is~~ are r_R and $r_R\cos(\psi)$ respectively, and the eccentricity is $\sin(\gamma)$.



(a) Cartesian representation.

(b) Polar representation.

Figure 1. The trajectory of a blade segment ~~close to the tip ($r = 45m$)~~ through a ~~waked wind field~~ wake inflow at varying yaw angles, represented in Cartesian coordinates (a) and unfolded polar coordinates (b) (generated using DWM, $U = 10ms^{-1}$, $x = 3D$).

2.1 Blade segment effective wind speed in an axisymmetric wake

This section introduces the concept of blade segment effective wind speed. For a blade segment located at radius, r_R, the blade segment effective wind speed, $\bar{U}(r_R)$, is defined as the expected value of wind speed experienced by the blade segment as it follows a trajectory through the wind field:

$$\bar{U}(r_R) = \mathbb{E}\left[U(r_m)\right] \tag{3}$$

where $\mathbb{E}[.]$ is the expected value function, and $U(r_m)$ is assumed to be axisymmetric as displayed in Fig. 1, and is therefore only a function of radius in meteorological coordinates. One way of expressing Eq. (3), assuming the blade segment trajectory is an ellipse as described in Fig. 1, is (~~Lemma A.1~~see Lemma A.1 in Appendix A):

$$\bar{U}(r_R) = \mathbb{E}\left[U(r_m)\right] = \underbrace{U(r_R)}_{\text{uniform velocity}} - \underbrace{\int\limits_{r_R\cos\gamma}^{r_R} \frac{dU(\rho)}{d\rho} F_{r_m}(\rho)d\rho}_{\text{added velocity}} \tag{4}$$

where $F_{r_m}(r_m)$ is the cumulative density function of r_m from equation (2) (~~Lemma A.2~~see Lemma A.2 in Appendix A):

$$F_{r_m}(r_m) = \begin{cases} 0 & r_m \leq r_R\cos\gamma \\ 1 - \frac{2}{\pi}\arccos\left(\sqrt{\frac{r_m^2 - r_R^2\cos^2\gamma}{r_m^2\sin^2\gamma}}\right) & r_R\cos\gamma < r_m < r_R \\ 1 & r_m \geq r_R \end{cases} \tag{5}$$

From the formulation in (4), it can be observed that the blade segment effective wind speed consists of two additive components. The uniform velocity depends on the wind speed at the rotor radius, whereas an added velocity component depends on radial variations (dU/dr) in the wind field. In a uniform wind field, where $dU/dr = 0$, the blade segment effective wind speed remains unchanged when the turbine is yawed. In a nonuniform wind field, the sign of dU/dr determines if the added velocity provides a surplus or a deficit to the blade segment effective wind speed. For instance, Fig. 2 shows the ~~waked wind field for various downstream distances~~radial variation of the wind speed and its derivative for different downstream positions in a wake generated by the DWM model. When the radial wind field function decreases with radius ($dU/dr < 0$), such as in the near wake, the blade segment effective wind speed increases. The opposite occurs when the radial wind field function increases with radius. Therefore, given $U(r_m)$, it is possible to explicitly calculate $\bar{U}(r_R)$ given a value of γ and r_R by solving (4).

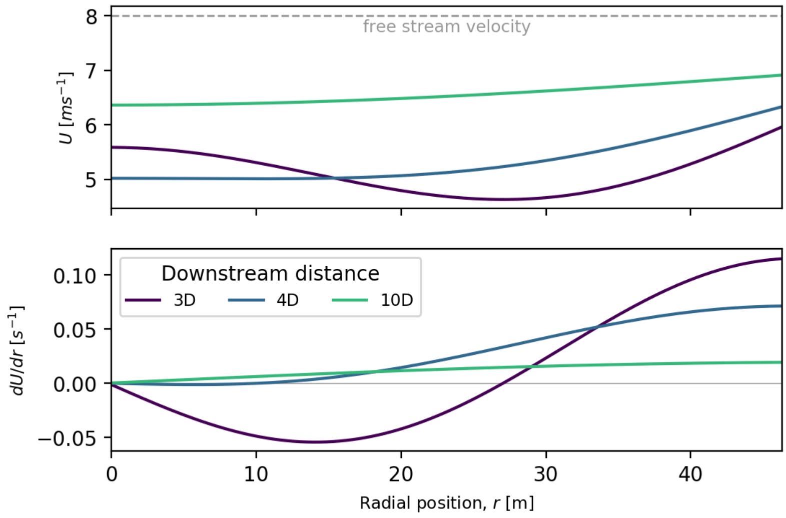


Figure 2. Radial functions of wind speed deficit and its derivative for varying ~~downstream~~ turbine spacing distances (generated using DWM).

2.2 Modified BEM formulation of wind turbine power in steady yaw and axisymmetric wake

Based ~~of~~ on Blade Element Momentum (BEM) theory,

$$\frac{\partial P}{\partial r_R} = A r_R U_\infty^3 \tag{6}$$

where $A = \frac{1}{2}\rho 2\pi 4a(1-a)^2$ is assumed to be constant, where ρ is the density of air and a is the axial induction. In order to stay consistent with the definition of α in (1), as well as the assumption that each blade segment experiences a blade segment effective wind speed, it is proposed that (6) is modified to:

$$\frac{\partial P}{\partial r_R} = A r_R \cos^{\alpha_0}\gamma \bar{U}_\gamma^3(r_R) \tag{7}$$

where α_0 is the ~~α fit of~~ power-yaw loss exponent from (1) for a turbine in free stream conditions. Integrating (7) over the length of the blade gives the total power output of the turbine:

$$P = A\cos^{\alpha_0}\gamma \int_0^{\sout{R}\,D/2} r_R \bar{U}_\gamma^3(r_R)\,dr_R \tag{8}$$

Therefore the power ratio defined on the left hand side of (1) is:

$$\frac{P_\gamma}{P_0} = \sout{\frac{\cos^{\alpha_0}\gamma \int_0^R r_R \bar{U}_\gamma^3(r_R)\,dr_R}{\int_0^R r_R \bar{U}_0^3(r_R)\,dr_R}}\ \frac{\cos^{\alpha_0}\gamma \int_0^{D/2} r_R \bar{U}_\gamma^3(r_R)\,dr_R}{\int_0^{D/2} r_R \bar{U}_0^3(r_R)\,dr_R} \tag{9}$$

It is therefore possible to determine a value of α from (1) which best fits (9) using curve fitting methods. This is achieved in this investigation using a least-squares optimisation:

$$\alpha(Dx) = \arg\min_{\alpha} \left(\frac{P_{\gamma|D}}{P_{0|D}} \frac{P_{\gamma|x}}{P_{0|x}} - \cos^{\alpha}\gamma \right)^2 \tag{10}$$

where $P_{\gamma|D}$ $P_{\gamma|x}$ is the power output if a turbine fora of a turbine for a yaw misalignment, γ and a downstream distance, Dturbine spacing, x. This analytical approximation of α gives an estimate for a turbine's power sensitivity to yawing while in full wake conditions. The inclusion of α_0 in (9) ensures P_γ/P_0 converges to the free stream value as turbine spacing becomes large and wake effects dissipate. Additionally, if the turbine faces a uniform wind field, then $\alpha = \alpha_0$.

3 Method

To determine the value of α for varying downstream distancesturbine spacing, four methods are used to estimate the relative power production when a turbine is yawed in a wake situation: (1) aeroelastic simulations with DWM-generated wakes, (2) aeroelastic simulations with LES-generated wakes (3) analytical model with DWM-generated wakes and (4) analytical model with LES-generated wakes.

The aeroelastic simulations are used to validate the results produced by the analytical model. Additionally, the inclusion of LES-generated wakes in this investigation verifies the results of the DWM-generated wake, which is unable to capture the behaviour of a wake in as much detail as LES. Each of the model-simulations model-simulation combinations aim to determine the power output $P_{\gamma|D}$$P_{\gamma|x}$, of the downstream wind turbine with yaw misalignment of γ in the full wake of an upstream turbine located distance D apart, at a distance, x, apart as illustrated in Fig. 3.

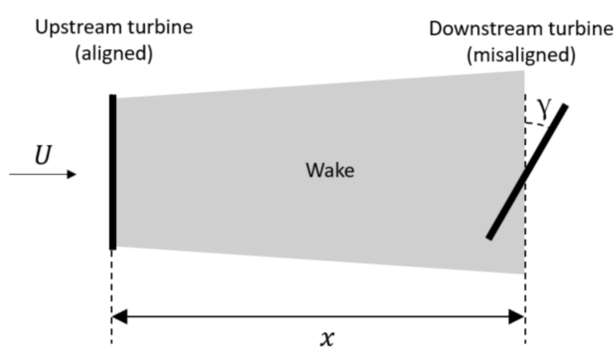


Figure 3. Wind turbine layout used in analysis.

To ensure that the combination of wake generation and simulation tools produce comparable results, the free stream wind speed is fixed at 8m/s $8ms^{-1}$ with an ambient turbulence intensity of 6% and a shear exponent of 0.14. The free variables, D x and γ, are varied over the ranges of $2D$ $3D$ to $14D$ and -30^o to 30^o, respectively. The α coefficient exponent is determined for each downstream turbine spacing distance for the four model-calculation combinations by performing the curve fitting in accordance with the definition of α in Eq. (1).

3.1 Aeroelastic Simulation

The aeroelastic simulations (1) and (2) are run using the aeroelastic code HAWC2 (Larsen and Hansen, 2007) using a 2.3MW turbine with a diameter of 96.2m and operating in full wake. The wake generating turbine is similar and has a fixed rotor speed of 1.37 rad/s rads^{-1} and blade pitch angle of -1^o to reflect the mean operating conditions at 8m/s$8ms^{-1}$, which was previously obtained based on the flow conditions defined below. The first set uses Simulation (1) use the DWM model to generate the wake on the target turbine as performed in Urbán et al. (2019). The second set uses a LES generated Simulation (2) use an LES-generated wake as the input wind field for the aeroelastic simulations which includes the wake dynamics. From the simulations, the mean power output is obtained for different downstream distances turbine spacing and misalignment angles. The results are used to calculate the α coefficient for each downstream distance power-yaw loss exponent for each turbine spacing using (10).

3.1.1 DWM DWM-generated wake

The dynamic wake meandering model, as described by Larsen et al. (2008) is used in combination with HAWC2to mimic the wake effects. The DWM model unifies three key components of wake generation in a computationally efficient manner. These components are the wake deficit profile, the added turbulence profile and wake meandering. The DWM model produces an axisymmetric wake profile for each downstream distance using the thrust properties of the upstream turbine. Added wake turbulence is superimposed over the wake profile, and the axisymmetric wake profile is translated to mimic the effects of wake meandering as described in Madsen et al. (2010), and shown in Fig. ??4(e). The implementation of the DWM model in HAWC2 has been validated against field data in Larsen et al. (2013). Additionally, a steady variation of the DWM model used in Section 3.2.1 has been validated in Keck (2015).

3.1.2 LES LES-generated wake

The turbine and its wake is simulated using the incompressible Navier-Stokes solver EllipSys3D coupled with the aeroelastic tool Flex5 through the actuator line method. EllipSys3D is based on a finite volume approach with general curvilinear co-ordinates , see Michelsen (1992) and by Sørensen (1995)(Michelsen (1992) and Sørensen (1995)). The actuator line method as developed by Sørensen and Shen (2002) applies body forces along rotating lines to simulate the presence of the turbine within the flow domain. The position of the rotating lines and applied body forces are determined through the aeroelastic tool, Flex5 by Øye (1996), which gives forces and deflections of the turbine. The effects of atmospheric boundary layer and inflow turbulence are also included using body forces. The atmospheric boundary layer is modelled with a shear exponent of 0.14. The inflow turbulence is generated using a Mann box ~~, see~~ (Mann (1994) and Mann (1998)). The boxes are generated using ~~$\alpha\epsilon^{2/3} = 0.01$,~~ the following inputs. $\phi\epsilon^{2/3} = 0.01$, where ϕ[1] is the spectral Kolmogorov constant ϵ is the specific rate of turbulent dissipation. Additionally, a turbulent length scale $L = 50$, and $\Gamma = 3.2$, which ~~results~~ describes the anisotropy of the generated turbulence, are used. These parameters result in a turbulence intensity of approximately 6%. For additional details on the numerical framework, please see Sørensen et al. (2015). The turbine and its wake is simulated in a domain of ~~$20R \times 20R \times 40R$~~ $10D \times 10D \times 20D$ in the lateral, vertical, and streamwise directions. The turbine is placed ~~in $(10R, 1.3729R, 11R)$~~ at $(5D, 0.6865D, 5.5D)$ and each blade is resolved by 27 cells. The wind fields consisting of all three velocity components are extracted for every ~~$1R$~~ $0.5D$ in the wake behind the turbine. These flow fields are used as input to HAWC2 and compared to the wind fields generated using the DWM model as described previously. The wake profiles extracted from the LES framework are expected to be more realistic given that it includes the asymmetric effects of shear on the wake as well as the nonlinear interactions in a dynamic wake inherent to the flow. These effect are visualized in ~~??~~Fig. 4(c), where the asymmetry and different turbulent structures are more realistic compared the DWM model in ~~??~~Fig. 4(e).

3.2 Analytical calculation

~~Using~~ Simulations (3) and (4) are performed using the wake profiles generated by a standalone version of the DWM model and the time-averaged LES wake deficit profiles, as well as the analytical formulation described in Section 2~~is solved to estimate α without the use of aeroelastic simulations~~. The radial wind function, $U(r)$ is extracted from the wake profiles ~~at varying downstream distances~~for varying turbine spacing, shown in ~~??. Equation~~ Fig. 4(a) and 4(b). Eq. (4) and (9) are solved using numerical differentiation and integration techniques, and the α fit is determined using Eq. (10).

[1] Usually, the spectral Kolmogorov constant is denoted by α, but has here been represented with ϕ to avoid confusion with the power-yaw loss exponent.

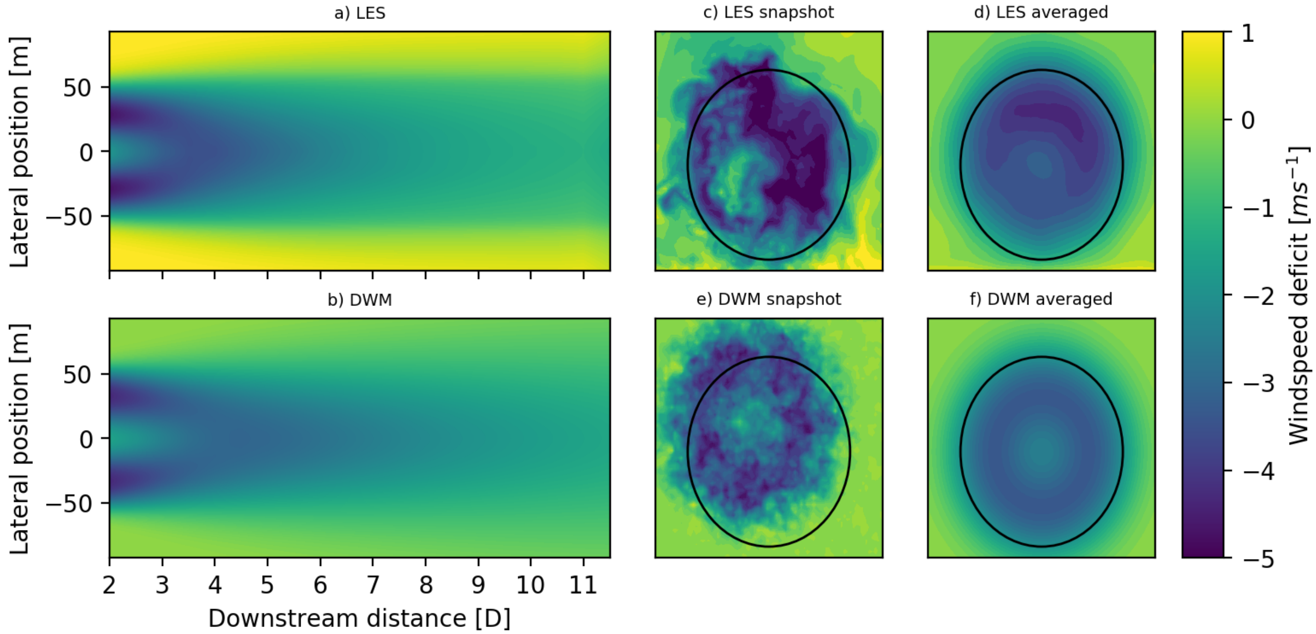


Figure 4. ~~Averaged~~ Visualizations of the LES and DWM wakes represented as the averaged wake downstream evolution (a, b), wake profile snapshots at $x = 3D$ (c, e), and wake averaged wake profiles at $x = 3D$ (d, f).
~~Wake profiles Visualisations of wake profiles generated using DWM and LES methods.~~

3.2.1 ~~DWM~~ DWM-generated wake

The DWM model, originally coded within HAWC2, has been externalized for its ~~further~~ use within optimization problems, which results in fast and accurate estimations of a wake profile for a given radial thrust distribution, ambient turbulence intensity and ~~downstream distance~~ turbine spacing (DTU Wind Energy, 2019). A steady wake profile, ~~$U_{DWM}(r)$~~ $U_{\mathrm{DWM}}(r)$ is obtained directly from the DWM code. To take into account meandering, ~~$U_{DWM}(r)$~~ $U_{\mathrm{DWM}}(r)$ is adjusted by applying a Gaussian smearing using a similar method to Keck (2015), where the spread of the Gaussian captures the standard deviation of the wake meandering motion~~.~~:

$$U_{meander}(r_m) = U_{DWM\,\mathrm{DWM}}(r_m) * \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{r^2}{2\sigma^2} - \frac{r_m^2}{2\sigma^2}}\right) \quad (11)$$

where $*$ is the linear convolution operator. Through a parametric study using the HAWC2 DWM model, the relation ~~$\sigma = 1.493D$~~ $\sigma = 1.493x$ was found to fit best when describing the standard deviation of the wake meandering path. As a result, an axisymmetric, time invariant wake profile is produced (Fig. ~~??~~4(f)), which is used in the analytical ~~formulation.~~ model.

3.2.2 ~~LES~~ LES-generated wake

The wake wind field is preconditioned before being used in the analytical formulation by removing the shear profile. This is achieved by subtracting the mean wind field ~~1D~~ $1D$ upstream of the wake generating turbine from the downstream wind field.

Unlike the DWM model, which fully describes the radial wind function, the LES wake at a particular downstream distance is described as a time varying 2 dimensional wind field, $f(x,y,t)$. The mean radial wind speed function is calculated by performing an azimuthal average as:

$$U(r_m) = \frac{1}{NM}\sum_{i=0}^{N}\sum_{j=0}^{M} f(r_m \sin\theta_j, r_m \cos\theta_j, t_i) \qquad \text{where } \theta_j = 2j\pi \tag{12}$$

where N is the number of time steps in the LES wind field, and ~~N~~ M is the desired azimuthal ~~discretisation~~ discretization (in this case, ~~$N = 500$~~$M = 500$). The time- and azimuthally-averaged wake profile, shown in Fig. ~~??~~4(d), produces a comparable wake profile to that generated by the DWM model~~.~~, however it can be seen in Fig. 4(a) and 4(b) that the LES wake dissipates at a slightly shorter downstream distance than the DWM wake.

4 Results

~~Figure~~ Fig. 5 presents the power output, ~~normalised~~ normalized with the aligned case, as a function of yaw angle. The expected concave relation between yaw angle and power output is observed and the cosine fit described in (10) is performed. All four methods presented in Fig. 5 present varying curvature of the ~~yaw-power relationship as downstream distance~~ power-yaw relationship as the turbine spacing changes. For instance, the difference in curvature can be clearly observed in the LES simulation results (right panels of Fig. 5). The same effect can be observed to a lesser extent for the DWM simulation results on the left of Fig. 5~~)~~.

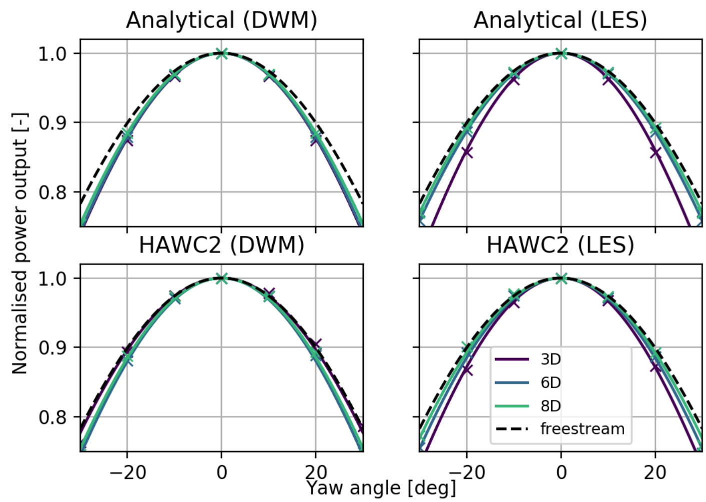


Figure 5. ~~Normalised~~ Normalized power output as function of yaw angle for different ~~downstream distances~~turbine spacing. Both calculation methods and wake generation tools are presented. The markers indicate the sample points used in the cosine fitting.

Although the variations of the power-yaw relation can be observed qualitatively in Fig. 5, it is insufficient at capturing the effect of ~~downstream distance~~ turbine spacing on the power-yaw relation in a quantitative sense. The value of the power-yaw loss exponent, α, is therefore presented in Fig. 6 as a function of ~~downstream distance~~turbine spacing. It is possible to observe that the maximum α value, for both DWM and LES generated wakes, is present at a low turbine spacing between ~~3D and 5D~~$3D$ and $4D$. As turbine spacing increases and the wake dissipates, α converges to the free stream value~~as the wake dissipates~~. The free-stream value of the power-yaw loss exponent was found to be 1.7 in the HAWC2 simulations, using both Mann generated and LES generated turbulence fields.

The analytical ~~formulation~~ model shows good overall agreement with the aeroelastic simulations. ~~At downstream distances between 5D and 8D~~for a turbine spacing between $5D$ and $8D$, the relative difference between the analytical estimation of α and its respective aeroelastic simulation result is up to 2.7%. For the far-wake region at distances larger than ~~8D~~$8D$, the analytical model and simulations show a lower relative difference in α of 0.4%. The agreement between aeroelastic simulations and the analytical model is weaker in the near wake scenarios, ~~but~~ however, such low turbine spacings are rare in practice. Nevertheless, the general trend of an increased ~~α~~ power-yaw loss exponent is still captured in this region for all four methods.

The maximum value of α at approximately ~~3D to 4D~~ $3D$ to $4D$ is due to the strong positive curvature of $U(r)$ ~~, leading to a maximum sensitivity to yaw misalignment~~at small turbine spacings. As the wake recovers further downstream, this positive slope diminishes, and so α slowly converges to its free stream value. The ~~downstream distance~~ turbine spacing at which α peaks is closely related to the breakdown point, where the wake transitions from near wake to far wake ~~, see Sørensen et al. (2015)~~

(Sørensen et al. (2015)). In terms of $U(r)$, this point approximately corresponds to the downstream distance at which there is no longer a negative slope. The location of the α peak is dependant on the inflow conditions. In particular, a higher turbulence intensity will cause α to reach a maximum value at a lower turbine spacing due to a faster breakdown of the wake.

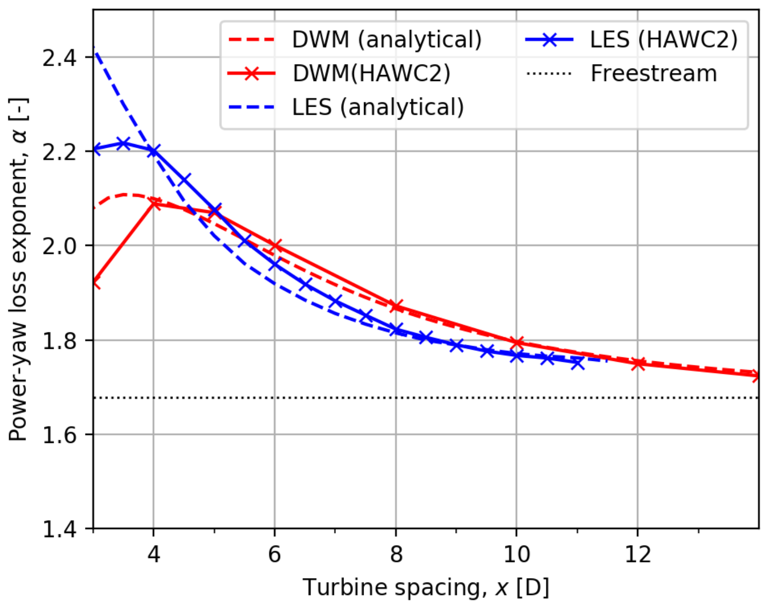


Figure 6. ~~α~~ Power-yaw loss exponent as a function of turbine spacing for the four power calculation methods presented in this investigation.

~~To highlight the significance of the results in Fig. 6,~~ An example of deviations in the power estimation that result from using a constant α compared to using the new, adapted α, is given in a wind farm layout consisting of two turbines with a spacing of ~~6D is considered~~$6D$. Table 1 compares the ~~normalised~~ normalized power output of the downstream turbine using $\alpha = 2.0$ based on the results shown in Fig. 6, and $\alpha = \alpha_0 = 1.7$, which corresponds to the free stream value shown previously. It can be seen that applying the free stream value of α causes an overestimation of the power output when a downstream turbine is yawed, which increases with increasing yaw misalignment. Using typical values of yaw misalignment during wake steering (McKay et al., 2013), it is possible to experience a 3.5% overestimation of the power output for a single turbine at 30^o yaw misalignment. Hence, this effect can significantly change the outcome of full wind farm layout optimizations when including the wind direction uncertainty and ~~particular when applying~~ particularly when attempting to develop wind farm control including intentionally yaw misaligned turbines in the interior of wind farms.

Table 1. Relative power output due to yaw misalignment for a downstream turbine located $6D$ downstream for $\alpha = 2.0$ and $\alpha = \alpha_0$

	Relative power (P_γ/P_0)		
Yaw misalignment	$\alpha = 1.7$ (free stream)	$\alpha = 2.0$	difference
10^o	97.5%	97.0%	-0.5%
20^o	90.1%	88.3%	-1.8%
30^o	78.5%	75.0%	-3.5%

5 Discussion

The estimation of α shows discrepancies in the near wake region depending on the choice of wake generation method (LES or DWM). There are a number of potential sources ~~of~~ for this discrepancy. Firstly, the two wake models, although having equal ambient conditions, present slight differences in the rate of mixing due to model differences. For this reason, the ~~break down~~ breakdown location of the LES wake appears at a shorter downstream distance than the DWM wake, which explains the α peak occurring at a ~~shorter downstream distance~~smaller turbine spacing. Secondly, The LES wake is subject to effects not present in the DWM wake, causing differences in the azimuthal and time averaging. These factors include tip vortices, wake rotation and ground effects.

Although the analytical ~~method~~ model presented does not consider some physical effects, such as tip losses or rotor induction, the method shows close agreement with aeroelastic simulations in estimating ~~α~~the power-yaw loss exponent. The results are further reinforced by being able to capture the behaviour of α for both medium and high fidelity wake ~~profile~~profiles. This provides a correction for which power output can be adjusted for better estimations. It should be noted that the effects of rotor induction on the ~~waked~~ wake inflow are not considered in the analytical model, HAWC2, as well as both DWM and LES generated wakes.

The investigation is limited to full wake situations~~,~~; however, by using the azimuthal-time averaging method described in Eq. (12), it is possible to extend the formulation for asymmetric, curled, or partial wake cases in future work.

6 Conclusions

This paper establishes the link between wake effects and the power sensitivity to yaw misalignment in a wind turbine, quantified by the ~~variable~~power-yaw loss exponent, α. A clear trend is found in α through the analysis of HAWC2 aeroelastic simulations using both DWM and LES generated wake flow fields. Namely, α is largest for turbines operating in the near wake region, and α converges to its free stream value as turbine spacing increases. These trends are correctly captured by ~~a theoretical formulation~~ the analytical model for α presented in this paper. The theoretical formulation correctly anticipates the peak value of α, where the wake breaks down, and also converges to the free stream conditions for large ~~downstream~~ turbine spacing distances. The model shows how neglecting the influence of the wake on α can result in power production overestimation up to 3.5% for a yaw misalignment of 30°.

The simplified model presented in this paper, provides a quick and reliable method to calculate α, which can be used for optimisation the optimization of wind farm layouts , which includes while including the uncertainty in the yaw misalignment of wind turbines operating in wake.

Appendix A: Blade segment effective wind speed derivation

~~Let $\Psi \in [0,\pi]$ be a uniformly distributed random variable with the cumulative density function,~~

$$F_\Psi(\psi) = P(\Psi \le \psi) = \begin{cases} 0 & \psi \le 0 \\ \frac{\psi}{\pi} & 0 < \psi < \pi \\ 1 & \psi \ge \pi \end{cases}$$

~~Let r_m be a random variable defined as $r_m = g(\Psi)$ where~~

$$g(\Psi) = \frac{R\cos\gamma}{\sqrt{1-\sin^2\gamma\cos^2\Psi}}$$

~~where $R \in \mathbb{R}^+$, $\gamma \in [-\pi,\pi]$. The cumulative distribution function, $F_{r_m}(\rho) = P(r_m \le \rho)$, of r_m is:~~

$$F_{r_m}(\rho) = \begin{cases} 0 & \rho \le R\cos\gamma \\ 1-\frac{2}{\pi}\arccos\left(\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) & R\cos\gamma < \rho < R \\ 1 & \rho \ge R \end{cases}$$

$$\begin{aligned}
F_{r_m}(\rho) &= P\left(r_m \le \rho\right) \\
&= P\left(g(\Psi) \le \rho\right) \\
&= P\left(\frac{R\cos\gamma}{\sqrt{1-\sin^2\gamma\cos^2\Psi}} \le \rho\right) \\
&= P\left(\cos^2\Psi \le \frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}\right) \\
&= P\left(-\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}} \le \cos\Psi \le \sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) \\
&= P\left(\arccos\left(-\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) \ge \Psi \ge \arccos\left(\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right)\right) \\
&= \frac{\arccos\left(-\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) - \arccos\left(\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right)}{\pi} \\
&= 1-\frac{2}{\pi}\arccos\left(\sqrt{\frac{\rho^2-R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right)
\end{aligned}$$

Lemma A.1. *The expected value, $\mathbb{E}[.]$, of a function, $U(r_m)$, is:*

$$\mathbb{E}\left[U(r_m)\right] = U(R\, r_R) - \int_{R\cos\gamma\; r_R\cos\gamma}^{R\; r_R} \frac{dU(\rho)}{d\rho} F_{r_m}(\rho)d\rho \qquad \text{(A1)}$$

where $F_{r_m}(r_m)$ is the cumulative density function of r_m.

Proof. The expected value of the random variable, U is defined as:

$$\mathbb{E}[U] = \int_{-\infty}^{\infty} U f_U(U) dU \tag{A2}$$

where $f_U(U)$ is the probability density function of U. Given that U is a function of radial position, $U(r_m)$, by the law of the unconscious statistician, Eq. (A2) can be written as:

$$\mathbb{E}[U(r_m)] = \int_{r_R \cos\gamma}^{r_R} U(\rho) \frac{dF_{r_m}(\rho)}{d\rho} d\rho \tag{A3}$$

The range of r_m is $(r_R \cos\gamma, r_R)$ as determined from the transformation in Eq. (2), hence the change in the integration limits. Integrating by parts gives:

$$\mathbb{E}[U(r_m)] = U(\rho) F_{r_m} \Big|_{r_R \cos\gamma}^{r_R} - \int_{r_R \cos\gamma}^{r_R} \frac{dU(\rho)}{d\rho} F_{r_m}(\rho) d\rho \tag{A4}$$

$$\mathbb{E}[U(r_m)] = U(r_R) - \int_{r_R \cos\gamma}^{r_R} \frac{dU(\rho)}{d\rho} F_{r_m}(\rho) d\rho \tag{A5}$$

□

Lemma A.2. *Let $\Psi \in [0, \pi]$ be a uniformly distributed random variable with the cumulative density function,*

$$F_\Psi(\psi) = P(\Psi \leq \psi) = \begin{cases} 0 & \psi \leq 0 \\ \frac{\psi}{\pi} & 0 < \psi < \pi \\ 1 & \psi \geq \pi \end{cases} \tag{A6}$$

Let r_m be a random variable defined as $r_m = g(\Psi)$ where

$$g(\Psi) = \frac{r_R \cos\gamma}{\sqrt{1 - \sin^2\gamma \cos^2\Psi}} \tag{A7}$$

where $r_R \in \mathbb{R}^+$, $\gamma \in [-\pi, \pi]$. The cumulative distribution function, $F_{r_m}(\rho) = P(r_m \leq \rho)$, of r_m is:

$$F_{r_m}(\rho) = \begin{cases} 0 & \rho \leq r_R \cos\gamma \\ 1 - \frac{2}{\pi} \arccos\left(\sqrt{\frac{\rho^2 - r_R^2 \cos^2\gamma}{\rho^2 \sin^2\gamma}}\right) & r_R \cos\gamma < \rho < r_R \\ 1 & \rho \geq r_R \end{cases} \tag{A8}$$

Proof. The cumulative distribution function of the random variable r_m is given by:

$$F_{r_m}(\rho) = P(r_m \leq \rho) \tag{A9}$$

$$= P(g(\Psi) \leq \rho) \tag{A10}$$

$$= P\left(\frac{r_R \cos\gamma}{\sqrt{1-\sin^2\gamma\cos^2\Psi}} \leq \rho\right) \tag{A11}$$

The steps to isolate Ψ yield the following:

$$F_{r_m}(\rho) = P\left(\cos^2\Psi \leq \frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}\right) \tag{A12}$$

$$= P\left(-\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}} \leq \cos\Psi \leq \sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) \tag{A13}$$

$$= P\left(\arccos\left(-\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) \geq \Psi \geq \arccos\left(\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right)\right) \tag{A14}$$

(A15)

From (A6):

$$F_{r_m}(\rho) = \frac{\arccos\left(-\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) - \arccos\left(\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right)}{\pi} \tag{A16}$$

(A17)

Using the identity $\arccos(-x) = \pi - \arccos(x)$ yields:

$$F_{r_m}(\rho) = 1 - \frac{2}{\pi}\arccos\left(\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) \tag{A18}$$

Finally, from (A7), the range of r_m is $[r_R\cos(\gamma), r_R]$, yielding the final expression:

$$F_{r_m}(\rho) = \begin{cases} 0 & \rho \leq r_R\cos\gamma \\ 1 - \frac{2}{\pi}\arccos\left(\sqrt{\frac{\rho^2 - r_R^2\cos^2\gamma}{\rho^2\sin^2\gamma}}\right) & r_R\cos\gamma < \rho < r_R \\ 1 & \rho \geq r_R \end{cases} \tag{A19}$$

□

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