Author's response to Referee #1

May 21, 2019

Thank you for the detailed review of the manuscript. In the following I will comment on each point. The referee's comments will be repeated in blue italic before the answer.

General comments

The manuscript by Mauz et al. describes in-situ measurements of wind turbine tip vortices with a UAS. From these measurements, circulation of the vortices is calculated using the Burnham-Hallock wake vortex model. These measurements are unique and I do not know of any other study in which a UAS operated in such close proximity to an operating wind turbine and even in its wake. The authors can convincingly show that wake vortices can be measured with the UAS. The presentation of the methods of analysis and the results however needs some significant improvement:

- *In the introduction and throughout the text I miss more thorough references to the state-of-the-art. For example, other methods to measure full-scale wind turbine wakes with remote sensing are not mentioned at all, but are carried out all the time and in multiple ways. What can a UAS do that is not possible with remote sensing?*
  Remote sensing methods (e.g. LIDAR) can not resolve turbulence in such a small scale as a UAS is capable of. LIDARs usually cycle cone measurements that resemble averages over a huge volume compared to a UAS line measurement. However, the method of operation allows for long term measurements whereas UAS excels at in-situ small scale measurements. Appropriate literature will be added. The lack of references is also contributed to the lack of publications of remote sensing experiments that try to resolve small scale turbulence in wakes. The wakes itself can be measured by LIDAR but focus of this manuscript is the identification of tip vortices that only have been verified qualitatively by Subramanian et al.

- *The structure of the text is sometimes confusing and much information is given in the results and discussion and outlook section that should have been introduced before. I provide details in the specific comments.*
  Thank you for the feedback. We will try to improve the structure of the text. Also by implementing the specified suggestions.

- *A major problem of the manuscript is that all the analysis is only done for a single sample of a wake vortex pair. If possible at all I would strongly urge the authors to investigate if they can maybe use some other flights. Maybe even flight levels above or below hub height could still be valuable.*
  Additional vortex measurements are available. In those flight legs usually the vortex pairs are visible. However, an evaluation of the core radius and vortex strength is mainly not possible due to the distance of the UAS being larger than r_c. From all the available data only one flight leg showed two vortices were the evaluation method presented in the manuscript was feasible.
  The aim of this manuscript is to establish an evaluation method for MASC-3 to later examine future wake measurements and then be able to establish a statistic for near wake vortex behaviour (e.g. turbulence and stratification dependent).

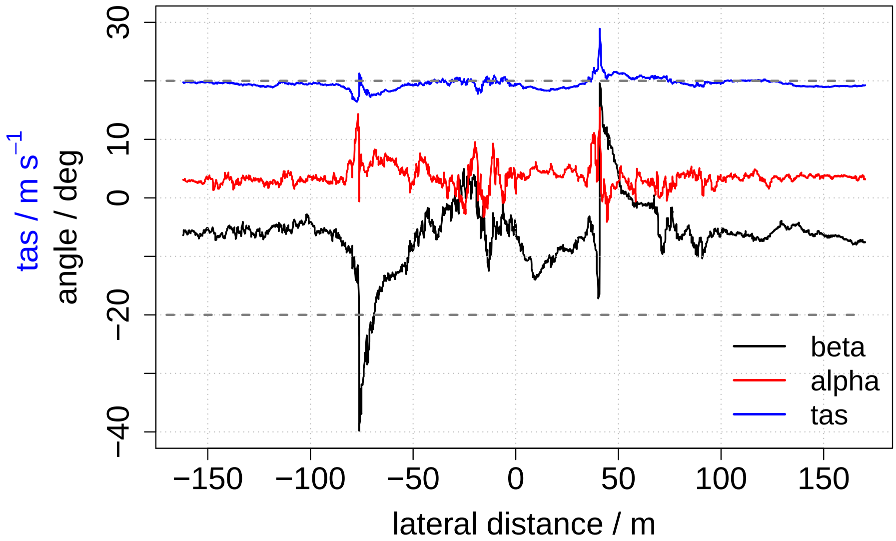

Figure 1: Wind angles and true air speed (TAS) of the 5-hole probe for the evaluated vortex measurements. Grey dashed line shows the calibration limit of -20 to 20 degrees. Overstepped angles are interpolated.

- *It is known that the estimation of the UAV attitude is a major source of error for the wind calculation. It is also known that navigation systems are less precise in dynamic flight manoeuvrers. I would therefore at least expect that attitude angles as well as airspeed and flow angles during the flight through the vortex are shown. The authors raise the issues themselves in the discussion, but I think it is necessary to include a proper analysis of this in the manuscript, including an estimation of uncertainty of the wind vector and thus the circulation.*
  The attitude angles of the UAS will be shown in an extra evaluation (e.g. subsection) also including an error estimation of the wind velocity and the measured circulation. See also Fig. 1 and Fig. 2 for a firs look.

- *I recommend some copy-editing to be done on English grammar and expressions.*
  We will look into that.

Specific comments

Abstract

The abstract should be rewritten with more statements about the hypotheses that were investigated in the study. A description of the results / conclusion is missing.
p.1,ll.1ff: wind converter should be either wind turbine or wind energy converter. Preferable: "rotor blade of a wind turbine". The whole first sentence is very hard to read and grammatically wrong. Starting with the relevance for numerical models in the abstract is misleading, because numerical models are not the topic of the paper.
p.1,l.6: what is the difference between atmospheric and meteorological quantities?
The abstract will be rewritten. All other annotations concerning the abstract have been implemented.

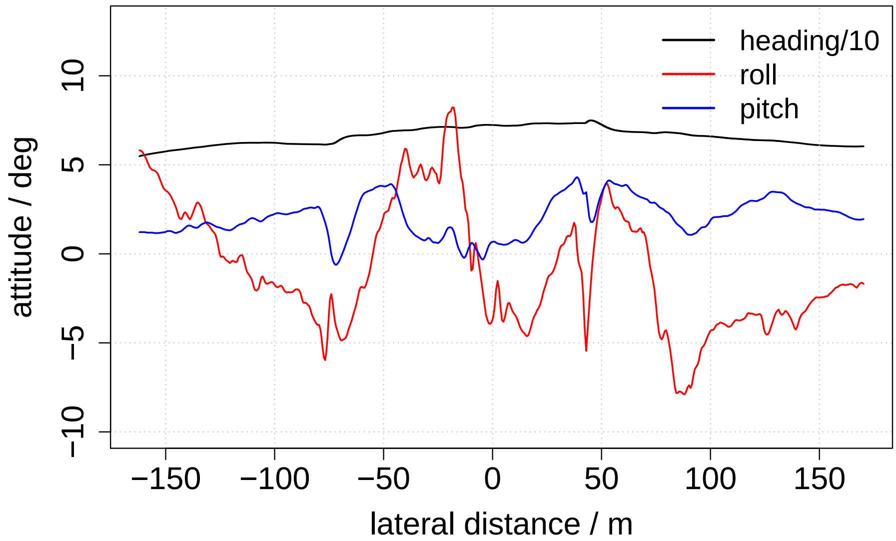

Figure 2: Attitude angles of the UAS for the vortex measurement flight. The entry and exit of the wake can be seen. The UAS remains on a stable flight path through the wake.

Introduction

p.2,ll.9-13: this paragraph should go into the description of the aircraft (Sect. 2.1)
p.2,ll.14-21: this paragraph could go in the experiment/site-description in Section 2
p.2,ll.22-31: here, a lot more literature should be referenced. Wake vortices are a major field of research and the state-of-the-art has not been evaluated at all.
All annotations have been addressed. Additional references and literature has been added.

Section 2

I miss a detailed description of the atmospheric conditions during the flight. Stratification, turbulence in the inflow, etc. which are important for the wake development are not given but should be available from the data.
The day the tip vortex measurement flights took place, only measurements at three different heights were made. The flight strategy was aiming on capturing tip vortices at hub height, and at the top and bottom of the wake. Since the distance to the nacelle is 0.25D or about 27 m the stratification of the atmosphere should not have a significant influence on the wake development. In an additional section a description of the atmosphere is now available, including turbulence.

A list of available flights and an explanation why only a single flight leg is analysed is missing.
A total of five flight legs at 0.25D are present. From these measurements only in one leg the criterion of $\Delta y < r_\mathrm{c}$ was met. Also a new subsection has been added 'Available data' where the data availability and the atmospheric condition are mentioned.

p.3,ll.3-8: Here, description of the aircraft is mixed with operational procedures. I feel like this should be separated.
Content has been separated.

p.3,l.11: Since RTK GPS is mentioned here and is not a standard feature of UAS in atmospheric research, some additional information would be appreciated: what kind of receiver is used (L1 or L1/L2 phase); is a local base station or RTCM-services used for correction data / what is the baseline? What

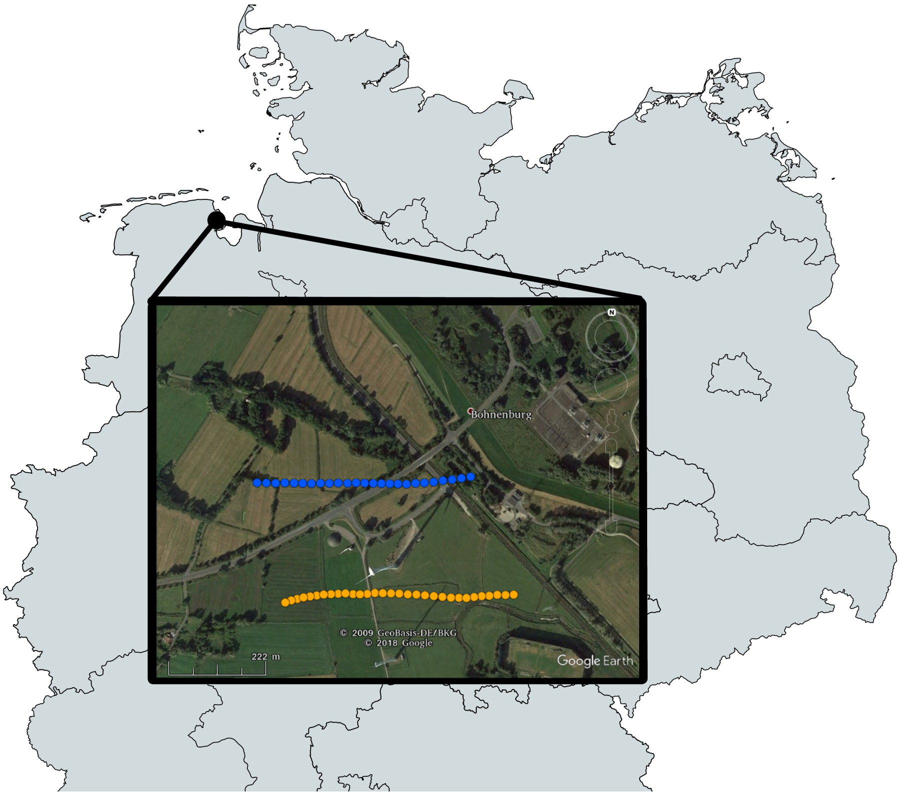

Figure 3: Location of the E-112 WEC in the north-west of Germany near the North Sea coast. MASC flight tracks in front (blue) and in the wake (orange) of the E-112 with northern main wind direction (5 degree north that day). On the Google Earth image the WEC is oriented toward south-easterly wind direction. Map created with mapchart.net

is the advantage of the very accurate flight path in atmospheric measurements?
Sorry, we made a mistake. RTK was not used during the flight and will not be mentioned in the manuscript any more.

Fig.3: I think a more schematic background (not Google Earth) with a better scale and legend would help
Google Earth map/image was replaced with a schematic map of Germany (cf. Fig. 3).

Section 3

Fig.6: It is unclear how TKE has been calculated. How large is the averaging window?
The TKE calculation serves a qualitative purpose. Therefore an averaging of 1 s (100 data points) has been used. The integral length scale was not calculated.

Fig.7: Nice figure, but watch out for which lines cross in front of or behind other lines to get the 3D visualization right. I think the red rectangle encloses the blue vortex, right? And "distance" should probably get a variable name or could even be left out.
First of all, thank you for the compliments. The line issue has been addressed. They should now support the viewer's perspective. 'Distance' has been removed. An updated figure will be found in the new manuscript (cf. Fig. 4).

p.9,l.5: Except that the vortices along the horizontal axis are not generated at the same time for the WEC.
In the specific line it is talked about the x-axis: Along the x axis, which is also the flight path of the UAS (pointing east with the main wind direction approximately 10° north), the vortices indeed show a little temporal delay. The first encountered vortex travelled a bit farther than the second vortex. This is now mentioned and could also explain the smaller core radius of the second vortex. We would like to

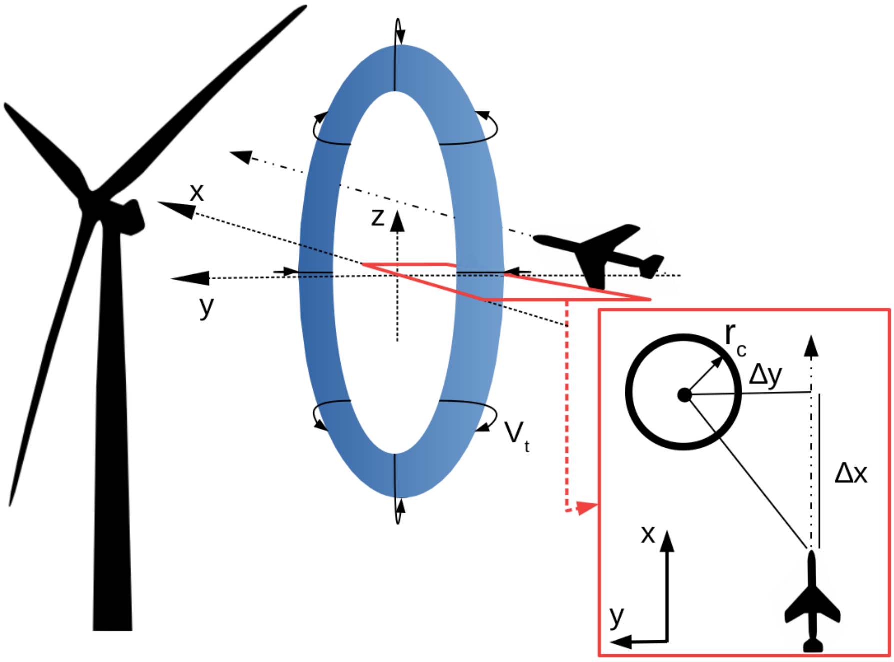

Figure 4: Simplified sketch of a vortex pair passed by the UAV to the right. In reality it would rather have a helical pattern than a ring shape. Velocities and axis according to meteorological standards, therefore axis and orientation according to the in-situ conditions. y axis pointing north, x axis pointing east. At hub height the w component (along z axis) vanishes. The red rectangle illustrates a top view of a tip vortex with distance Δy to the UAS.

argue for simplicity reasons that the flight path was perpendicular to the wake and therefore no huge 'ageing' differences occur.

Section 4

p.14,l.1: What is the vortex coordinate system and which angles are used for the rotation? This has not been introduced before.
Thank you for this comment. 'Vortex coordinate system' might really be the wrong expression here. What we were trying to say is that the 'horizontal wind data' have been rotated into the vortex rotational plane. So after this rotation the horizontal wind plane is parallel to the rotational plane of the tip vortex. The rotation was accomplished by rotating the x and y axis (u and v wind component respectively). This information will be added in the new version of the manuscript.

p.14, ll.26ff: this needs to be introduced and explained in more details in the methods section. Why can this correction not be done for other flight levels to increase the number of samples of tip vortices?
In principle it is possible to look at different altitudes. The flight strategy was to concentrate on measurements at hub height, since the probability to hit a tip vortex is the highest at this level. Also the introduced simplification of the vortex only rotating in two dimensions is mainly true at hub height. The rotation into the vortex rotational plane only makes sense, when the vortex has passed with the necessary criterion ($\Delta y < r_c$). Otherwise an evaluation of Γ and r_c is not possible.

Fig.16: I think this figure is not necessary.
This figure has been removed.

p.15, l.5: "rotated slightly" → what does that mean?
Done. The wind is rotated into the wake direction.

p.15, l.10ff: It is said that the wind speed deficit plays an important role for the tip vortex, but this is not discussed any further. Is BH even an appropriate model under these circumstances?

The short answer is 'yes'. The u component of the model is not affected by the deficit, especially since the data has been rotated into the wake direction. Also the peak position (on the x axis) for the v component in the BH model is also not affected. Only the slope and magnitude.

Fig.18: not sure if this figure is necessary
Will be dropped from the manuscript. It was simply thought to be a visualisation of the applied correction.

Section 5

p.19,l.7f: These are too strong statements for an experiment with a single sample

p.19,l.15ff: The issues that are raised here are not insignificant and call for some more analysis and quantification.
Attitude angles and true air speed variations will be analysed and the results addressed accordingly in the new manuscript.

Section 6

p.19,l.21: What is the equation by Sorensen et al. (2014) that is meant here?
Done. Eq. 20. Reference has been added.

p.19,l.23: "aggravates an evaluation" - what do you mean by that?
The evaluation is done mainly graphically. The second tip vortex is embedded in a relatively high level of turbulence (wake deficit, shear, etc.). The second tip vortex does not show a clear border to the undisturbed atmosphere as tip vortex 1 does. The reference velocity levels for the evaluation are therefore harder to extract from the measurements.

p.19,ll.27ff: The information about the 5-hole probe calibration range and why other flights could not be used should be given in Section 2 already. In section 2, it was said that the UAV operated with RTK GPS which is contradicted here. The subsection 'Data availability' has been added in Section 2 where we explain briefly why no other vortex examples are available. The RTK GPS mentioning will be stripped. We will link to a recent MASC-3 paper by Rautenberg et al. 2019.

Technical corrections

p.1,l.1ff: wind converter should be either wind turbine or wind energy converter. Preferable: "rotor blade of a wind turbine". The whole first sentence is very hard to read and grammatically wrong. The second sentence raises problems of numerical simulations that are not the topic of the paper.
p.1,l.6: what is the difference between atmospheric and meteorological quantities?
p.1,l.8: u,v,w, italic please
p.2,l.3: underestimate
p.2,l.4: "Another way" or "Another method"
p.8,l.7: "several analytical models are available"!?
Fig.8: x-axis should be labelled r
p.11,l.18: L is not proportional to the velocity ratio → The velocity ratio is a function of L.
p.14,l.10f: "In Fig.14 shown as a solid purple line." What is?
p.14,l.21: "clear and sharp jump" - strange expression
p.14,l.22: Fig. 16 appears before Fig. 15 in the text.
Fig.17: What is u and what is v should be mentioned in the caption. the same line style should be used for the same analysis method, i.e. dashed line for simple BH, and dotted for corrected version for example.
All corrections have been adopted and implemented in the new manuscript.

p.1,l.18: "their individual capacity and diversity" (what do you mean by diversity?)

Here we wanted to point out that there are different WEC designs for different terrain (complex vs. homogeneous) with all their challenges (high wind speeds, high turbulence and increased stress on blade structures etc.) The paragraph has been rewritten.

Author's response to Referee #2

May 21, 2019

Thank you for the detailed review of the manuscript. In the following I will comment on each point. The referee's comments will be repeated in blue italic before the answer.

General comments

I agree with the comments from RC1, so I will focus on some of the other things I noted. I have written down some general comments below, followed by a list of specific comments. As it is, I found the article hard to follow, which is unfortunate since the results are interesting. The authors should focus on guiding the reader through their thoughts and results, in a clear and concise way, reducing the length of the paper. This will require a major revision of the paper. I also believe the authors should spend an important amount on time correcting the language of the article. I've highlighted a couple of paragraph and sentences that

- *I also believe the authors should spend an important amount on time correcting the language of the article. I've highlighted a couple of paragraph and sentences that could be improved up to page 11, after which I stopped mentioning these. The authors should still work on the text after page 11.*
  We will look into that.

- *Regarding the title, I would probably not support using the word 'first' in the title since it brings a competitive touch to it that is unnecessary in my opinion. Also in light of the following publication, it may unfortunately not be justified to claim this 'first' attribute: F. Carbajo Fuertes et al - 2019 - 'Multirotor UAV based platform for the measurement of atmospheric turbulence: validation and signature detect ion of tip vortices of wind turbine blades.'. The author may also consider the studies from, Kocer et al. 2011 and Reuder and Jonassen 2012 cited in the above reference. (Please note that I'm not an author of any of these papers). Yet, I leave this up to the authors and the editor to decide whether to change the title.*
  Thank you for your valuable input there. The paper by Carbajo Fuertes was put online on March 28 2019. So there was no possibility to the authors to have known from these results.
  Following your suggestion I had a look at Fuertes 2019. The results look good. It is nice to see that two different approaches and measurement systems come to a similar result and output. Fuertes also derives a circulation of 50 m^2/s from his measurements. Given that he measured at a different wind turbine, the circulation strength is of the same order of magnitude. However, I could not find any information about the vortex fit he is using for his measurements as well as a detailed information about the circulation calculation.

- *I would personally prefer the equations to be closer to the text. As it is now, the equations are usually floating at the end of the paragraph which can make the discussions hard to follow.*
  I will consider this point to be mainly to be an editorial problem. I will see, to put equations closer to the text. However, in the later two-column layout everything will change again. So I might need input from the editor at this point.

- *It appears that the method presented can be attributed to Fischenberg, and the author may need to be clearer when highlighting if something is new or unique in their approach compared to what was already published (apart from the measurement campaign). The model using a regularized vortex cannot really be seen as a new contribution or method. The experimental data though are of high value.*
  It is correct that the method and model is not new. But as we state that the model has its origin in aviation, it is new to be used to describe wind turbine wake vortices properties. We added a paragraph to highlight this new approach in WEC wake studying.
  Also the described method illustrates how to obtain results from in-situ line measurements of a UAS, including systematic measurement uncertainties like skewed/canted vortex hoses etc.

- *The amount of measurements appear unclear, and some statistical analysis and information about the ensemble of results available could be valuable. Reading the article, it seems that only one vortex was analysed. Further data with different distances downstream should be incorporated since according to section 2 more data was acquired. Statistical tools should also be used to mention the uncertainty on the fitted parameters and to quantify the error between the model and the measurements.*
  A new subsection '2.3 Available data' has been added. We also state why only one pair of vortices could be used for an evaluation. A simple vortex measurement in the wake is not sufficient, also the criterion $\Delta y < r_c$ must be met which is a rare condition. We comment on this in the 'specific comment' section below in more detail.

- *The figures are usually clear. The authors could yet reduce the number of figures, particularly in the first 10 pages, or by combining the measurements with the fitted model in figures 10-17.*
  We will take out some figures from the manuscript. But also some new ones will be added. Mainly to do the uncertainty evaluation (wind angles, true air speed, etc.). We will also try if some figures could be combined. But that might add to the readers confusion.

- *I hope my numerous comments will not discourage the authors, and I strongly encourage them to further work on this paper. As I mentioned earlier, the article has some great potential, it just needs some additional work to reach the point of publication.*
  Highly appreciated emphasis. Thank you!

Specific comments

Abstract

The statement in the abstract 'the BH model can be used to describe wake vortices' is probably too strong and would need to be moderated since this simpler model is not capturing all the dynamics. I will comment more on this later.
We will add the simplifications that have to be made. The BH model is also only a tool to describe the vortex geometry in a 2D cut. This point might need additional stressing.

Introduction

I would think that bringing the context of Germany appears too specific, since the wind energy sector is growing in other countries.
This is indeed true. The introduction has been 'internationalised'.

p1 l21: "In research..." this sentence and the following two are hard to read and could be reformulated
The abstract as well as the introduction were rewritten.

p2 l8: The scaling problem of wind tunnel measurements could be mentioned here
Thank you for the hint. It will be mentioned.

p2 l10: make sure the acronym for UAS (and other acronyms) is made explicit in the introduction
Has been implemented.

p2 l15: Could you mention the arguments for the offshore comparison. It probably relies on arguments on the boundary layer when the flow comes from the shore, but wave exciting the turbines and the surface roughness may be different.
Yes, there is definitely an influence through the surface roughness compared to the sea. It can also be seen in Fig. 5 that the horizontal wind in a certain altitude experiences much more fluctuations. So there is an internal boundary (surface) layer that shows increased turbulence with a more stable marine layer on top.
This was only mentioned since for the briefly presented HeliOW project it would be desirable to fly and measure behind an actual off-shore wind turbine. But for several practical reasons this was simply not feasible.
The influence of turbulence is also a good point for future measurements to get a large amount of vortex measurements to do some statistics.

p2 l18: "The project aims for safe helicopter flight paths in off-shore wind energy parks" needs reformulation
Will be changed to "The goal of the project is also to determine safe helicopter flight paths in off-shore wind energy parks".

*p2 l19, l21 *University of* Munich, University *of* Stuttgart.*
Has been changed accordingly.

p2 l23: the wind turbine also generates strong coherent vorticies, can these be considered turbulence?
Coherent (in this case artificial turbulent) structures are not turbulence covered by Kolmogorov 1941. The coherent vortices might be considered as a large scale anisotrop turbulence. The surrounding atmosphere in the best case scenario can be considered as isotrop. In the far wake these coherent structures decay and add to the isotrop turbulence.

p2 l30: could you highlight more the difference between the study from Subramanian and yours?
Subramanian et al.detected tip vortices by a pressure signal along a longitudinal flight path. The vortices were also not quantified (vortex strength, core radius). We will add some more details to the manuscript and highlight the main differences.

Section 2

p3 l7: Wind speed and directions could have changed during this 15min period, do you have access to measurements to support this assumption?
Unfortunately, we didn't have access to highly resolved SCADA data for the measurement time which could have given us precise information about wind speed and direction changes. Moreover, we were flying about 27 m behind a 114 m diameter WEC. If the wind direction would have changed significantly we might have crashed into the rotor blades. For a single vortex measurement it is also not important, if the wind direction varies a bit. The wind velocity component rotation later in the manuscript was done for the single measurement. The mean wind direction is only important to set up the flight trajectory for the UAS.

Section 3

p5 l11: You could mention that the ring vortex is an approximation of the wake vorticity at high tip-speed ratio.
Thank you for the input. We will mention that.

p5 l14: This line can be reformulated to mention that this result is true under the vortex ring assumption.
The line has been modified.

p6 Figure 4: "recreation" may not be the correct word in the caption, maybe "model", or "reproduction" would be more accurate?
Done.

Figure 7: Instead of using north/east for the axis wouldn't it be easier in that case to use an orientation in the frame of the turbine, with y pointing upstream against the main wind direction? You could then remove the sentence at the end of page 6.
This was actual the intent when planing the flight path at the field campaign. The wind was slowly turning. So Figure 7 shows the actual flight path (direction) of the UAS. So y pointing north is almost into the main wind direction. But at the time of the measurement the wind direction was already a bit off. Also the post-processing software of the UAS data always writes the wind velocities according to the meteorological standards u pointing east, v pointing north and w pointing upwards. So the data in the end always has to be rotated into the main wind direction. And this is necessary to later make $\overline{u'} = 0$ in the wake.

p7 line1-12: The potential flow assumption probably appears too early in this paragraph and the paragraph could be reformulated. The definition of circulation as function of the vorticity is independent of this assumption. Equation (3) only uses an axi-symmetric assumption. It is yet true that the circulation of a vortex makes more sense in inviscid flows where the vorticity is condensed to confined singular regions.
We will look into a possible reformulation and implementation of the introduction of potential flow.

p10 l1-4: This paragraph needs should be reformulated, the language improved.
The paragraph has been rewritten.

p10 l9-15 and p11 l1-8: While reading the text I was confused since figure 9b didn't appear to be mentioned. The explanation could be improved by clearly explaining both figures and both scenario, before mentioning figure 10. Alternatively, you could tell the reader to focus only on figure 9a for now and figure 9b will be explained later. Also, equations 8-10 could be introduced first before drawing the conclusions that there is no unique solution. Further, the way the equations are introduced can be improved by telling to the reader what is coming, e.g. "The velocities at point 1 and 2 are...". Right now, they appear in the text announced.
Figure 9b is mentioned on page 11 l7. I agree that the two different cases of passing a vortex in the UAS measurements needs additional addressing to not lose the reader. The paragraphs have been modified.

p11 l9: "This double peak can be lead back to passing the maximum tangential velocity at $r = r_c$ at position 1 and 2". The sentence may need reformulation
Changed to: This double peak is caused by passing the maximum tangential velocity at $r = r_c$ at position 1 and 2".

p11 l12-15: Similar to the previous remark, the equations can be introduced before drawing the conclusions.
We think this comes down to personal preference. We would like to explain what's happening when passing the vortex at $\Delta y < r_\mathrm{c}$ and follow up with what that means for the previously introduces equations.

p11: "As shown above the presence and identification of a vortex (or a pair of vortices) is measurable". The language needs to be reformulated (one cannot measure an identification). Also, the previous section seemed to show that in some cases the determination was not possible, which would imply that the identification is not always possible.
Fair point. "As shown above the presence is measurable and a later identification of a vortex (or a pair of vortices) possible when the $\Delta y < r_\mathrm{c}$ is met."

Section 4

p13 l8: It would help the reader to provide some information about the measurement campaigns (how many samples were selected, what were the mean conditions), and some introduction about the samples you selected (and why you selected them) to present in the paper.
This issue will be addressed with the additional subsection about data availability.

p14 l26: It is not clear why you mention the skewed vortex at this stage. You may need to guide the reader. Also, the definition of the skewed vortex on figure 15 appears unclear. From the figure it seems that the vortex is simply rotated. I'm not sure this qualifies as a skewed vortex. Also the 2D cut appears to have some 3D aspect to it, which can be confusing. Introducing a coordinate system on the 3D vortex on the 2D cut can help the understanding.
Maybe 'canted' or 'oblique' vortex is the more precise expression. The additional 3D features are meant to help the reader. That is why the blue is more transparent. It could be removed, if it confuses more than it helps?!
We have to introduce the oblique vortex hose, since the real (measurement) world is different from the analytical approximation (ring vortex). We will try to make that more clear.

p14 l30: I am wondering if "analytical reconstruction" is the proper term and how this "reconstruction" is different from the previous section. The parameters you extracted from the measurements were fitted to an analytical model. The "reconstructed" vortex is this analytically modelled vortex, and it is intrinsically part of the results you presented in table 2. If I understand this correctly, it could make sense to have the modelled vortex directly on the figures 13-15, as "fitted vortices".
In Section 3 we want to present the theory/method how to "reconstruct" the vortex from GPS and pressure data. In section 4 the individual fit is calculated. One might consider to just remove this subsection, e.g. Fuertes 2019 never introduces any vortex model of the WEC and just "fitted" the data. As for the second point, in fact the GPS positioning standard deviation is in the range of the vortex core radius. So the double peak is not visible and overall the accuracy is not up to the in-situ, in-line pressure measurement. Thus, to keep an step-by-step approach and to use the analytical model, as a proof of concept, we prefer to keep it separate from the measurement.

p15 l9: What is meant by "artificially induced drop" of velocity? Does this refer to drop of velocity in the turbine wake? If this is so, the drop in velocity should be a function of the thrust coefficient at the rotor, and a value of 65% may not be comparable to other measurements unless they are at the same operating conditions.
You are correct, the statement has been changed. The actual deficit in percent or m/s is only important for the correction of the modelled v component.

Section 5

p18 eq20: You may have to introduce all symbols closer to the equation, even if these are obvious.
We will try to do so.

p19 l1-4: It appears surprising that the authors do not have more information about the turbine (thrust curve, pitch curve). Earlier in the text, it was mentioned that a model of the turbine was done. These quantities can then be obtained from a Blade Element Momentum code.
The argument here may simply be that most turbine have a pitch angle around +/- 1 degree below rated, and in the absence of data, you picked 0 degree. It is also not clear where the pitch angle enters in the equation. Most likely an argument of the thrust coefficient, but usually you'll have a thrust coefficient vs wind speed curve available for that turbine.
The engineering sector of wind energy converters is a highly competitive market. Although the University of Stuttgart has a generic recreation of the wind turbine, it is reserved for wake computations. The resulting aerodynamic properties of the turbine are however strongly confidential. That's why those properties, namely here the thrust coefficient, had to be estimated. As stated on p. 19 l1, we approximated the thrust coefficient vs. wind speed with an NREL 5 MW turbine. Since the E112 and NREL 5MW wind turbines both have similar rotor diameters and produce a similar amount of electricity, the approximation was judged reasonable.

p19 l10-11: These sentences need to be moderated. First, it appears that the study was only done on one vortex at a given operating conditions and a more quantitative analysis would be required. Second, it appears wrong to state that the wake of a turbine is described by two vortices. You could clarify your discussions based on the following considerations. The fact that two vortices are crossed on the trajectory of the drone is due to the likelihood of crossing the tip-vortices from different blades. This likelihood increases as the number of blades or the tip-speed ratio increases. When it is such, the wake

vorticity surface can be approximated to a vortex cylinder, in which case any trajectory of the drone will indeed cross the tip-vorticity surface twice. This cylindrical surface does not resemble two vortices spinning in opposite direction and the wake dynamics cannot be described by assuming that it consists of two vortices. What the author probably means is that the velocity field across a tip-vortex (or a tip-vorticity surface) resembles the one of a regularized point vortex. This analogy (which is natural given the different analytical vortex wake models of wind turbines) cannot be used to "describe" the wake, but it can be used to "estimate" some of the wake properties, that is, the tip-vortex core radius and circulation.
We agree. This paragraph needs to be more precise. We will implement your suggestions.

Section 6

p19 l21: The identification of one vortex strength does not appear to be enough to draw a conclusion, or the conclusion needs to be moderated. Also, it may not be necessary to attribute this equation to Sorensen et al. and instead it can be mentioned where this formula comes from: circulation for a rotor of constant thrust coefficient (This should also be mentioned earlier in the text p18 l1-6).
The conclusions will be readdressed and the statement moderated. Also we want to repeat at this point (and later in the new version of the conclusions) that the fact that we hit two (!) consecutive vortices in one flight leg at the crucial criterion ($\Delta y < r_c$) might also be considered lucky. That both measurements, after consideration of the non-ideal conditions (canted vortex hose) and therefore the rotation of the horizontal measurements into the vortex plane is giving two circulations strengths that match the theoretical calculation, needs an appropriate (strong) statement.

First Identification and Quantification of Detached Tip Vortices Behind a WEC Using Fixed Wing UAS

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Abstract. In the present study, blade-tip vortices have been experimentally identified in the wake of a commercial wind turbine using the MASC Mk 3 (Multi-purpose Airborne Sensor Carrier Mark 3) UAS(Unmanned Aircraft System) of the University of Tuebingen. By evaluation of the wind components, detached blade-tip vortices were identified in the time series. From these measurements the circulation and core radius of a pair of detached blade-tip vortices is calculated using the Burnham-Hallock (BH) wake vortex model. The presented data were captured under a dominating marine stratification about 2 km from the North Sea coast line with northern wind direction. The measured vortices are also compared to the analytical solution of the BH model for two vortices spinning in opposite direction. The model has its origin in aviation, where it describes two aircraft wake vortices spinning in opposite direction.

An evaluation method is presented to measure detached tip vortices with a fixed wing UAS. The BH model will be used to describe wake vortex properties behind a wind energy converter (WEC). The circulation and core radius of detached blade-tip vortices will be calculated. Also proposition of the model in WEC wake use will be made to describe two independent co-rotating vortices. Quantifying blade-tip vortices helps to understand the process of vortices detaching from a rotor blade of a wind turbine, their development in the wake until finally dissipating in the far wake and contributing to overall turbulence. This is especially interesting for set-ups of numerical simulations when setting the spatial resolution of the simulation grid.

1 Introduction

The wind energy sector has been growing world wide for decades and the produced power from wind energy is still growing. Not only the amount of installed wind energy converters (WECs) is increasing but also their single capacity. Also the field of application has widely increased with WEC. There are systems available for homogeneous terrain, off or near shore, or even complex terrain with a high amount of additional turbulence stress that is induced onto the wind turbine's blades.

A modern off-shore WEC delivers up to 12 MW of power in ideal conditions. In wind energy research numerical simulations of the wind velocity field of a WEC and its produced turbulence are important tools that give valuable informations. Pressure and velocity distributions around a turbine blade and nacelle as well as in the wake can be studied. A numerical model increases its validity when it is backed by real world in-situ data. Not least, because of improvements that can flow back into the model, once measured data has revealed some possible tweaks and enhancement to the prior model. Numerical simulation might underestimate peak vorticity and radii of wake vortices, especially when the grid size of the simulation is not sufficient (Kim et al., 2016). Another way of studying WEC wakes are wind tunnel experiments that try to recreate wake patterns in a smaller scale (e.g. Bartl et al. (2012) or Vermeer (1992)). While in the early days of wind tunnel experiments the wake has been visualised by smoke trails, PIV (Particle Image Velocimetry) measurements have increased the resolution and accuracy of wind tunnel experiments drastically (e.g. resolving Reynolds shear stress and turbulent kinetic energy, Zhang et al. (2012)). But a common issue with wind tunnel measurements is that they usually suffer from scaling problems (Wang et al., 1996). Remote sensing techniques like LIDAR have also found their way into WEC wake evaluations. Various measurement strategies were developed to visualise WEC wakes e.g. in complex terrain (Barthelmie et al., 2018). LIDAR scans provide a long term measurement of a probed volume or plane. The spatial resolution ($25\ -50\ \mathrm{m}$), however, is comparably coarse. LIDARs can provide a continuous monitoring of WEC wake structures (e.g. wake centre, direction and wind velocity deficit) (Bodini et al., 2017) in homogeneous or even in complex terrain (Wildmann et al., 2018). UAS (Unmanned Aircraft System) measurements can provide in-situ line measurements, covering a small volume but with a high temporal and spatial resolution in (deca-) centimetre range. The coverage of these scales is important to measure detached tip vortices in the near wake of a WEC.

A WEC, especially in a stable marine ABL (atmospheric boundary layer), acts as a turbulence generator. The added turbulence has two main sources. On the one hand the increased wind shear in the wake that results from the wind deficit in the near wake and the low pressure bulb that develops behind the WEC nacelle. On the other hand turbulence is created by expansion and dissipation of detached blade-tip vortices that transfer their kinetic energy to the surrounding flow. A proper understanding of these vortices and their induced load onto the converter blade is of great importance for future enhancement of life span and working loads of wind energy converters in wind farms. Blade-tip vortices follow a helical pattern into the wake, detaching from each converter blade. These detached eddies can be measured with the mounted five-hole-probe on the MASC UAS. Subramanian et al. (2015) detected tip vortices via pressure fluctuations qualitatively in a flight pattern along the wake, also using a small UAS. In this study an evaluation method is presented to measure the core radius r_c, circulation Γ and maximum tangential velocity $V_{\mathrm{t,max}}$ of a tip vortex using in-situ wind measurements from UAS flights perpendicular to the mean wind velocity.

2 Measurement system and measurement site

2.1 Research aircraft

The research UAS MASC Mk 3 (cf. Fig. 1 and Tab. 1) is a fixed wing airborne measurement system of the University of Tübingen that has been used in several measurement campaigns and has been described by Wildmann et al. (2014a, b). The third iteration of this platform features some changes to the fuselage. The electrical pusher motor has been moved from a centre position behind the wings to the tail, accelerating the aircraft along the centre axis and increasing flight stability. The MASC

Figure 1. Research UAS MASC Mk 3 shortly before lift-off. (Photo taken by the author).

Table 1. Characteristics of the MASC Mk 3 UAS at the HeliOW campaign.

wingspan	4 m
total weight	$\approx$ 7 kg
sci. payload	$\approx$ 1 kg
cruising speed	19 m s^{-1}
endurance	up to 2.5 h
propulsion	electrical pusher engine
take-off	bungee or winch

Mk 3 system allows in-situ high frequency measurements of the atmospheric flow and its transported properties. A detailed description of the improved UAS and its instruments can be found in Rautenberg et al. (2019b). The latest iteration MASC Mk 3 is using an improved IMU (Inertial Measurement Unit) and positioning system.

Aside the changes in fuselage design, the former ROCS autopilot operating on the MASC Mk 2 system has been changed to the Pixhawk 2.1 autopilot. This is an independent open-hardware and open-source autopilot project (www.pixhawk.org).

2.2 Measurement site

Figure 3 shows the location of the measurement site in the north-west of Germany and the flight tracks of the MASC Mk 3 UAV around the Enercon E-112 converter. Both tracks are part of a rectangular flight pattern around the WEC in anti-clockwise direction. For the wake data evaluation only the data captured along the southern flight tracks (orange path in Fig. 3) are used. The E-112 WEC is the most powerful converter in the Jade-Wind-Park north of Wilhemshaven, Germany. The particular converter is a former near-shore prototype with a rotor diameter D of 114 m delivering up to 4.5 MW of electrical power and thus comparable to an actual off-shore WEC. The Jade-Wind-Park is located about 2 km from the North Sea coast line and a

Figure 2. Research UAS MASC Mk 3 in front of an Enercon wind energy converter at the Jade Wind Park in July 2018. (Photo taken by the author).

maritime influence in the wind profile can be expected.

Apart from surrounding WECs (to the south of the E-112 WEC) power lines to the east and north and industrial buildings to the north and north-east (not in the picture) restricted the flight path to the ones depicted in Fig. 3.

For this study of the near wake of a WEC, the wind turbine described above, has been chosen. This specific converter and its location near the coast is comparable with off-shore converters in marine flow which was a requirement when choosing the WEC. The measurements are part of the HeliOW project, in which the atmospheric turbulence in front of and in the wake of a WEC are the foundation of a chain of numerical simulations. The goal of the project is also to determine safe helicopter flight paths in off-shore wind energy parks. The numerical simulation chain also includes CFD simulations of the wind turbine (University of Stuttgart) which are injected in flight-mechanical simulations of a helicopter (provided by Technical University of Munich and DLR Braunschweig). Thus, the tip vortex measurements are an important contribution to the validation of later numerical simulations of the flow.

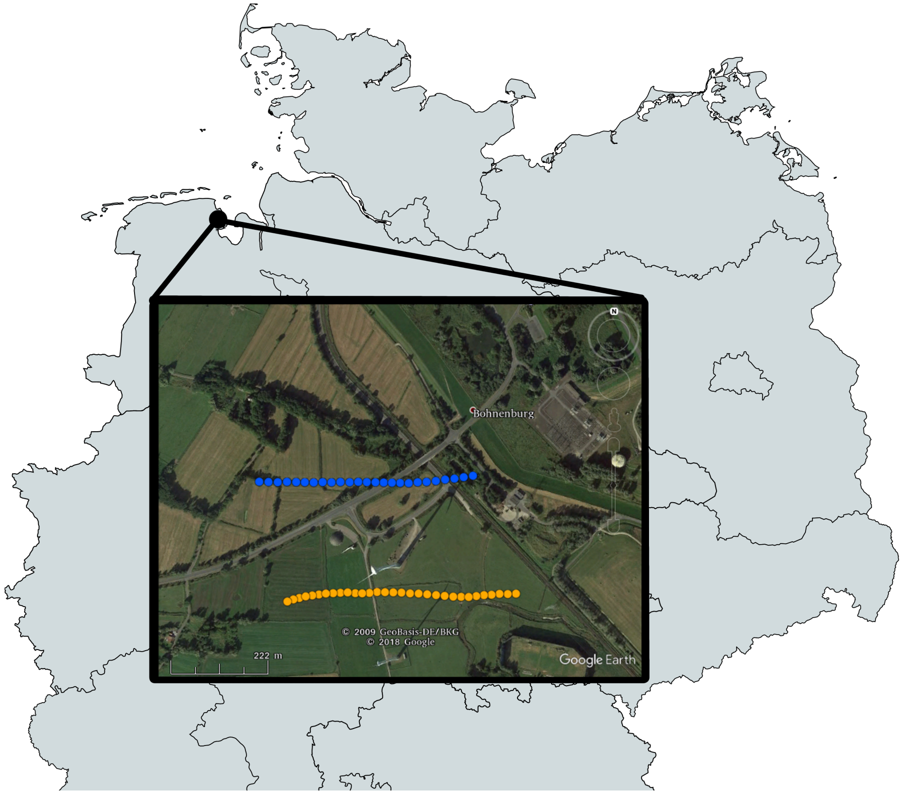


Figure 3. Location of the E-112 WEC in the north-west of Germany north of Wilhelmshaven near the North Sea coast. MASC flight tracks in front (blue) and in the wake (orange) of the E-112 with northern main wind direction (5 degree north that day). On the Google Earth image the WEC is oriented toward south-easterly wind direction. Map created with mapchart.net

2.3 Available data

For the tip vortex evaluation five flight legs (straight and level fly-by's) are available 0.25 D downstream the WEC rotor plane. Only one of these legs shows the necessary criterion for the circulation and core radius calculation (cf. the following sections). This one leg (two vortex measurements) that fits the criterion will be shown exemplary to present the evaluation method and the analytical solution using the BH model approach, including also the approach by Sørensen et al. (2014).

The presented data was captured within 15 minutes. It can be expected that atmospheric conditions (wind direction and speed, thermal stratification, turbulence intensity) did not change significantly during this period. The average wind speed in the inflow was $8.8\ \mathrm{m\ s^{-1}}$ with a turbulence kinetic energy (TKE) of $0.085\ \mathrm{m^2\ s^{-2}}$ at hub height. These values have been calculated from a ten second measurement in the undisturbed atmosphere.

3 Methods

With the goal to measure detached tip vortices behind a WEC, it is helpful to have at first an understanding of the behaviour of those vortices. Fig. 4 shows the helical vortex pattern forming behind a WEC, by representing the iso-surfaces of the λ_2-criterion of detached tip vortices from CFD simulation. The fully resolved URANS simulation has been performed by the University of Stuttgart with the compressible flow solver FLOWer (Kroll and Fassbender, 2005), using the Menter SST (Menter, 1994) turbulence model. The modelled rotor is a stand alone generic model of the Enercon E-112 WEC rotor, based on free access airfoil data. For more details regarding the numerical methods, please refer to Cormier et al. (2018) in which the same methods have been applied and described. Figures 5 and 6 give a qualitative impression of the presence of the WEC wake. In both, horizontal wind velocity and turbulence kinetic energy (TKE), the wake and its effects are visible. Farther downstream the helical pattern will start to meander and the symmetrical pattern will dissipate into turbulence. In the near vicinity of the WEC nacelle, these vortices follow a helical pattern. The helical structure is shown simplified by a ring vortex in Fig. 7 which is an approximation of the wake vorticity at high tip-speed ratio. The tangential velocity in this sketch can be split in its horizontal components at hub height (nacelle height). Here the y axis points north (ideally antiparallel to the main wind direction) similar to the conditions at the HeliOW campaign (cf. Fig. 3) and the x axis points east along the UAS flight path. Note that, at hub height, the tip vortex ideally has no w component (Fig. 7) under the vortex ring assumption. Thus, at this height, the tangential velocity can be split into its horizontal components u and v. The red rectangle indicates a change of perspective, showing a top view of a vortex spinning in the $x-y$ plane. In reality, from planing flight paths until take-off of the UAS and the actual measurement, the wind direction changes slightly. Therefore, for later evaluations the coordinate system has been rotated into the main wind direction.

3.1 Vortex model

To measure and evaluate tip vortices from UAS data an analytical vortex model has to be found. Previous efforts to define a vortex were reviewed e.g. by Jeong and Hussain (1995), comparing several definitions with data from direct numerical simulations and exact solutions of the Navier-Stokes Equations. A universal definition of a vortex or a generally applicable model does not exist. Assuming incompressible flow and an irrotational velocity field, where the curl of the gradient of the velocity is zero, the circulation Γ, representing the strength of a vortex around a contour C, can be connected to the vorticity flux by Stoke's theorem. For any surface S that spans the curve C and $d\boldsymbol{I}$ being an infinitesimal tangential element along C,

$$\Gamma = \oint_C \boldsymbol{V}_\mathbf{t} \cdot \mathrm{d}\boldsymbol{I} = \int_S \boldsymbol{\omega} \cdot \boldsymbol{n} \, \mathrm{d}S. \tag{1}$$

The circulation Γ is the line integral of the tangential velocity along the curve C which is equal to the vorticity flux $\omega = \nabla \times V_\mathrm{t}$ through the surface S, with $\boldsymbol{n}$ being the normal vector of the surface.

A circular integration in a cylindrical polar coordinate system with the azimuthal angle ϕ and the radius r yields:

$$\Gamma(r) = \int_0^{2\pi} \int_0^{r} \boldsymbol{\omega}(r,\phi) r \; dr \; d\phi \tag{2}$$

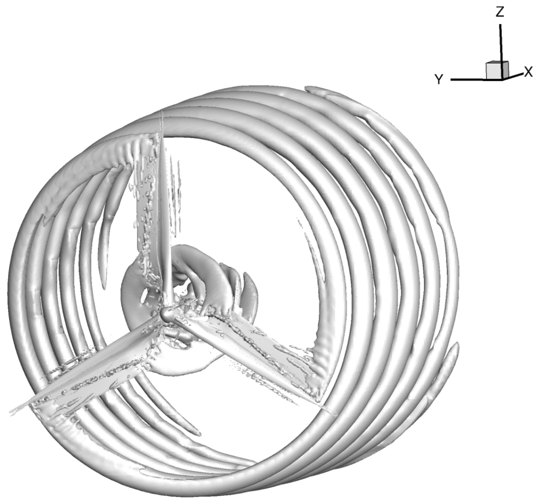


Figure 4. Iso-surfaces of detached blade-tip and root vortices following the λ_2 criterion for vortex identification. Here the x axis follows the main wind direction. Numerical simulation of a generic model of an E-112 4.5 MW converter.

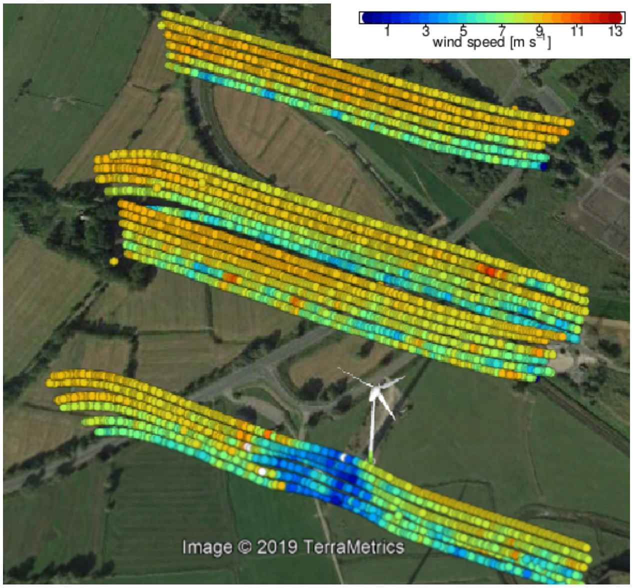


Figure 5. Visualisation of the horizontal wind measurements at different flight leg altitudes (from 85 m to 185 m above ground in 20 m steps) and different distances to the WEC (1D, -1D, -2D and -4D). Significant wind deficit 1 D behind the WEC E-112. At this day the wind direction was about 30° north. Image generated in Google Earth.

For a two dimensional, axisymmetric vortex, the circulation

$$\Gamma(r) = 2\pi r V_{\mathrm{t}}(r) \tag{3}$$

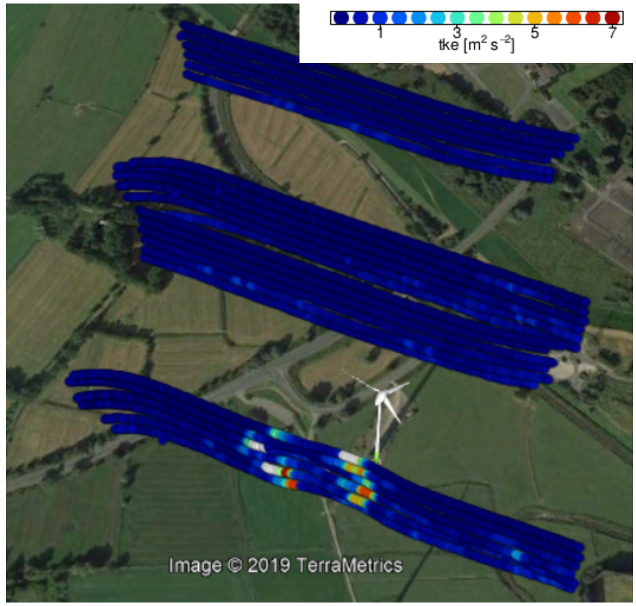


Figure 6. Visualisation of the turbulence kinetic energy (TKE) from the same measurements as in Fig. 5. Blue areas represent low turbulence and red the highest measured. At 1D the presence of the wake and the produced turbulence by the tip vortices is visible. The averaging window for the for TKE calculation is $1\ \mathrm{s}$, corresponding to 100 data points, and suits only for qualitative reading only. Image generated in Google Earth.

is a simple function of the radius and the tangential velocity V_{t}. Since real vortices in fluids experience viscous effects, the structure of detaching tip vortices of the blade of WEC cannot be sufficiently described by Equation 3. Close to the centre of the vortex, lower tangential velocities persist, increasing to their maximum at the core radius r_c of the vortex and decreasing again for farther distances r. To account for that, in the context of WEC and also for detaching tip vortices from the wings of aircraft, an analytical model is necessary.

Since in this study detached vortices of a WEC converter are treated similar to aircraft wake vortices, a few similar model approaches were possible. A comparison of analytical vortex models for tip vortices created by aircraft has been done by Ahmad et al. (2014). Also Fischenberg (2011) measured wake vortices created by the VFW 614 ATTAS manned aircraft (DLR Braunschweig) and compared the results to two similar vortex models proposed by Lamb (1939) and Burnham and Hallock (1982). Fischenberg concludes that both models show the ageing processes of a vortex wake known from theory. In general the model by Burnham-Hallock shows a slightly better agreement in circulation and tangential velocity to the conducted measurements by Fischenberg. Also Vermeer (1992) uses the Burnham-Hallock vortex model successfully to describe WEC wake vortices. According to these findings and its simplicity it has been decided to use the analytical solution for wake vortices by Burnham-Hallock in this study. While the two counter rotating vortices in the BH model used in aviation interact with each other, the two opposite vortices in a WEC wake do not do that. This is an important detail to point out. So for the identification of the vortex parameters (Γ, r_{c}) a model of two counter-spinning vortices is not necessary. Here, a stand alone vortex is

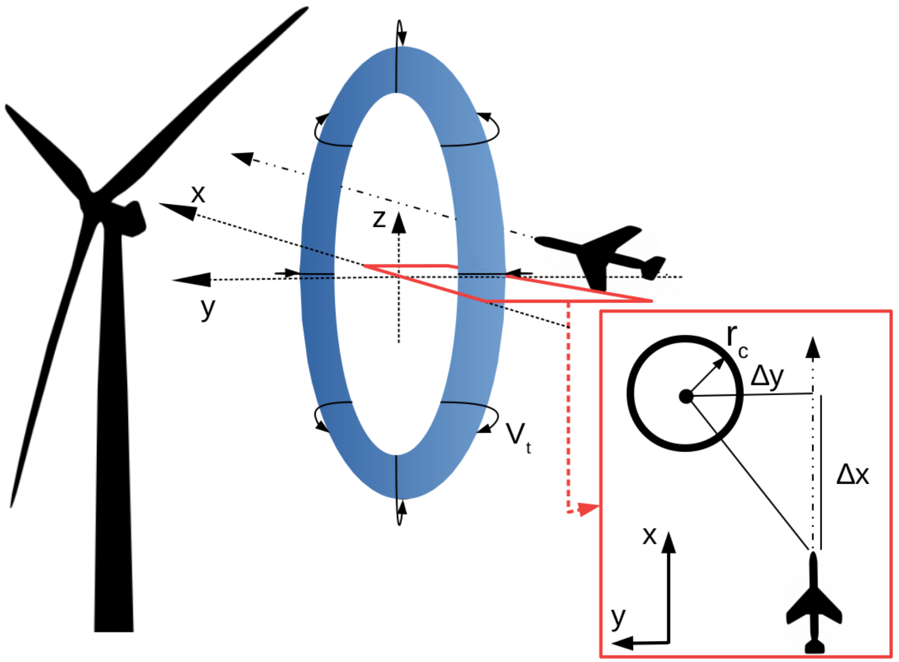


Figure 7. Simplified sketch of a vortex pair passed by the UAS to the right. In reality it would rather have a helical pattern than a ring shape. Velocities and axis according to meteorological standards, therefore axis and orientation according to the in-situ conditions. y axis pointing north, x axis pointing east. At hub height the w component (along z axis) vanishes. The red rectangle illustrates a top view of a tip vortex with distance Δy to the UAS.

considered. For the later analytical solution of the whole flight path perpendicular to the WEC wake, the BH model for two vortices is consulted.

The BH model does not provide a solution for the whole wake structure, but for an idealised 2D cut. Describing two (independent) counter rotating wake vortices with a simple analytical model and comparing it to in-situ measurements is a new approach in studying wind turbine wake structures.

Having a look at the Burnham-Hallock model, a vortex is described by its circulation Γ, tangential velocity V_t and its core radius r_c. The tangential velocity is the velocity of the air circling the vortex centre and is a function of the distance r to the vortex core.

$$V_\mathrm{t}(r) = \frac{\Gamma}{2\pi}\frac{r}{r_\mathrm{c}^2 + r^2}. \tag{4}$$

The core radius r_c is defined as the distance from the vortex centre (or core) at which the tangential velocity is at its maximum (circular symmetry). So the radius r_c is also the radius at which the surface integral (cf. Eq. 1) is maximal, considering a circular surface. For $r = r_\mathrm{c}$ the maximum tangential velocity becomes (Eq. 5)

$$V_\mathrm{t,\,max} = \frac{\Gamma}{4\pi r_\mathrm{c}}. \tag{5}$$

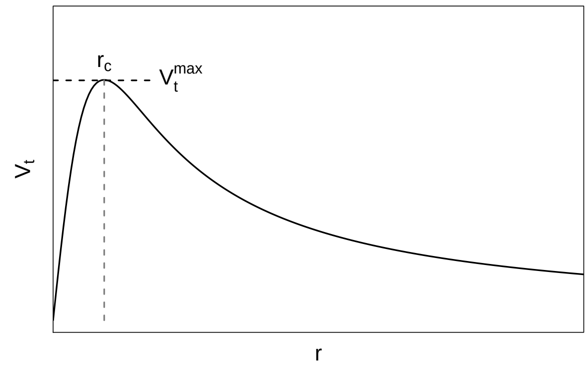


Figure 8. Qualitative plot of the tangential velocity from the vortex core outwards. The tangential velocity increases from zero (left) to a maximum at a distance r_c and decreases to zero for large distances (to the right).

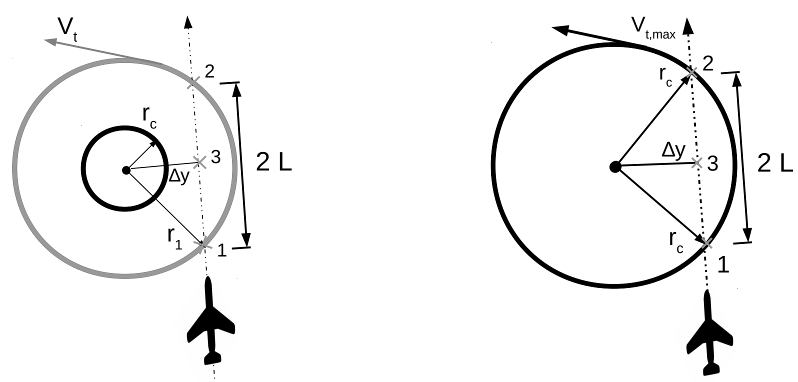


Figure 9. Schematic of the UAS passing a vortex in the horizontal plane (top down view). Two different cases have to be distinguished. The closest distance to the vortex $\Delta y > r_\mathrm{c}$ (a) and the passing distance being $\Delta y < r_\mathrm{c}$ (b).

Figure 8 shows the tangential velocity V_t distribution of a Burnham-Hallock modelled vortex with the highest tangential velocity at the distance $r = r_\mathrm{c}$. The distribution is circle symmetric with the vortex core ($r = 0$) in its centre.

In order to estimate the circulation and size of r_c from transects through the vortices with MASC in the wake of a WEC, the following procedure is proposed.

3.2 Evaluation method

As shown above, it is likely to measure tip vortices at hub height. At this height a simplification of the two vortices can be made. The blade-tip vortices can be considered as two dimensional vortices of circular shape in the horizontal plane and ideally the w component can be neglected. After subtracting the mean wind v_∞ the vortex tangential velocity is

$$\boldsymbol{v} - \boldsymbol{v_\infty} = \boldsymbol{v'} = (u', v', 0). \tag{6}$$

The norm of the tangential velocity then is

$$V_\mathrm{t} = \sqrt{u'^2 + v'^2}. \tag{7}$$

When measuring with a UAS the measurement can be considered a snap shot of the in-situ conditions. Figure 9 differentiates between two different scenarios of the UAS passing a vortex. Both shown from a top view. Both scenarios will be explained in detail in the following paragraphs, with first focussing on Fig. 9a. Here the UAS passes the vortex at its closest distance (Δy), marked point 3 in the sketch, with $\Delta y > r_\mathrm{c}$, thus the vortex core radius is not reached. Point 1 and 2 mark the position of two corresponding tangential velocities of identical absolute value, when approaching the vortex and moving away from it again. The measured signal is similar to the dashed black line in Fig. 10 that is an example for $\Delta y = 2r_\mathrm{c}$. From such data only point 3 can be identified, since it is the point at which the measured tangential velocity is at its maximum. Point 1 and 2 are somewhere left and right of the maximum with L being unknown. There are indefinite combinations of Γ and r_c that could describe the vortex using Eq. 4.

$$V_{\mathrm{t},2} = V_{\mathrm{t},1} = \frac{\Gamma}{2\pi}\frac{r_1}{r_\mathrm{c}^2 + r_1^2} \tag{8}$$

$$V_{\mathrm{t},\Delta \mathrm{y}} = \frac{\Gamma}{2\pi}\frac{\Delta y}{r_\mathrm{c}^2 + \Delta y^2} \tag{9}$$

$$r_1^2 = r_2^2 = L^2 + \Delta y^2 \text{ (Pythagorean theorem)} \tag{10}$$

The three equations 8, 9, 10 are known to describe the velocities and geometry of the measurement. $V_{\mathrm{t},1}$ ($V_{\mathrm{t},2}$) is the tangential velocity at the point 1 (and 2). Since there are four unknown parameters Γ, r_c, L, and $r_{1,2}$ the problem is not solvable.

Now we consider the case, when the UAS passes a vortex at $\Delta y < r_\mathrm{c}$, as shown in Fig. 9b. The measured tangential velocity now provides a distinct feature; a double peak in the horizontal wind measurement. This double peak is caused by passing the maximum tangential velocity at $r = r_\mathrm{c}$ at position 1 and 2. Since the tangential velocity decreases from that point inwards (towards the vortex core), the velocity at point 3 is a local minimum, leading to a visible 'dent' in the data (cf. red line in Fig. 10). Additionally the ground speed of the UAS is known, hence the distance L can be calculated. The three equations previously described above, then become:

$$V_{\mathrm{t},2} = V_{\mathrm{t},1} = V_{\mathrm{t,max}} = \frac{\Gamma}{2\pi}\frac{r_\mathrm{c}}{r_\mathrm{c}^2 + r_\mathrm{c}^2} = \frac{\Gamma}{4\pi r_\mathrm{c}} \tag{11}$$

$$V_{\mathrm{t},\Delta \mathrm{y}} = \frac{\Gamma}{2\pi}\frac{\Delta y}{r_\mathrm{c}^2 + \Delta y^2} \tag{12}$$

$$r_1^2 = r_2^2 = L^2 + \Delta y^2 = r_\mathrm{c}^2 \longleftrightarrow \Delta y^2 = r_\mathrm{c}^2 - L^2 \tag{13}$$

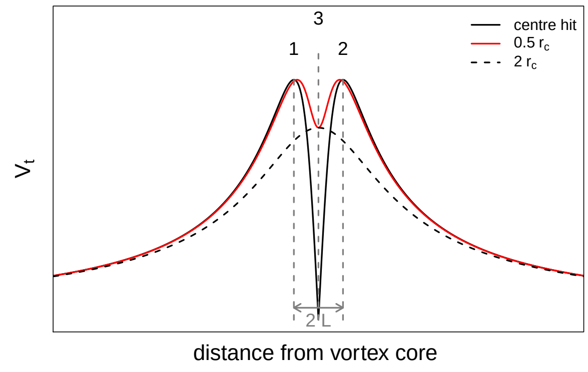


Figure 10. Analytical solution of a UAS passing the vortex at a path crossing the centre (black solid line), passing at $r = 0.5r_{\mathrm{c}}$ (red line) and at a distance double the core radius (black dashed line). The peak to peak distance is 2 L (cf. Fig. 9), above illustrated for the black solid line.

With now only 3 (Γ, r_{c} and Δy) unknown parameters it is possible to solve the equations.

Dividing Eq. 12 by Eq. 11 eliminates Γ. Inserting Eq. 13 gives:

$$\frac{V_{\mathrm{t,\Delta y}}}{V_{\mathrm{t,max}}} = \frac{r_{\mathrm{c}}\sqrt{r_{\mathrm{c}}^2 - L^2}}{r_{\mathrm{c}}^2 - \frac{L^2}{2}} \tag{14}$$

Equation 14 describes a tangential velocity ratio that is a function of L. Also L is known to range from 0 to r_{c}. A dimensionless relationship $L\, r_{\mathrm{c}}^{-1}$ can be plotted and is shown in Fig. 11. By passing the vortex with $\Delta y < r_{\mathrm{c}}$, and plotting the measured V_{t} against the distance to the vortex (Fig. 10), we can determine $L, V_{\mathrm{t,max}}, V_{\mathrm{t}}, \Delta y$. Using diagram Fig.11, we finally determine $L\, r_{\mathrm{c}}^{-1}$ and thus r_{c}.

3.3 Analytical reconstruction

As shown above, blade-tip vortices are theoretically measurable. They can be identified by their distinct 'dent' feature when the $\Delta y < r_{\mathrm{c}}$ criterion is met. Basic geometry and the Burnham-Hallock model further allow for a reconstruction (analytical solution) of the individually measured vortex, which is helpful to verify the measurements and evaluation technique. With Eq. 15 and 16 the distance to each vortex core (centre), to and along the UAV flight path, can be calculated (Fischenberg, 2011). In Fig. 12 the distances of the UAS to two vortices spinning in opposite direction are shown. In the figure, vortex 1 is passed through its core and vortex 2 is passed with a slight off-set. The flight path of the UAS is indicated with a red dashed line. Those distances are inserted into Eq. 17 and 18 using the relation of Eq. 4 the tangential velocity along the meteorological *x* axis (*u'* component) and *y* axis (*v'* component) can be calculated:

$$d_1 = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x - x_{\text{Vortex1}})^2 + (y - y_{\text{Vortex1}})^2} \tag{15}$$

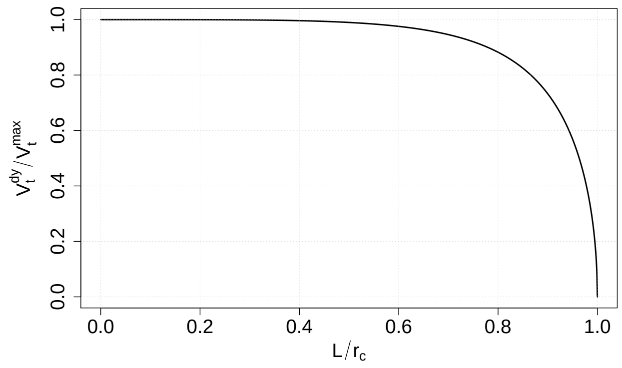


Figure 11. Dimensionless relationship between the ratio of the minimum (dent) tangential velocity and the maximum tangential velocity versus half the peak to peak distance (L), in percentage of r_c.

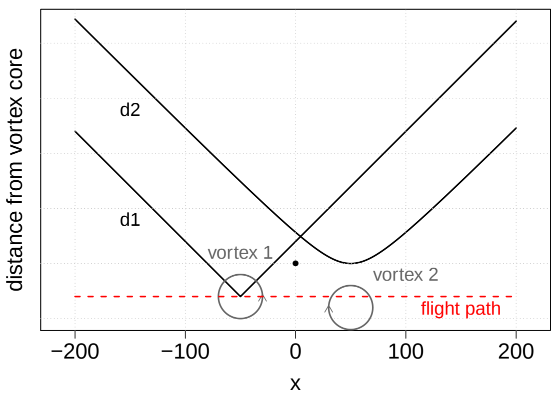


Figure 12. Qualitative example of an ideal flight path (vortex 1) and a passing with a little off-set (vortex 2) of the UAS. For the field measurement the distances $d1$ and $d2$ are calculated from the UAS GPS position and the location (off-set) of the vortex in relative coordinates with WEC at $(0, 0)$, indicated with a black dot. The vortex position can be derived from the extent of the tangential velocity V_t measured by the UAS and the peak to peak distance, explained in the previous sections. In this example $d_{1,2} = \sqrt{\Delta x^2 + \Delta y^2}$ with $\Delta y_1 = 0$ for $d1$ and $\Delta y_2 = \text{const.} \neq 0$ for $d2$.

$$d_2 = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x - x_{\text{Vortex2}})^2 + (y - y_{\text{Vortex2}})^2} \tag{16}$$

While the y coordinate can be derived from the measurement (using Δy and the UAS position, s.a. chapter 4.3) the x coordinate of the vortex $x_{\mathrm{Vortex1,2}}$ is the x coordinate of the flight path at the position '3', e.g. Fig. 9.

$$u' = V_{\mathrm{t}}(d_1)\left(\frac{y - y_{\mathrm{Vortex1}}}{d_1}\right) - V_{\mathrm{t}}(d_2)\left(\frac{y - y_{\mathrm{Vortex2}}}{d_2}\right) \tag{17}$$

$$v' = -V_{\mathrm{t}}(d_1)\left(\frac{x - x_{\mathrm{Vortex1}}}{d_1}\right) + V_{\mathrm{t}}(d_2)\left(\frac{x - x_{\mathrm{Vortex2}}}{d_2}\right) \tag{18}$$

4 Results

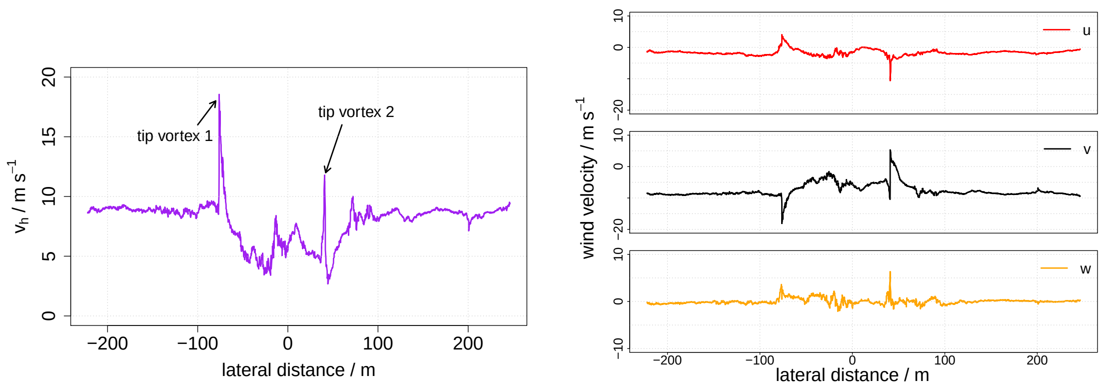
4.1 Vortex measurement

Figure 13a shows the $v_{\mathrm{h}} = \sqrt{u^2 + v^2}$ component of the wind measurement behind the WEC at hub height. The data reveals several (near) wake specific features. This flight leg shows two measurements of a tip vortex, indicated by the arrows in Fig. 13a. In-between those two peaks the wake deficit is measurable by a significant drop of the horizontal wind velocity. Due to the near vicinity to the nacelle, the wake deficit is dominated by turbulence created by the blade root vortices.

Figure 15 shows a zoomed-in look at the measured vortices depicted in Fig. 13. Figure 15a shows the vortex measured while entering the wake (vortex 1) and 15b while leaving the wake (vortex 2). Both, (a) and (b), are the plain UAS measurements. Figure 15c and Fig. 15d show the same measurement but the UAS coordinate system is rotated into the vortex rotational plane. This ensures that the rotational energy of the vortex is entirely captured by the u and v component, thus becoming the velocity components of the two dimensional rotational plane of the vortex, shown in Fig. 15 as a solid purple line. Examining both vortices, the velocity distribution pattern of the UAS passing at distance $r < r_{\mathrm{c}}$ is visible in the v_{h} measurement. The horizontal wind velocity v_{h} is a superposition of the tangential velocity, turbulence and the horizontal wind of the undisturbed inflow. The characteristics of the tangential velocity of vortex 1 (Fig. 15a,c) is almost solely determined by the *v* component, while in Fig. 15b,d the *u* component inheres an equal part. In the plain UAS measurement (i.e. before coordinate transformation) vortex 2 has a significant non-zero w component (Fig. 15b), indicating that the vortex did not rotate in the $x - y$ plane, hence the coordinate transformation into the vortex coordinate system. Especially Fig. 15d shows a significant reduction of the w component after the coordinate transformation. Purple dashed lines indicate the velocity deficit dV_{t} (dent), grey dashed lines the peak-to-peak distance. The dot-dashed purple line can be interpreted as an extension of the horizontal wind velocity by the w component, essentially giving the norm of the wind vector:

$$|\boldsymbol{v}| = \sqrt{u^2 + v^2 + w^2} \tag{19}$$

The described coordinate system rotation was applied with respect to the area between the grey dashed lines (Fig. 15) i.e. between entering and leaving the vortex. A good indicator that the data rotation was successful is when the norm of the wind vector (purple dashed line) and the v_{h} in-between the grey dashed lines are about the same magnitude. Then it can be



(a)

(b)

Figure 13. (a) Horizontal wind v_{h} measurement at hub height in the WEC wake at a distance of 0.25 D to the nacelle. The two tip vortices are indicated by the arrows. (b) The same measurement split into the three wind components u, v, w.

concluded that the two dimensional vortex rotation (u and v components) includes the entire kinetic energy, i.e. the vertical wind component is now neglectable.

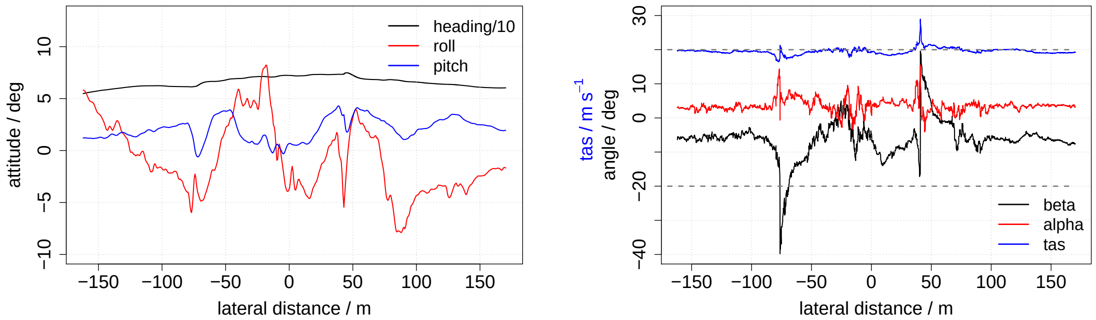
Table 2 shows the derived parameters from the vortices depicted in Fig. 15. It has to be mentioned that vortex 1 made for a better and clearer measurement, since vortex 2 is influenced by the wind deficit and turbulence inside the wake. Vortex 1 shows a sharp jump in the tangential velocity which makes it easier to obtain the necessary quantities and provides more reliable results. The average of the obtained circulations is $\overline{\Gamma} = 74.17\ \mathrm{m^2\ s^{-1}}$, the average core radius is $\overline{r}_{\mathrm{c}} = 0.61\ \mathrm{m}$.

Figure 16 shows a two dimensional cut through a skewed or canted vortex that results in an ellipse where the peak to peak distance is $2\ L'$. This peak to peak distance is under-predicted ($2\ L' < 2\ L$). The introduced error $\Delta y'$ is visualised in Fig. 16 by dotted red lines. To overcome this issue the measured data are rotated into the vortex hose if necessary. This simulates the UAS canting to follow the oblique vortex hose.

4.2 Quality control and error estimation

The wake of a WEC, especially as close as 0.25 D behind the nacelle, is a highly turbulent region. When measuring with an autonomous UAS, it is of interest whether the UAS is capable of manoeuvring stably in such an environment and if the measurement instrument (e.g. 5-hole probe) is operating within its operational specifications. Figure 14 shows the attitude of the UAS (a) while passing the WEC for the consulted flight leg and the angle of attack, sideslip and true air speed (b). The UAS is affected by the wake entry and exit. The motions of the UAS are well recognised by the IMU and auto-pilot (cf. Fig. 14a) and taken into account for the later post-processing. The UAS handles these motions without loss of control.

Grey dashed lines in Fig. 14b indicate the limit of the calibrated range of $\pm 20°$ of the 5-hole probe. Passing a tip vortex at $\Delta y < r_{\mathrm{c}}$ is an extreme event, not only for the aircraft, but also for the pressure probe. Angle of attack and sideslip are within the calibrated ranges with one exception of vortex 1. Here, the sideslip is extrapolated. An examination of the true air speed



(a) (b)

Figure 14. (a) Attitude angles of the UAS from flight leg taken for the tip vortex evaluation. (b) Angle of attack (alpha) and sideslip (β) at the 5-hole probe. Grey dashed lines indicate the calibration range of the 5-hole probe. Overstepped angles are extrapolated in post-processing. In blue the true air speed of the UAS.

Table 2. Determined parameters from vortex measurements.

Vortex	dV_{t} $[\mathrm{m\,s^{-1}}]$	$V_{\mathrm{t,max}}$ $[\mathrm{m\,s^{-1}}]$	$V_{\mathrm{t,\Delta y}}$ $[\mathrm{m\,s^{-1}}]$	$V_{\mathrm{t,\Delta y}}/V_{\mathrm{t,max}}$ $[-]$	L [m]	$L/r_{\mathrm{c}}[-]$	r_{c} [m]	Γ $[\mathrm{m^2\,s^{-1}}]$
Vortex 1	3.4	9.6	6.2	0.65	0.61	0.93	0.66	81.30
Vortex 2	0.2	9.7	9.5	0.98	0.3	0.55	0.55	67.04

(TAS) for the measurement (blue line in Fig. 14b) shows clearly the entry and exit of the wake. Changes in true air speed cannot be avoided. Usually small deviations from the calibrated TAS value of the 5-hole probe do not result in significant changes in the calculated wind speed. The peaks visible in the TAS measurement however, will have an effect on the wind velocity calculation. The influence of different air speed calibrations on UAS measurements is studied by Rautenberg et al. (2019a). There it is concluded that the deviation from the "true" wind speed is about 10% or at most $1\ \mathrm{m\,s^{-1}}$, e.g. for a TAS error measured at vortex 2 of about $8\ \mathrm{m\,s^{-1}}$. So the peak velocities may be underestimated by $1\ \mathrm{m\,s^{-1}}$.

While this error has no significant influence on the ratio $V_{\mathrm{t,\Delta y}}/V_{\mathrm{t,max}}$ it is significant when calculating the circulation Γ from Eq. 5. In the presented case the circulation of vortex 2 is under predicted by about 10%.

4.3 Vortex reconstruction

The BH model provides a solution for two vortices spinning in opposite direction, as, for example, found in an aircraft wake. A similar constellation of vortex pairs can be found in a WEC wake at hub-height (cf. Fig. 7), with their vortex cores positioned along the x axis. This approximation can only be done, when the flight path is perpendicular to the wind (wake) direction to assure that the measured vortices are of the same age.

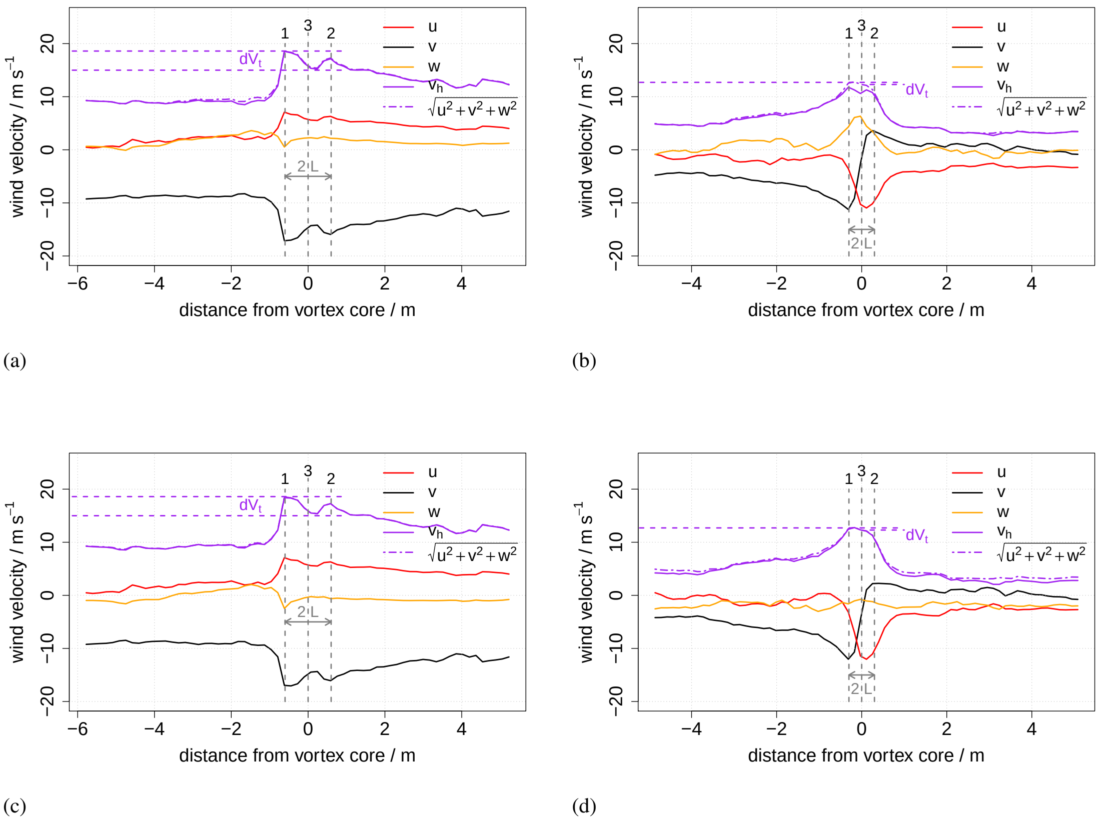


Figure 15. Measured tip vortex 1 and tip vortex 2 from Fig. 13a. Purple dashed lines indicate the velocity deficit (dent), grey dashed lines the peak-to-peak distance. The horizontal wind velocity v_{h} is a superposition of the tangential velocity and the horizontal wind of the inflow/surroundings. To eliminate the w component the data has been rotated into the vortex coordinate system. This is necessary to measure the vortex correctly. The sub-figures (a) and (b) show the plain UAS measurement of the vortices. In sub-figures (c) and (d) the UAS has been rotated into the vortex coordinate system (vortex plane) to capture the whole two dimensional rotation.

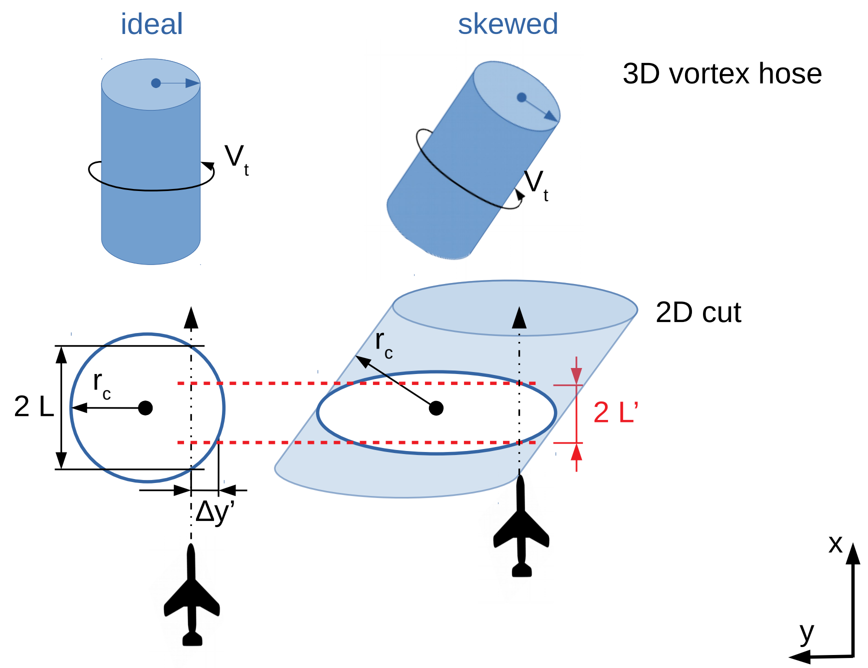
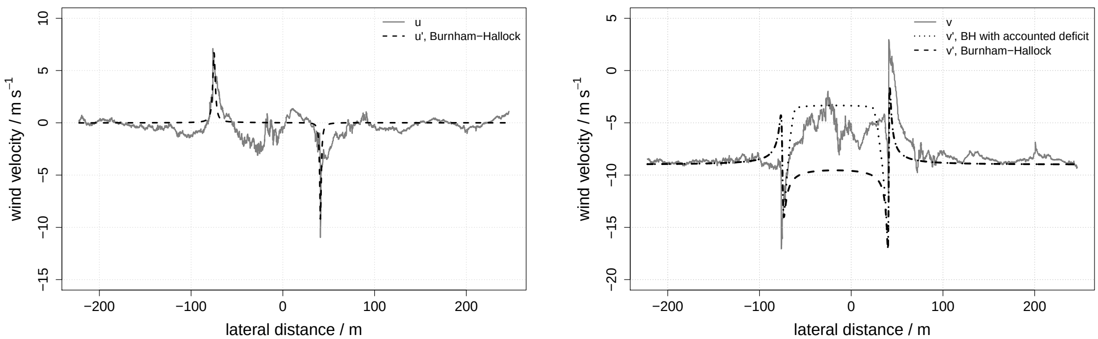


Figure 16. Sketch of an ideal and skewed (exaggerated) vortex hose at hub height. The simplifications in the evaluation method only consider components in the $x-y$ plane which leads to an under-prediction of the real peak to peak distance. The fact that the real 'r_c' does not lie in the $x-y$ plane leads to an error. A horizontal cut though the vortex has an ellipsoidal geometry instead of a circular one, as in ideal measurement conditions.

With the average values $\overline{\Gamma}$ and $\overline{r}_c$ retrieved from Tab. 2 the minimum measured tangential velocity between the two peaks (position '3' in Fig. 15) as well as a distance Δy can be derived. The resulting distance to the vortex core Δy can then be fed to a model, based on the Burnham-Hallock approach. Figure 17 shows the analytical solution of u' and v' overlain measured data of *u* an *v*. Overlain to the in-situ data the tangential velocity still contains the mean horizontal wind $V_t = \sqrt{u'^2 + v'^2}$. For the analytical solution the measured data has been rotated slightly (ca. $10°$) into the mean wind direction to fit the meteorological coordinate system with the vortex coordinate system, so the *u* component equals zero in average and *v* is the predominant horizontal wind direction. In addition to the solely Burnham-Hallock solution for the v' component (dotted line in Fig. 17b), the long dashed line shows the same solution, but multiplied with a correction factor to satisfy for the wind deficit in the wake. The general vortex model does not consider the mean horizontal velocity, so it needs to be accounted for, especially when there is a artificially induced drop behind the WEC in the wake (wake deficit). In the present case the velocity deficit was measured to be about 65 %. It is visible as a jump in the mean horizontal wind between the two measured vortices. Similar deficits were already measured by Wildmann et al. (2014a) or Bartl et al. (2012). The velocity correction function is simply an upside down Tukey window. The analytical solution remains uncorrected until entering the wake of the WEC. After incorporating a deficit correction to the analytical solution, it is visible that the deficit in the wake plays an important role to the structure (placement,



(a) (b)

Figure 17. Analytical solution (dashed lines) for u' and v' of the two vortices from the parameter evaluation. The corresponding wind components u and v from (Fig. 13b) from the UAS measurements in grey. The long dashed solution in (b) additionally accounts for the deficit in the wake.

intensity, etc.) of the vortex. Especially since the two vortices do not interact with each other, as the two vortices in the BH model for aircraft wakes do.

5 Discussion

Here we compare the airborne measured circulation Γ with data of the WEC itself. Equation 20 allows for a calculation of the blade-tip vortex strength by given parameters and describes the circulation for a rotor of constant thrust coefficient, e.g. (Sørensen et al., 2014):

$$\Gamma = \frac{\pi v_\infty^2 C_T}{\Omega N_\mathrm{b}} \approx 66.2\ \mathrm{m}^2\ \mathrm{s} \tag{20}$$

N_b being the number of blades and Ω the rotational velocity provided by the owner of the WEC. For the determination of the thrust coefficient C_T the following estimation is done:

The relatively low wind speed ($v_\infty = 8.8\ \mathrm{m\ s}^{-1}$ by UAS measurement) implies a pitch angle of $\beta = 0°$ when approximating the E-112 with the NREL 5 MW offshore WEC (Jonkman et al., 2009).

The tip-speed ratio ($TSR = \frac{\Omega R}{v_\infty}$) can also be calculated and thus a thrust coefficient $C_T \approx 0.8$ can be estimated from the C_T to TSR relationship by Al-Solihat and Nahon (2018).

The calculated value for Γ from WEC specific and atmospheric parameters is similar to the vortex strength that was extracted from the vortex measurements (average $\overline{\Gamma} = 74.17\ \mathrm{m}^2\ \mathrm{s}^{-1}$). The presented method, to calculate Gamma from UAS data, provides reasonable results, and the geometric simplification of the tip vortex and the application of the Burnham-Hallock vortex model both are valid.

The BH vortex model does work for aircraft induced vortices as shown by Ahmad et al. (2014) as well as Fischenberg (2011) and as the results imply, it can be used to describe WEC wake vortex properties. Not least, both phenomena can be described by two vortices spinning in opposite direction, yet there is no interaction of the two opposite vortices, as in the aircraft wake model usually intended. Vortex patterns of a WEC wake show higher complexity than aircraft wake vortices. The whole wake is in motion and different turbulence and shear forces interact with each other. Therefore, for the wake vortices some simplifications had to be made, e.g. the shown evaluation method is only valid for a 2-D cut of the whole vortex hose. Also the blade root vortex was not analysed any further.

In this study also the fact that the UAS experiences a change in true air speed (TAS) when entering the wake is addressed. Theoretically the calibration range of the used five hole probe is for a fixed air speed which changes when entering the wake. Since this evaluation uses the ratio of two velocities the influence of a different calibration for the five hole probe does not lead to a significant error. For the calculation of the circulation Γ, however, absolute velocities are necessary and a small error can be expected due to a change in TAS when entering and leaving the wake velocity deficit. The error is estimated to be $\pm 10\ \%$ for the calculated wind velocities (Rautenberg et al., 2019a). An error estimation is given in Section 4.2.

6 Conclusions and outlook

The resulting circulation strength Γ derived from UAS data shows good accordance to the results obtained from Eq. 20. It can be concluded that the evaluation method, using the basic geometrical properties of a vortex, can be used to derive vortex properties in a WEC wake. Turbulence acting on the vortex and on the surrounding atmospheric flow can aggravate an evaluation since the evaluation is done mainly graphically. For example the second tip vortex is embedded in a relatively high level of turbulence (wake deficit, shear, etc.). It also does not show a clear border to the undisturbed atmosphere as tip vortex 1 does. The reference velocity levels for the evaluation are therefore harder to extract from the measurements. Also a hit of a blade-tip vortex in flight changes the true air speed (TAS) locally and temporally, resulting in a (small) error in the velocity measurement.

In addition, this method still has to be proven at larger distances to the WEC nacelle, where the vortices might begin to meander and get unstable. However, to our knowledge, this is the first quantitative analysis of WEC tip vortices using in situ measured turbulence data by a fixed wing UAS.

The MASC Mk 3 system is capable of measuring detached tip vortices in the wake of a WEC. The spatial and temporal resolution is sufficient to detect vortex patterns in the measurements. However, on many occurrences, the measured sideslip β left the calibration range of the 5-hole probe and in conclusion, those measurements could not been used. For future measurements the calibration of the 5-hole probes could simply be expanded to larger angles. This also allows for a lower TAS (true air speed) of the UAV, which in turn results in a better spacial resolution of the data. The path accuracy of the UAS will be upped by using an RTK (real time kinematic) GPS.

The proposed analytical vortex model by Burnham and Hallock is capable of describing WEC wake vortices. Yet, as for most analytical models, the analytical solution shown in this paper can and should be improved. E.g. to better fit the WEC wake (velocity deficit, blade root vortex near the nacelle). This evaluation was conducted with data obtained at 0.25 D from the nacelle. For a future additional field campaign blade-tip vortices in the farther wake shall be investigated.

Competing interests. The author declares that he has no competing interests.

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First Identification and Quantification of Detached Tip Vortices Behind a WEC Using Fixed Wing UAS

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Abstract.

Quantifying blade tip vortices helps to understand the process of vortices detaching from the wind converter blade and their development in the wake until finally dissipating in the far wake , contributing to overall turbulence. This is especially interesting for set-ups of numerical simulations when setting the spatial resolution of the simulation grid.

The MASC MK In the present study, blade-tip vortices have been experimentally identified in the wake of a commercial wind turbine using the MASC Mk 3 (Multi-purpose Airborne Sensor Carrier Mark 3) UAS(Unmanned Aircraft System) by the University of Tübingen measured atmospheric and meteorological quantities during the HeliOW campaign in July 2018 data behind a wind energy converter (WEC) (Enercon E-112) north of Wilhelmshaven, Germany, at the Jade Wind Park. Aside turbulence distribution, air temperature, humidity and the three wind components u, v, w in front of the WEC and in the wake were measuredof the University of Tuebingen. By evaluation of the wind components, detached blade tip blade-tip vortices were identified in the time series. From these measurements the circulation and core radius of a pair of detached blade-tip vortices is calculated using the Burnham-Hallock (BH) wake vortex model. The presented data were captured under a dominating marine stratification about 2 km from the North Sea coast line with northern wind direction. The measured vortices are also compared to the analytical Burnham-Hallock solution of the BH model for two vortices spinning in opposite direction. The model has its origin in aviation, where it describes two aircraft wake vortices . It will be shown that the BH model can be used to describe wake vortices behind a WEC. spinning in opposite direction.

An evaluation method is presented to measure detached tip vortices with a fixed wing UAS. Also an improvement for The BH model will be used to describe wake vortex properties behind a wind energy converter (WEC). The circulation and core radius of detached blade-tip vortices will be calculated. Also proposition of the model in WEC wake use will be proposedmade to describe two independent co-rotating vortices. Quantifying blade-tip vortices helps to understand the process of vortices detaching from a rotor blade of a wind turbine, their development in the wake until finally dissipating in the far wake and contributing to overall turbulence. This is especially interesting for set-ups of numerical simulations when setting the spatial resolution of the simulation grid.

1 Introduction

~~Since the politically induced 'Energiewende' (exit from nuclear and fossil energy sources) in the late 1990s in Germany, the~~ The wind energy sector has been growing world wide for decades and the produced power from wind energy is still growing. Not only ~~in~~ the amount of installed wind energy converters (WECs) ~~but also in~~ is increasing but also their single capacity~~and diversity.~~ . Also the field of application has widely increased with WEC. There are systems available for homogeneous terrain, off or near shore, or even complex terrain with a high amount of additional turbulence stress that is induced onto the wind turbine's blades.

A modern off-shore WEC delivers up to ~~9~~ 12 MW of power in ideal conditions. ~~The constant enhancements of the concept of harvesting energy from wind lead to a wide field of applications for WECs, e.g. in homogeneous conditions off-shore as well as in complex terrain further inland. In research and development numerical simulation~~ In wind energy research numerical simulations of the wind velocity field of a WEC and its produced turbulence ~~is an important tool that gives valuable insights of pressure~~ are important tools that give valuable informations. Pressure and velocity distributions around a turbine blade and nacelle as well as in the wake can be studied. A numerical model increases its validity when it is backed by real world in-situ data. Not least, because of improvements that can flow back into the model, once measured data has revealed some possible tweaks and enhancement to the prior model. Numerical simulation might ~~under predict~~ underestimate peak vorticity and radii of wake vortices, especially when the grid size of the simulation is not sufficient (Kim et al., 2016). Another ~~mean~~ way of studying WEC wakes are wind tunnel experiments that try to recreate wake patterns in a smaller scale (e.g. Bartl et al. (2012) or Vermeer (1992)). While in the early days of wind tunnel experiments the wake has been visualised by smoke trails, PIV (Particle Image Velocimetry) measurements have increased the resolution and accuracy of wind tunnel experiments drastically (e.g. resolving Reynolds shear stress and turbulent kinetic energy, Zhang et al. (2012)).

~~The MASC Mk 3 system allows in-situ high frequency measurements of the atmospheric flow and its transported properties. A detailed description of the UAS and its instruments can be found in Wildmann et al. (2014a) and Wildmann et al. (2014b). The latest iteration MASC Mk 3 is using an improved IMU (Inertial Measurement Unit) and positioning system. The lateral positioning accuracy is now in~~ But a common issue with wind tunnel measurements is that they usually suffer from scaling problems (Wang et al., 1996). Remote sensing techniques like LIDAR have also found their way into WEC wake evaluations. Various measurement strategies were developed to visualise WEC wakes e.g. in complex terrain (Barthelmie et al., 2018). LIDAR scans provide a long term measurement of a ~~range of 2 , whilst the vertical path accuracy has also improved by approximately 0.45 m through implementing revised design elements in the fuselage allowing a more stable flight path and using a more precise GPS.~~

~~For this study of the near wake of a WEC an Enercon E-112 prototype located at the Jade Wind Park north of Wilhelmshaven at the German North Sea coast has been chosen.This specific converter and its location near the coast is comparable with off-shore converters in marine flow which was a requirement when choosing the WEC. The measurementsare part of the HeliOW project, in which the atmospheric turbulence in front of and in the wake of a WEC are the foundation of a chain of numerical simulations. The project aims for save helicopter flight paths in off-shore wind energy parks. The numerical~~

simulation chain also includes flight-mechanical simulations (provided by Technical University Munich and DLR Braunschweig) of helicopters interacting with turbulence generated by WECs. The helicopter flight simulations are coupled with a numerical simulation of the marine flow over an off-shore 4.5 MW wind energy converter. The WECsimulations are done by the Univerity Stuttgartprobed volume or plane. The spatial resolution $(25 - 50\,\mathrm{m})$, however, is comparably coarse. LIDARs can provide a continuous monitoring of WEC wake structures (e.g. wake centre, direction and wind velocity deficit) (Bodini et al., 2017) in homogeneous or even in complex terrain (Wildmann et al., 2018). UAS (Unmanned Aircraft System) measurements can provide in-situ line measurements, covering a small volume but with a high temporal and spatial resolution in (deca-) centimetre range. The coverage of these scales is important to measure detached tip vortices in the near wake of a WEC.

A WEC, especially in a stable marine ABL (atmospheric boundary layer), acts as a turbulence generator. The added turbulence has two main sources. On the one hand the increased wind shear in the wake that results from the wind deficit in the near wake and the low pressure bulb that develops behind the WEC nacelle. On the other hand turbulence is created by expansion and dissipation of detached blade tip blade-tip vortices that transfer their kinetic energy to the surrounding flow. A proper understanding of these vortices and their inducing induced load onto the converter blade is of great importance for future enhancement of life span and working loads of wind energy converters . Blade tip in wind farms. Blade-tip vortices follow a helical pattern into the wake, detaching from each converter blade. These detached eddies can be measured with the mounted five-hole-probe on the MASC UAS. Subramanian et al. (2015) measured the presence of detected tip vortices via pressure fluctuations qualitatively in a flight pattern along the wake, also using a small UAS. In this study an evaluation method is presented to measure the core radius r_c, circulation Γ and maximum tangential velocity $V_{t,max}$ of a tip vortex r_c using in-situ wind measurements from flight patterns UAS flights perpendicular to the mean wind velocity.

2 Measurement system and measurement site

2.1 Research aircraft

The research UAS MASC Mk 3 (cf. Fig. 1 and Tab. 1) is a fixed wing airborne measurement system of the University of Tübingen that has been used in several measurement campaigns and has been described by Wildmann et al. (2014a, b) Wildmann et al. (2014a, b). The third iteration of this platform features some changes to the fuselage. The electrical pusher motor has been moved from a centre position behind the wings to the tail, accelerating the aircraft along the centre axis and increasing flight stability. The time span in which the presented data was captured was about 15 minutes. It can be expected that atmospheric conditions (wind direction and speed) did not change during this period. MASC Mk 3 system allows in-situ high frequency measurements of the atmospheric flow and its transported properties. A detailed description of the improved UAS and its instruments can be found in Rautenberg et al. (2019b). The latest iteration MASC Mk 3 is using an improved IMU (Inertial Measurement Unit) and positioning system.

Aside the manual changes to the platform changes in fuselage design, the former ROCS autopilot operating on the MASC Mk 2 system has been changed to the Pixhawk 2.1 autopilot. This is an independent open-hardware and open-source autopi-

Figure 1. Research UAS MASC Mk 3 shortly before lift-off. (Photo taken by the author).

Table 1. Characteristics of the MASC Mk 3 UAS at the HeliOW campaign.

wingspan	4 m
total weight	≈ 7 kg
sci. payload	≈ 1 kg
cruising speed	19 m s^{-1}
endurance	up to ~~2~~ 2.5 h
propulsion	electrical pusher engine
take-off	bungee or winch

lot project (www.pixhawk.org). ~~The Pixhawk autopilot is equipped with RTK GPS, which enables a track accuracy in the centimetre range.~~

2.2 Measurement site

Figure 3 shows the location of the measurement site in the north-west of Germany and the flight tracks of the MASC Mk 3 UAV around the Enercon E-112 converter. Both tracks are part of a rectangular flight pattern around the WEC in anti-clockwise direction. For the wake data evaluation only the data captured along the southern flight tracks (orange path in Fig. 3) are used. The E-112 WEC is the most powerful converter in the Jade-Wind-Park north of Wilhemshaven, Germany. The particular converter is a former near-shore prototype with a ~~diameter~~ rotor diameter D of 114 m delivering up to 4.5 MW of electrical power and thus comparable to an actual off-shore WEC. The Jade-Wind-Park is located about 2 km from the North Sea coast line and a maritime influence in the wind profile can be expected.

Apart from surrounding WECs (to the south of the E-112 WEC) power lines to the east and north and industrial buildings to the north and north-east (not in the picture) restricted the flight path to the ones depicted in Fig. 3.

Figure 2. Research UAS MASC Mk 3 in front of an Enercon wind energy converter at the Jade Wind Park in July 2018. (Photo taken by the author).

For this study of the near wake of a WEC, the wind turbine described above, has been chosen. This specific converter and its location near the coast is comparable with off-shore converters in marine flow which was a requirement when choosing the WEC. The measurements are part of the HeliOW project, in which the atmospheric turbulence in front of and in the wake of a WEC are the foundation of a chain of numerical simulations. The goal of the project is also to determine safe helicopter flight paths in off-shore wind energy parks. The numerical simulation chain also includes CFD simulations of the wind turbine (University of Stuttgart) which are injected in flight-mechanical simulations of a helicopter (provided by Technical University of Munich and DLR Braunschweig). Thus, the tip vortex measurements are an important contribution to the validation of later numerical simulations of the flow.

2.3 Available data

For the tip vortex evaluation five flight legs (straight and level fly-by's) are available 0.25 D downstream the WEC rotor plane. Only one of these legs shows the necessary criterion for the circulation and core radius calculation (cf. the following sections). This one leg (two vortex measurements) that fits the criterion will be shown exemplary to present the evaluation method and

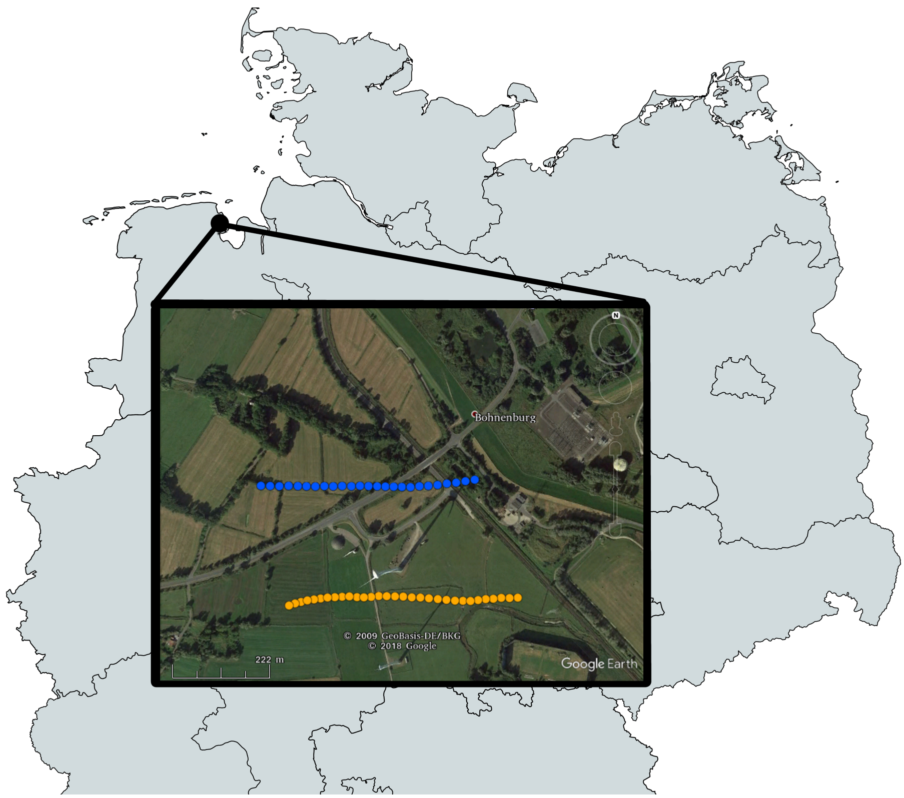


Figure 3. Location of the E-112 WEC in the north-west of Germany north of Wilhelmshaven near the North Sea coast. MASC flight tracks in front (blue) and in the wake (orange) of the E-112 with northern main wind direction (5 degree north that day). On the Google Earth image the WEC is oriented toward south-easterly wind direction. Map created with mapchart.net

the analytical solution using the BH model approach, including also the approach by Sørensen et al. (2014).

The presented data was captured within 15 minutes. It can be expected that atmospheric conditions (wind direction and speed, thermal stratification, turbulence intensity) did not change significantly during this period. The average wind speed in the inflow was $8.8\ \mathrm{m\ s^{-1}}$ with a turbulence kinetic energy (TKE) of $0.085\ \mathrm{m^2\ s^{-2}}$ at hub height. These values have been calculated from a ten second measurement in the undisturbed atmosphere.

3 Methods

With the goal to measure detached tip vortices behind a WEC, it is helpful to have ~~a first~~ at first an understanding of the behaviour of those vortices. Fig. 4 shows the helical vortex pattern forming behind a WEC, by representing the iso-surfaces of the λ_2-criterion of detached tip vortices from CFD simulation. The fully resolved URANS simulation has been performed ~~at~~ by the University of Stuttgart with the compressible flow solver FLOWer (Kroll and Fassbender, 2005), using the Menter SST (Menter, 1994) turbulence model. The modelled rotor is a stand alone generic ~~recreation~~ model of the Enercon E-112 WEC

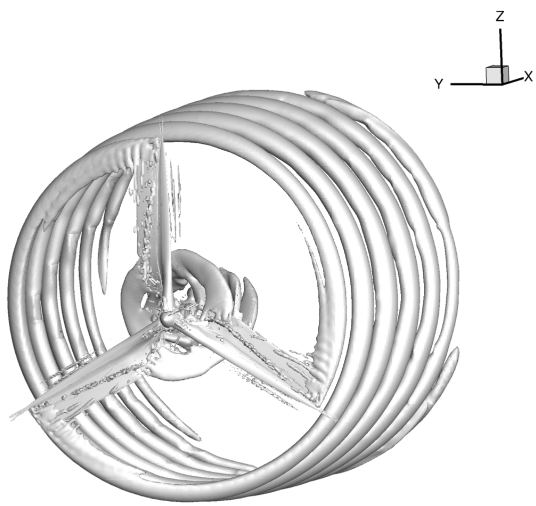


Figure 4. Iso-surfaces of detached ~~blade tip~~ blade-tip and root vortices following the λ_2 criterion for vortex identification. Here the x axis follows the main wind direction. Numerical simulation of a generic ~~recreation~~ model of an E-112 4.5 MW converter.

rotor, based on free access airfoil data. For more details regarding the numerical methods, please refer to Cormier et al. (2018) in which the same methods have been applied and described. Figures 5 and 6 give a qualitative impression of the presence of the WEC wake. In both, horizontal wind velocity and turbulence kinetic energy (TKE), the wake and its effects are visible. Farther downstream the helical pattern will start to meander and the symmetrical pattern will dissipate into turbulence. In the near vicinity of the WEC nacelle, these vortices follow a helical pattern. The helical structure is shown simplified by a ring vortex in Fig. 7 ~~. Note that the~~ which is an approximation of the wake vorticity at high tip-speed ratio. The tangential velocity in this sketch can be split in its horizontal components at hub height (nacelle height). Here the y axis points north (~~parallel to~~ ideally antiparallel to the main wind direction) similar to the conditions at the HeliOW campaign (cf. Fig. 3) and the x axis points east along the UAS flight path. ~~When measuring~~ Note that, at hub height, the tip vortex ~~has ideally~~ ideally has no w component (Fig. 7) under the vortex ring assumption. Thus, at this height, the tangential velocity can be split into its horizontal components u and v. The red rectangle indicates a change of perspective, showing a top view of a vortex spinning in the $x-y$ plane. ~~For~~ In reality, from planing flight paths until take-off of the UAS and the actual measurement, the wind direction changes slightly. Therefore, for later evaluations the coordinate system has been rotated into the main wind direction.

3.1 Vortex model

To measure and evaluate tip vortices from UAS data an analytical vortex model has to be found. Previous efforts to define a vortex were reviewed e.g. by Jeong and Hussain (1995), comparing several definitions with data from direct numerical simulations and exact solutions of the ~~Navier Stokes~~ Navier-Stokes Equations. A universal definition of a vortex or a generally applicable model does not exist. Assuming incompressible flow and an irrotational velocity field, where the curl of the gradient

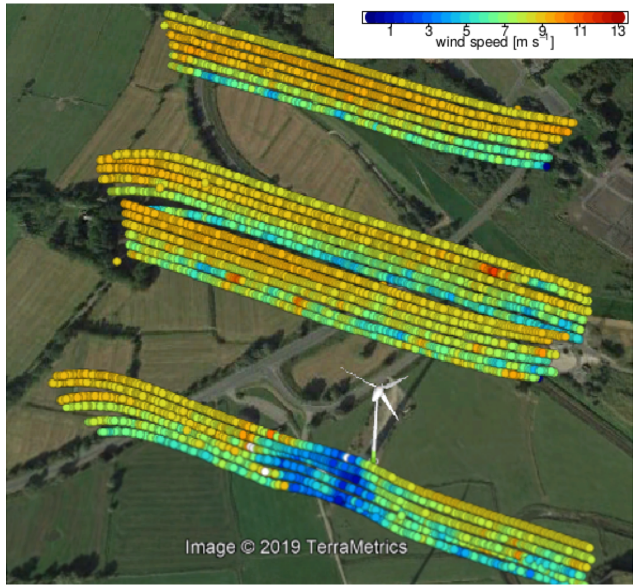


Figure 5. Visualisation of the horizontal wind measurements at different flight leg altitudes (from 85 m to 185 m above ground in 20 m steps) and different distances to the WEC (1D, -1D, -2D and -4D). Significant wind deficit 1 D behind the WEC E-112. At this day the wind direction was about 30° north. Image generated in Google Earth.

of the velocity is zero, ~~potential theory is applicable. Considering potential flow,~~ the circulation Γ, representing the strength of a vortex ~~,~~ around a contour C, can be connected to the vorticity flux by ~~Stokes'~~ Stoke's theorem. For any surface S that spans the curve C and $d\boldsymbol{I}$ being an infinitesimal tangential element along C,

$$\Gamma = \oint_C \boldsymbol{V_\mathrm{t}} \cdot \mathrm{d}\boldsymbol{I} = \int_S \boldsymbol{\omega} \cdot \boldsymbol{n}\, \mathrm{d}S. \tag{1}$$

The circulation Γ is the line integral of the tangential velocity along the curve C which is equal to the vorticity flux $\omega = \nabla \times V_\mathrm{t}$ through the surface S, with $\boldsymbol{n}$ being the normal vector of the surface.

A circular integration in a cylindrical polar coordinate system with the azimuthal angle ϕ and the radius r yields:

$$\Gamma(r) = \int_0^{2\pi} \int_0^{r} \boldsymbol{\omega}(r, \phi) r \, dr \, d\phi \tag{2}$$

For a two dimensional, axisymmetric vortex, the circulation

$$\Gamma(r) = 2\pi r V_\mathrm{t}(r) \tag{3}$$

is a simple function of the radius and the tangential velocity V_t. Since real vortices in fluids experience viscous effects, the structure of detaching tip vortices of the blade of WEC cannot be sufficiently described by Equation 3. Close to the centre of the vortex, lower tangential velocities persist, increasing to their maximum at the core radius r_c of the vortex and decreasing

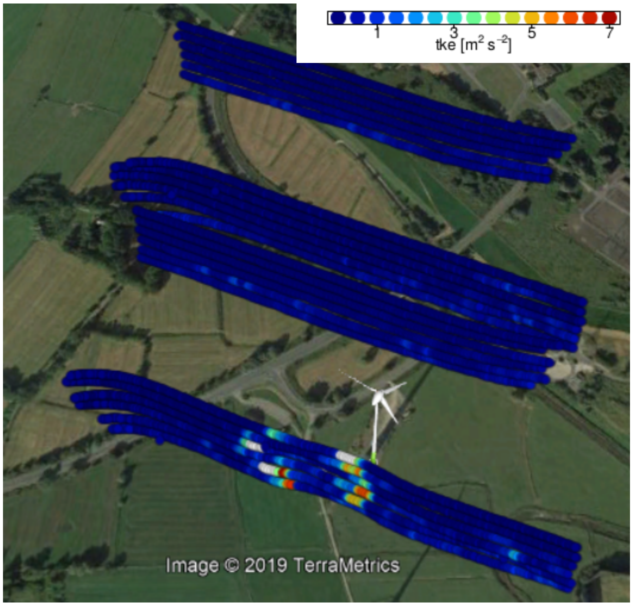


Figure 6. Visualisation of the turbulence kinetic energy (TKE) from the same measurements as in Fig. 5. Blue areas represent low turbulence and red the highest measured. At 1D the presence of the wake and the produces produced turbulence by the tip vortices is visible. The averaging window for the for TKE calculation is 1 s, corresponding to 100 data points, and suits only for qualitative reading only. Image generated in Google Earth.

again for further farther distances r. To account for that, in the context of WEC and also for detaching tip vortices from the wings of aircraft, several analytical models were suitablean analytical model is necessary.

Since in this study detached vortices of a WEC converter are treated similar to aircraft wake vortices, a few similar model approaches were possible. A comparison of analytical vortex models for tip vortices created by aircraft has been done by Ahmad et al. (2014). Also Fischenberg (2011) measured wake vortices created by the VFW 614 ATTAS manned aircraft (DLR Braunschweig) and compared the results to two similar vortex models proposed by Lamb (1939) and Burnham and Hallock (1982). Fischenberg concludes that both models show the ageing processes of a vortex wake known from theory. In general the model by Burnham-Hallock shows a slightly better agreement in circulation and tangential velocity to the conducted measurements by Fischenberg. Also Vermeer (1992) uses the Burnham-Hallock BH vortex model successfully to describe WEC wake vortices. According to these findings and its simplicity it has been decided to use the analytical solution for wake vortices by Burnham-Hallock in this study. While the two counter rotating vortices in the BH model used in aviation interact with each other, the two opposite vortices in a WEC wake do not do that. This is an important detail to point out. So for the identification of the vortex parameters (Γ, r_c) a model of two counter-spinning vortices is not necessary. Here, a stand alone vortex is considered. For the later analytical solution of the whole flight path perpendicular to the WEC wake, the BH model for two vortices is consulted.

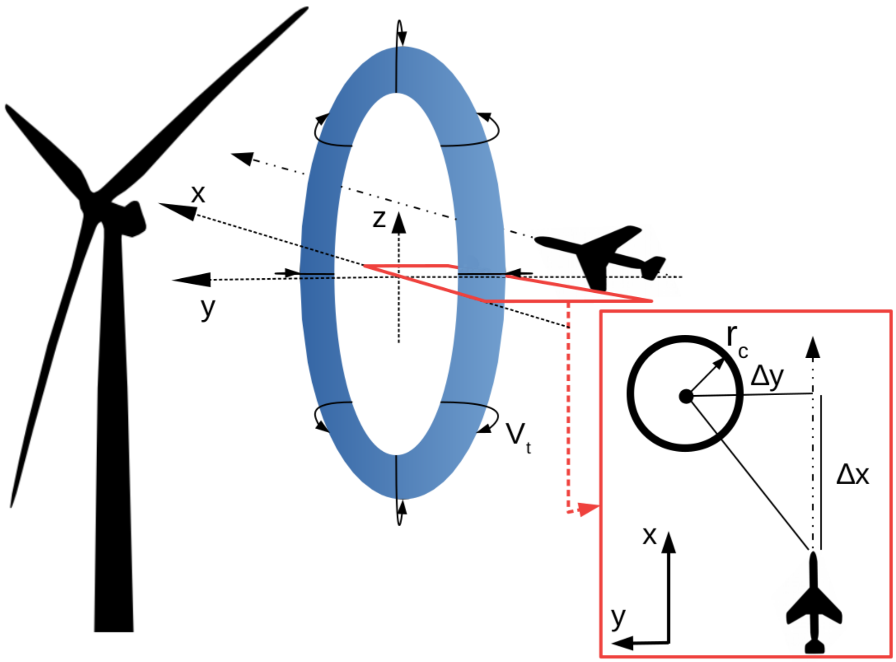


Figure 7. Simplified sketch of a vortex pair passed by the ~~UAV~~ UAS to the right. In reality it would rather have a helical pattern than a ring shape. Velocities and axis according to meteorological standards, therefore axis and orientation according to the in-situ conditions. y axis pointing north, x axis pointing east. At hub height the w component (along z axis) vanishes. The red rectangle illustrates a top view of a tip vortex with distance Δy to the UAS.

~~The Burnham-Hallock model provides a solution for two vortices spinning in opposite direction, as found in an aircraft wake . A similar constellation of vortex pairs can be found in a WEC wake at hub-height (cf. Fig. 7) along the x axis~~The BH model does not provide a solution for the whole wake structure, but for an idealised 2D cut. Describing two (independent) counter rotating wake vortices with a simple analytical model and comparing it to in-situ measurements is a new approach in studying wind turbine wake structures.

~~Following the Burnham-Hallock model~~Having a look at the BH model, a vortex is described by its circulation Γ, tangential velocity V_t and its core radius r_c. The tangential velocity is the velocity of the air circling the vortex centre and is a function of the distance r to the vortex core.

$$V_\mathrm{t}(r) = \frac{\Gamma}{2\pi} \frac{r}{r_\mathrm{c}^2 + r^2}. \tag{4}$$

The core radius r_c is defined as the distance from the vortex centre (or core) at which the tangential velocity is at its maximum (circular symmetry). So the radius r_c is also the radius at which the surface integral (cf. Eq. 1) is maximal, considering a circular surface. For $r = r_\mathrm{c}$ the maximum tangential velocity becomes (Eq. 5)

$$V_\mathrm{t,\,max} = \frac{\Gamma}{4\pi r_\mathrm{c}}. \tag{5}$$

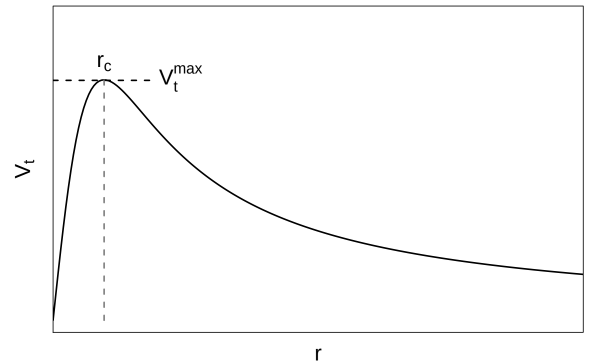
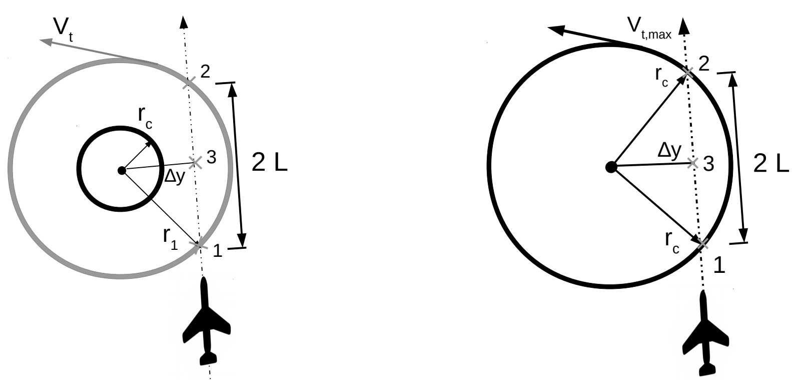


Figure 8. Qualitative plot of the tangential velocity from the vortex core outwards. The tangential velocity increases from zero (left) to a maximum at a distance r_c and decreases to zero for large distances (to the right).



(a) (b)

Figure 9. Schematic of the UAS passing a vortex in the horizontal plane (top down view). Two different cases have to be distinguished. The closest distance to the vortex $\Delta y > r_\mathrm{c}$ (a) and the passing distance being $\Delta y < r_\mathrm{c}$ (b).

Figure 8 shows the tangential velocity V_t distribution of a Burnham-Hallock BH modelled vortex with the highest tangential velocity at the distance $r = r_\mathrm{c}$. The distribution is circle symmetric with the vortex core ($r = 0$) in its centre.

In order to estimate the circulation and size of r_c from transects through the vortices with MASC in the wake of a WEC, the following procedure is proposed.

3.2 Evaluation method

As shown above, it is likely to measure tip vortices ~~in hub heightthat reduce to a two dimensional problem, simply due to their physical presence and orientation at hub height. The vortices can therefore be considered~~ at hub height. At this height a simplification of the two vortices can be made. The blade-tip vortices can be considered as two dimensional vortices of circular shape ~~rotating~~ in the horizontal plane ~~thus the vertical component~~ and ideally the w ~~vanishes. So after~~ component can be neglected. After subtracting the mean wind v_∞ the vortex tangential velocity is

$$\boldsymbol{v} - \boldsymbol{v_\infty} = \boldsymbol{v'} = (u', v', 0). \tag{6}$$

The norm of the tangential velocity then is

$$V_\mathrm{t} = \sqrt{u'^2 + v'^2}. \tag{7}$$

When measuring with a UAS the measurement can be considered a snap shot of the in-situ conditions. Figure 9 differentiates between two different scenarios of the UAS passing a vortex. Both shown from a top view. ~~In~~ Both scenarios will be explained in detail in the following paragraphs, with first focussing on Fig. 9a. Here the UAS passes the vortex at its closest distance (Δy), marked point 3 in the sketch, with $\Delta y > r_\mathrm{c}$, thus the vortex core radius is not reached. Point 1 and 2 mark the position of two corresponding tangential velocities of identical absolute value, when approaching the vortex and moving away from it again. The measured signal is similar to the dashed black line in Fig. 10 that is an example for $\Delta y = 2r_\mathrm{c}$. From such data only point 3 can be identified, since it is the point at which the measured tangential velocity is at its maximum. Point 1 and 2 are somewhere left and right of the maximum with L being unknown. There are indefinite combinations of Γ and r_c that could describe the vortex using Eq. 4.

$$V_{\mathrm{t},2} = V_{\mathrm{t},1} = \frac{\Gamma}{2\pi}\frac{r_1}{r_\mathrm{c}^2 + r_1^2} \tag{8}$$

$$V_{\mathrm{t},\Delta\mathrm{y}} = \frac{\Gamma}{2\pi}\frac{\Delta y}{r_\mathrm{c}^2 + \Delta y^2} \tag{9}$$

$$r_1^2 = r_2^2 = L^2 + \Delta y^2 \text{ (Pythagorean theorem)} \tag{10}$$

The three equations 8, 9, 10 are known to describe the velocities and geometry of the measurement. $V_{\mathrm{t},1}$ ($V_{\mathrm{t},2}$) is the tangential velocity at the point 1 (and 2). Since there are four unknown parameters Γ, r_c, L, and $r_{1,2}$ the problem is not solvable.

~~In case $\Delta y < r_\mathrm{c}$ the sketch~~ Now we consider the case, when the UAS passes a vortex at $\Delta y < r_\mathrm{c}$, as shown in Fig. 9b~~applies~~. The measured tangential velocity now provides a distinct feature; a double peak in the horizontal wind measurement. This double peak ~~can be lead back to~~ is caused by passing the maximum tangential velocity at $r = r_\mathrm{c}$ at position 1 and 2. Since the tangential velocity decreases from that point inwards (towards the vortex core), the velocity at point 3 is a local minimum, leading to a visible 'dent' in the data (cf. red line in Fig. 10). Additionally the ground speed of the UAS is known, hence the

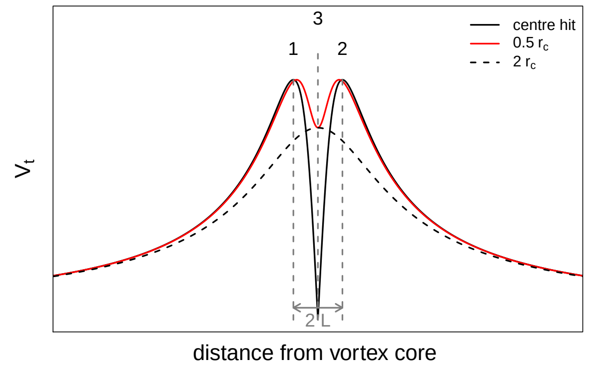


Figure 10. Analytical solution of a UAS passing the vortex at a path crossing the centre (black solid line), passing at $r = 0.5r_\mathrm{c}$ (red line) and at a distance double the core radius (black dashed line). The peak to peak distance is 2 L (cf. Fig. 9), above illustrated for the black solid line.

distance L can be calculated. The three equations previously described above, then become:

$$V_{\mathrm{t},2} = V_{\mathrm{t},1} = V_{\mathrm{t,max}} = \frac{\Gamma}{2\pi}\,\frac{r_\mathrm{c}}{r_\mathrm{c}^2 + r_\mathrm{c}^2} = \frac{\Gamma}{4\pi r_\mathrm{c}} \tag{11}$$

$$V_{\mathrm{t},\Delta\mathrm{y}} = \frac{\Gamma}{2\pi}\,\frac{\Delta y}{r_\mathrm{c}^2 + \Delta y^2} \tag{12}$$

$$r_1^2 = r_2^2 = L^2 + \Delta y^2 = r_\mathrm{c}^2 \longleftrightarrow \Delta y^2 = r_\mathrm{c}^2 - L^2 \tag{13}$$

With now only 3 (Γ, r_c and Δy) unknown parameters it is possible to solve the equations.

Dividing Eq. 12 by Eq. 11 eliminates Γ. Inserting Eq. 13 gives:

$$\frac{V_{\mathrm{t},\Delta\mathrm{y}}}{V_{\mathrm{t,max}}} = \frac{r_\mathrm{c}\sqrt{r_\mathrm{c}^2 - L^2}}{r_\mathrm{c}^2 - \frac{L^2}{2}} \tag{14}$$

Equation 14 describes a tangential velocity ratio that is ~~proportional to~~ a function of L. Also L is known to range from 0 to r_c. A dimensionless relationship $L\,r_\mathrm{c}^{-1}$ can be plotted and is shown in Fig. 11. By passing the vortex with $\Delta y < r_\mathrm{c}$, and plotting the measured V_t against the distance to the vortex (Fig. 10), we can determine $L, V_{\mathrm{t,max}}, V_\mathrm{t}, \Delta y$. Using diagram Fig.11, we finally determine $L\,r_\mathrm{c}^{-1}$ and thus r_c.

3.3 Analytical reconstruction

As shown above~~the presence and identification of a vortex (or a pair of vortices) is measurable.~~ , blade-tip vortices are theoretically measurable. They can be identified by their distinct 'dent' feature when the $\Delta y < r_\mathrm{c}$ criterion is met. Basic geometry and the ~~Burnham-Hallock~~ BH model further allow for a reconstruction (analytical solution) of the individually measured vortex, which is helpful to verify the measurements and evaluation technique. With Eq. 15 and 16 the distance to

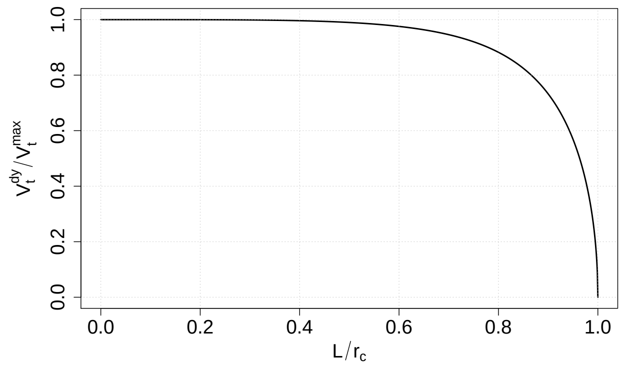


Figure 11. Dimensionless relationship between the ratio of the minimum (dent) tangential velocity and the maximum tangential velocity versus half the peak to peak distance (L), in percentage of r_c.

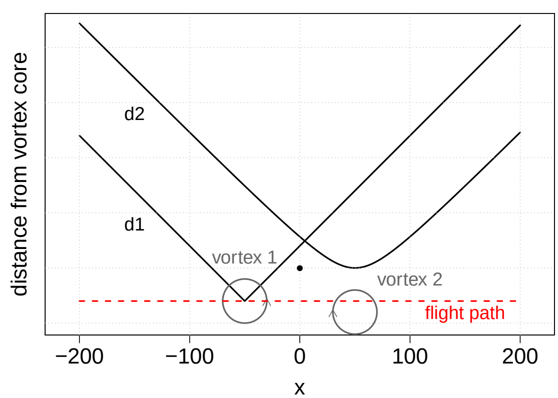


Figure 12. Qualitative example of an ideal flight path (vortex 1) and a passing with a little off-set (vortex 2) of the UAS. ~~In a~~ For the field measurement the distances $d1$ and $d2$ are calculated from the UAS GPS position and the location (off-set) of the vortex in relative coordinates with WEC at $(0,0)$, indicated with a black dot. The vortex position can be derived from the extent of the tangential velocity V_t measured by the UAS and the peak to peak distance, explained in the previous sections. In this example $d_{1,2} = \sqrt{\Delta x^2 + \Delta y^2}$ with $\Delta y_1 = 0$ for $d1$ and $\Delta y_2 = \text{const.} \neq 0$ for $d2$.

each vortex core (centre), to and along the UAV flight path, can be calculated (Fischenberg, 2011). In Fig. 12 the distances of the UAS to two vortices spinning in opposite direction are shown. In the figure, vortex 1 is passed through its core and vortex 2 is passed with a slight off-set. The flight path of the UAS is indicated with a red dashed line. Those distances are inserted into Eq. 17 and 18 using the relation of Eq. 4 the tangential velocity along the meteorological *x* axis (*u'* component) and *y* axis (*v'* component) can be calculated:

$$d_1 = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x - x_{\text{Vortex1}})^2 + (y - y_{\text{Vortex1}})^2} \tag{15}$$

$$d_2 = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x - x_{\text{Vortex2}})^2 + (y - y_{\text{Vortex2}})^2} \tag{16}$$

While the y coordinate can be derived from the measurement (using Δy and the UAS position, s.a. chapter 4.3) the x coordinate of the vortex $x_{\mathrm{Vortex1,2}}$ is the x coordinate of the flight path at the position '3', e.g. Fig. 9.

$$u' = V_\mathrm{t}(d_1)\left(\frac{y - y_{\text{Vortex1}}}{d_1}\right) - V_\mathrm{t}(d_2)\left(\frac{y - y_{\text{Vortex2}}}{d_2}\right) \tag{17}$$

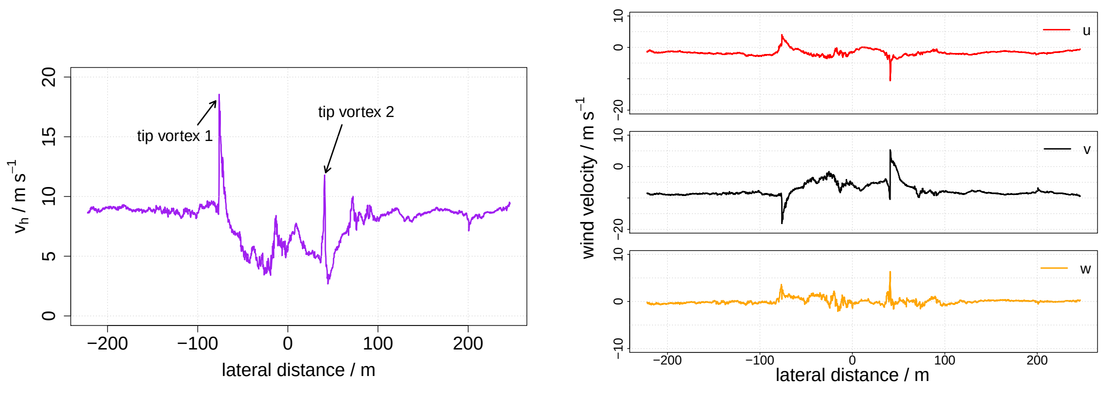
$$v' = -V_\mathrm{t}(d_1)\left(\frac{x - x_{\text{Vortex1}}}{d_1}\right) + V_\mathrm{t}(d_2)\left(\frac{x - x_{\text{Vortex2}}}{d_2}\right) \tag{18}$$

4 Results

4.1 Vortex measurement

Figure 13a shows the $v_\mathrm{h} = \sqrt{u^2 + v^2}$ component of the wind measurement behind the WEC at hub height. The data reveals several (near) wake specific features. This ~~measurement~~ flight leg shows two measurements of a tip vortex, indicated by the arrows in Fig. 13a. In-between those two peaks the wake deficit is measurable by a significant drop of the horizontal wind velocity. Due to the near vicinity to the nacelle, the wake deficit is dominated by turbulence created by the blade root vortices.

Figure 15 shows a zoomed-in look at the measured vortices depicted in Fig. 13. Figure 15a shows the vortex measured while entering the wake (vortex 1) and 15b while leaving the wake (vortex 2). Both, (a) and (b), are the plain UAS measurements. Figure 15c and Fig. 15d show the same measurement but the UAS coordinate system is rotated into the vortex ~~coordinate system~~rotational plane. This ensures that the rotational energy of the vortex is entirely captured by the u and v component, thus becoming the velocity components of the two dimensional rotational plane of the vortex, shown in Fig. 15 as a solid purple line. Examining both vortices, the velocity distribution pattern of the UAS passing at distance $r < r_\mathrm{c}$ is visible in the v_h measurement. The horizontal wind velocity v_h is a superposition of the tangential velocity, turbulence and the horizontal wind of the undisturbed inflow. The characteristics of the tangential velocity of vortex 1 (Fig. 15a,c) is almost solely determined by the v component, while in Fig. 15b,d the u component inheres an equal part. In the plain UAS measurement (i.e. before coordinate transformation) vortex 2 has a significant non-zero w component (Fig. 15b), indicating that the vortex did not rotate in the $x - y$ plane, hence the coordinate transformation into the vortex coordinate system. Especially Fig. 15d shows a significant reduction of the w component after the coordinate transformation. Purple dashed lines indicate the velocity deficit dV_t (dent), grey dashed lines the peak-to-peak distance. ~~In Fig. 15 shown as a solid purple line.~~ The dot-dashed purple line



(a) (b)

Figure 13. (a) Horizontal wind v_h measurement at hub height in the WEC wake at a distance of 0.25 D to the nacelle. The two tip vortices are indicated by the arrows. (b) The same measurement split into the three wind components u, v, w.

can be interpreted as an extension of the horizontal wind velocity by the w component, essentially giving the norm of the wind vector:

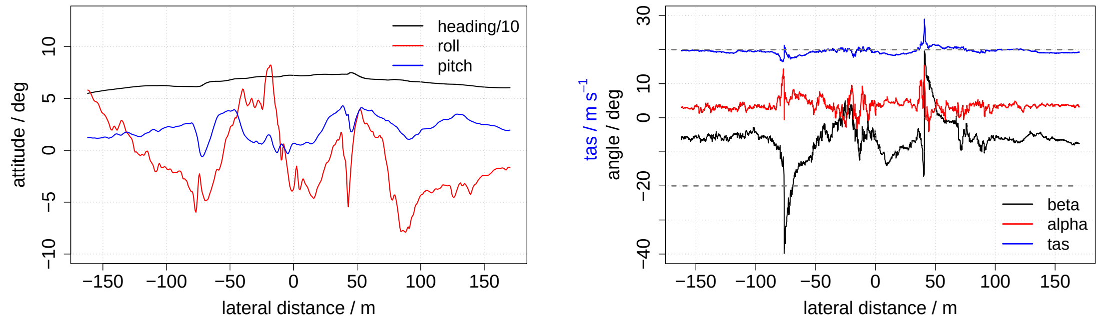
$$|\boldsymbol{v}| = \sqrt{u^2 + v^2 + w^2} \qquad (19)$$

The described coordinate system rotation was applied with respect to the area between the grey dashed lines (Fig. 15) i.e. between entering and leaving the vortex. A good indicator that the data rotation was successful is when the norm of the wind vector (purple dashed line) and the v_h in-between the grey dashed lines are about the same magnitude. Then it can be concluded that the two dimensional vortex rotation (u and v components) includes the entire kinetic energy, i.e. the vertical wind component is now neglectable.

Table 2 shows the derived parameters from the vortices depicted in Fig. 15. It has to be mentioned that vortex 1 made for a better and clearer measurement, since vortex 2 is influenced by the wind deficit and turbulence inside the wake. Vortex 1 shows a ~~clear and~~ sharp jump in the tangential velocity which makes it easier to obtain the necessary quantities and provides ~~clearer results. Figure ??a shows the relationship between the velocity ratio (Eq. 14) at point 3 (cf. Fig. 10) and the peak to peak distance (2 L) for the flight measurement. The obtained solutions for vortex 1 and 2 are marked with grey dashed lines. Equation 5 allows a calculation of Γ from the parameters extracted from Tab. 2.~~ more reliable results. The average of the obtained circulations is $\overline{\Gamma} = 74.17\ \mathrm{m}^2\ \mathrm{s}^{-1}$, the average core radius is $\overline{r}_\mathrm{c} = 0.61\ \mathrm{m}$.

Figure 16 shows a two dimensional cut through a skewed or canted vortex that results in an ellipse where the peak to peak distance is 2 L'. This peak to peak distance is under-predicted (2 $L' <$ 2 L). The introduced error $\Delta y'$ is visualised in Fig. 16 by dotted red lines. To overcome this issue the measured data are rotated into the vortex hose if necessary. This simulates the UAS canting to follow the ~~skewed~~ oblique vortex hose.

4.2 Quality control and error estimation



(a) (b)

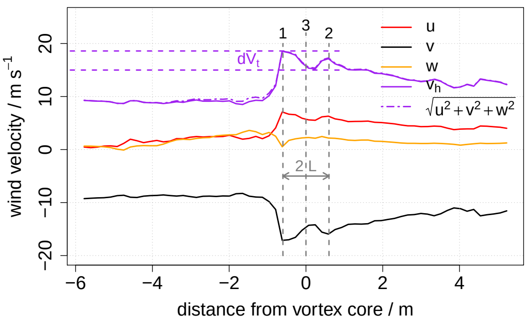
Figure 14. (a) Attitude angles of the UAS from flight leg taken for the tip vortex evaluation. (b) Angle of attack (alpha) and sideslip (β) at the 5-hole probe. Grey dashed lines indicate the calibration range of the 5-hole probe. Overstepped angles are extrapolated in post-processing. In blue the true air speed of the UAS.

The wake of a WEC, especially as close as 0.25 D behind the nacelle, is a highly turbulent region. When measuring with an autonomous UAS, it is of interest whether the UAS is capable of manoeuvring stably in such an environment and if the measurement instrument (e.g. 5-hole probe) is operating within its operational specifications. Figure 14 shows the attitude of the UAS (a) while passing the WEC for the consulted flight leg and the angle of attack, sideslip and true air speed (b). The UAS is affected by the wake entry and exit. The motions of the UAS are well recognised by the IMU and auto-pilot (cf. Fig. 14a) and taken into account for the later post-processing. The UAS handles these motions without loss of control.

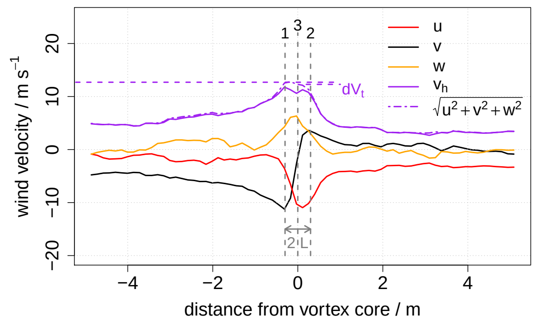
Grey dashed lines in Fig. 14b indicate the limit of the calibrated range of $\pm 20°$ of the 5-hole probe. Passing a tip vortex at $\Delta y < r_c$ is an extreme event, not only for the aircraft, but also for the pressure probe. Angle of attack and sideslip are within the calibrated ranges with one exception of vortex 1. Here, the sideslip is extrapolated. An examination of the true air speed (TAS) for the measurement (blue line in Fig. 14b) shows clearly the entry and exit of the wake. Changes in true air speed cannot be avoided. Usually small deviations from the calibrated TAS value of the 5-hole probe do not result in significant changes in the calculated wind speed. The peaks visible in the TAS measurement however, will have an effect on the wind velocity calculation. The influence of different air speed calibrations on UAS measurements is studied by Rautenberg et al. (2019a) . There it is concluded that the deviation from the "true" wind speed is about 10% or at most $1\ \mathrm{m\,s^{-1}}$, e.g. for a TAS error measured at vortex 2 of about $8\ \mathrm{m\,s^{-1}}$. So the peak velocities may be underestimated by $1\ \mathrm{m\,s^{-1}}$.

While this error has no significant influence on the ratio $V_{t,\Delta y}/V_{t,max}$ it is significant when calculating the circulation Γ from Eq. 5. In the presented case the circulation of vortex 2 is under predicted by about 10%.

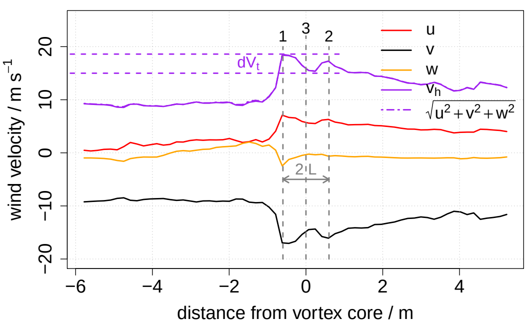
4.3 Vortex reconstruction



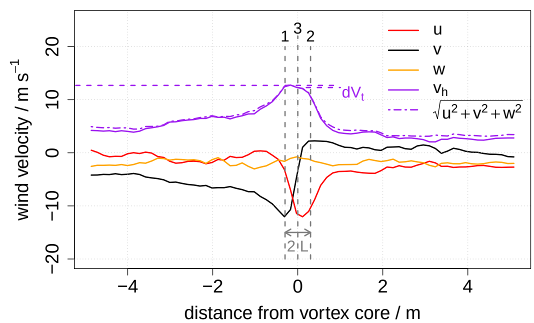
(a)



(b)



(c)



(d)

Figure 15. Measured tip vortex 1 and tip vortex 2 from Fig. 13a. Purple dashed lines indicate the velocity deficit (dent), grey dashed lines the peak-to-peak distance. The horizontal wind velocity v_h is a superposition of the tangential velocity and the horizontal wind of the inflow/surroundings. To eliminate the w component the data has been rotated into the vortex coordinate system. This is necessary to measure the vortex correctly. The sub-figures (a) and (b) show the plain UAS measurement of the vortices. In sub-figures (c) and (d) the UAS has been rotated into the vortex coordinate system (vortex plane) to capture the whole two dimensional rotation.

Table 2. Determined parameters from vortex measurements.

Vortex	dV_t $[\mathrm{m\,s^{-1}}]$	$V_\mathrm{t,max}$ $[\mathrm{m\,s^{-1}}]$	$V_\mathrm{t,\Delta y}$ $[\mathrm{m\,s^{-1}}]$	$V_\mathrm{t,\Delta y}/V_\mathrm{t,max}$ $[-]$	L [m]	$L/r_\mathrm{c}[-]$	r_c [m]	Γ $[\mathrm{m^2\,s^{-1}}]$
Vortex 1	3.4	9.6	6.2	0.65	0.61	0.93	0.66	81.30
Vortex 2	0.2	9.7	9.5	0.98	0.3	0.55	0.55	67.04

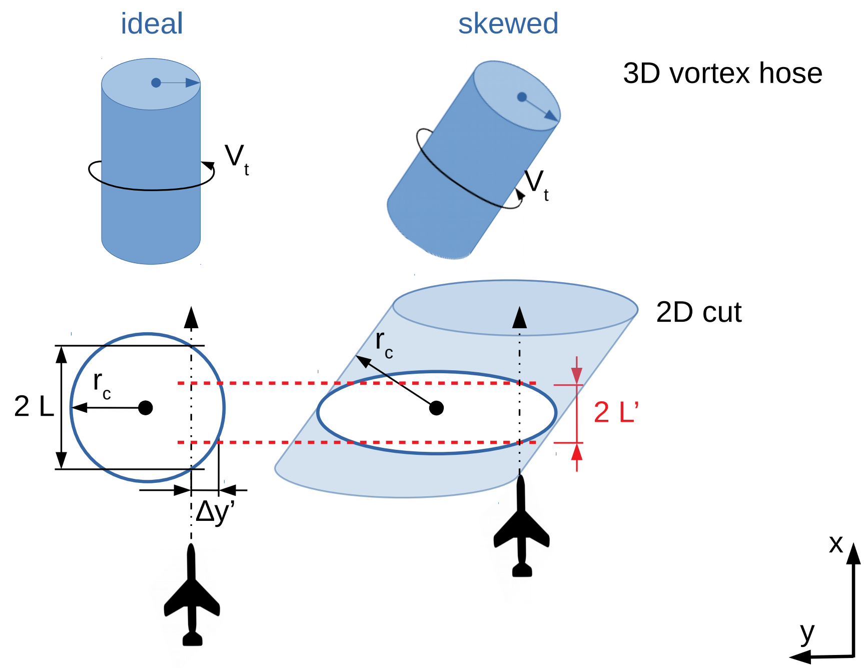
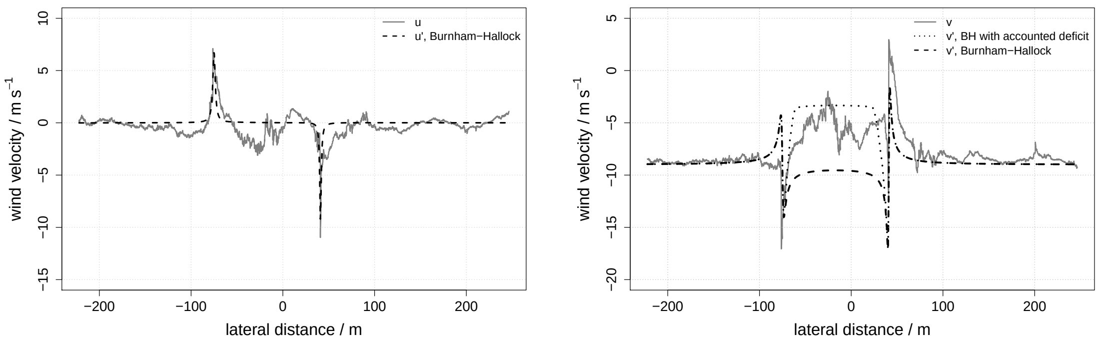


Figure 16. Sketch of an ideal and skewed (exaggerated) vortex hose at hub height. The simplifications in the evaluation method only consider components in the $x-y$ plane which leads to an under-prediction of the real peak to peak distance. The fact that the real 'r_c' does not lie in the $x-y$ plane leads to an error. A horizontal cut though the vortex has an ellipsoidal geometry instead of a circular one, as in ideal measurement conditions.

~~With all necessary quantities obtained from the measurement an analytical reconstruction of the vortices can be done. Figure ??b shows the distribution of the tangential velocity of a typical vortex at 0.25 D from the WEC derived from the measurement of vortex 1 (Fig. 15a) . With~~

The BH model provides a solution for two vortices spinning in opposite direction, as, for example, found in an aircraft wake. A similar constellation of vortex pairs can be found in a WEC wake at hub-height (cf. Fig. 7), with their vortex cores positioned along the x axis. This approximation can only be done, when the flight path is perpendicular to the wind (wake) direction to assure that the measured vortices are of the same age.



(a) (b)

Figure 17. Analytical solution (dashed lines) for u' and v' of the two vortices from the parameter evaluation. The corresponding wind components u and v from (Fig. 1513b) from the UAS measurements in grey. The long dashed solution in (b) additionally accounts for the deficit in the wake.

With the average values $\overline{\Gamma}$ and $\overline{r}_c$ retrieved from Tab. 2 the minimum measured tangential velocity between the two peaks (position '3' in Fig. 15) , as well as a distance Δy can be derivedby comparing the measured tangential velocity with the analytical solution shown in Fig. ??b. The resulting distance to the vortex core Δy can then be fed to a model, based on the Burnham-Hallock BH approach. Figure 17 shows the analytical solution of u' and v' overlain measured data of *u* an *v*. Overlain to the in-situ data the tangential velocity still contains the mean horizontal wind $V_t = \sqrt{u'^2 + v'^2}$. For the analytical solution the measured data has been rotated slightly (ca. 10°) into the mean wind direction to fit the meteorological coordinate system with the vortex coordinate system, so the *u* component equals zero in average and *v* is the predominant horizontal wind direction. In addition to the solely Burnham-Hallock BH solution for the v' component (dotted line in Fig. 17b), the long dashed line shows the same solution, but multiplied with a correction factor to satisfy for the wind deficit in the wake. The general vortex model does not consider the mean horizontal velocity, so it needs to be accounted for, especially when there is a artificially induced drop behind the WEC in the wake (wake deficit). In the present case the velocity deficit was measured to be about 65 %. It is visible as a jump in the mean horizontal wind between the two measured vortices. This deficit is in agreement with the findings Similar deficits were already measured by Wildmann et al. (2014a) or Bartl et al. (2012). The velocity correction function is shown in Fig. ?? and is simply an upside down Tukey window. The analytical solution remains uncorrected until entering the wake of the WEC. After incorporating a deficit correction to the analytical solution, it is visible that the deficit in the wake plays an important role to the structure (placement, intensity, etc.) of the vortex.

(a) Obtaining the correction factor for L for vortex 1 and 2 using the relationship described above. (b) Analytical solution for the tangential velocity of a vortex in the wake at a distance of 0.25 D. Γ and r_c averages from measurements from Tab. 2. Especially since the two vortices do not interact with each other, as the two vortices in the BH model for aircraft wakes do.

Velocity correction function for the v' component of the analytical solution when entering the wake (grey).

5 Discussion

Here we compare the airborne measured circulation Γ with data of the WEC itselfand with data from an LES simulation. Equation 20 allows for a calculation of the blade tip blade-tip vortex strength by given parameters and describes the circulation for a rotor of constant thrust coefficient, e.g. (Sørensen et al., 2014):

$$\Gamma = \frac{\pi v_\infty^2 C_T}{\Omega N_\mathrm{b}} \approx 66.2\ \mathrm{m^2\ s} \tag{20}$$

The rotational speed N_b being the number of blades and Ω is the rotational velocity provided by the owner of the WEC. For the determination of the thrust coefficient C_T the following estimation is done:

The relatively low wind speed ($v_\infty = 8.8\ \mathrm{m\ s^{-1}}$ by UAS measurement) implies a pitch angle of $\beta = 0°$ when approximating the E-112 with the NREL 5 MW offshore WEC (Jonkman et al., 2009).

The tip-speed ratio ($TSR = \frac{\Omega R}{v_\infty}$) can also be calculated and thus a thrust coefficient $C_T \approx 0.8$ can be estimated from the C_T to TSR relationship by Al-Solihat and Nahon (2018). With further N_b being the number of blades, Γ can be calculated.

The calculated value for Γ from WEC specific and atmospheric parameters is similar to the vortex strength that was extracted from the vortex measurement ($\overline{\Gamma} = 73.17\ \mathrm{m^2\ s^{-1}}$). This shows that the method presented here (calculation of measurements (average $\overline{\Gamma} = 74.17\ \mathrm{m^2\ s^{-1}}$). The presented method, to calculate Gamma from UAS data) , provides reasonable results, and that the geometric simplification of the tip vortex and the application of the Burnham-Hallock BH vortex model both are valid.

The Burnham-Hallock BH vortex model does work for aircraft induced vortices as shown by Ahmad et al. (2014) as well as Fischenberg (2011) and as the results imply, it can be used to describe WEC wake verticesvortex properties. Not least, both phenomena can be described by two vortices spinning in opposite direction, yet there is no interaction of the two opposite vortices, as in the aircraft wake model usually intended. Vortex patterns of a WEC wake show higher complexity than aircraft wake vortices. The whole wake is in motion and different turbulence and shear forces interact with each other. Therefore, for the wake vortices some simplifications have had to be made, e.g. the shown evaluation method is only valid for a 2-D cut of the whole vortex hose. Also the blade root vortex was not analysed any further.

In this study also the fact that the UAS experiences a change in true air speed (TAS) when entering the wake is ignoredaddressed. Theoretically the calibration range of the used five hole probe is for a fixed air speed which changes when entering the wake. Since this evaluation uses the ratio of two velocities the influence of a different calibration for the five hole probe should does not lead to a significant error. For the calculation of the circulation Γ, however, absolute velocities are necessary and a small error can be expected due to a change in TAS when entering the wake. and leaving the wake velocity deficit. The error is estimated to be $\pm 10\ \%$ for the calculated wind velocities (Rautenberg et al., 2019a). An error estimation is given in Section 4.2.

6 Conclusions and outlook

The resulting circulation strength Γ derived from ~~the~~ UAS data shows good accordance to the ~~equation by Sørensen et al. (2014) . This shows~~ results obtained from Eq. 20. It can be concluded that the evaluation method, using the basic geometrical properties of a vortex, can be used to derive vortex properties in a ~~flow~~WEC wake. Turbulence acting on the vortex ~~often aggravates an evaluation . So~~ and on the surrounding atmospheric flow can aggravate an evaluation since the evaluation is done mainly graphically. For example the second tip vortex is embedded in a relatively high level of turbulence (wake deficit, shear, etc.). It also does not show a clear border to the undisturbed atmosphere as tip vortex 1 does. The reference velocity levels for the evaluation are therefore harder to extract from the measurements. Also a hit of a blade-tip vortex in flight changes the true air speed (TAS) locally and temporally, resulting in a (small) error in the velocity measurement.

In addition, this method still has to be proven at larger distances to the WEC nacelle, where the vortices might begin to meander and get unstable. However, to our knowledge, this is the first quantitative analysis of WEC tip vortices using in situ measured turbulence data by a fixed wing UAS.

The MASC Mk 3 system is capable of measuring detached tip vortices in the wake of a WEC. The spatial and temporal resolution is sufficient to detect vortex patterns in the measurements. However, on many occurrences~~the measured side slip angle beta overstepped the calibration .~~, the measured sideslip β left the calibration range of the 5-hole probe and in conclusion, those measurements could not been used. ~~This adds uncertainties to the wind measurement. For the measurements shown in this study the angles of the 5-hole probe were not exceeded.~~ For future measurements the calibration of the 5-hole probes could simply be expanded to larger angles. This also allows for a lower TAS (true air speed) of the UAV, which in turn results in a better spacial resolution of the data. The path accuracy of the UAS will be upped by using an RTK ~~GPS as originally intended~~(real time kinematic) GPS.

The proposed analytical vortex model by Burnham and Hallock is capable of describing WEC wake vortices. Yet, as for most analytical models, the analytical solution shown in this paper can and should be improved. E.g. to better fit the WEC wake (velocity deficit, blade root vortex near the nacelle). This evaluation was conducted with data obtained at 0.25 D from the nacelle. For a future additional field campaign ~~blade tip~~ blade-tip vortices in the farther wake shall be investigated.

Competing interests. The author declares that he has no competing interests.

Acknowledgements. We acknowledge support by Projektträger Jülich and the BMWi (Federal Ministry for Economic Affairs and Energy) that funded the HeliOW project. We thank Enercon GmbH for cooperation and WRD GmbH for the provision of a generic recreation of the E-112 geometry for the numerical simulations. We also acknowledge support by Deutsche Forschungsgemeinschaft and Open Access Publishing Fund of University of Tübingen. For extensive technical support at the field campaign ~~I~~ we want to thank Martin Schön and Patrick Manz.

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Major changes in the manuscript:

- Abstract and Introduction rewritten
- New subsection “Available data” and Paragraph to research UAS have been rewritten
- Improved precision of certain statements all over the text. E.g. in the method section and discussion/outlook
- Several Images have been removed
- New subsection “Quality control and error estimation” has been added:
  --> error estimation from UAS attitude and flow angles
- Language and grammar improvements

Author's response to Referee #1

May 21, 2019

Thank you for the detailed review of the manuscript. In the following I will comment on each point. The referee's comments will be repeated in blue italic before the answer.

General comments

The manuscript by Mauz et al. describes in-situ measurements of wind turbine tip vortices with a UAS. From these measurements, circulation of the vortices is calculated using the Burnham-Hallock wake vortex model. These measurements are unique and I do not know of any other study in which a UAS operated in such close proximity to an operating wind turbine and even in its wake. The authors can convincingly show that wake vortices can be measured with the UAS. The presentation of the methods of analysis and the results however needs some significant improvement:

- *In the introduction and throughout the text I miss more thorough references to the state-of-the-art. For example, other methods to measure full-scale wind turbine wakes with remote sensing are not mentioned at all, but are carried out all the time and in multiple ways. What can a UAS do that is not possible with remote sensing?*
  Remote sensing methods (e.g. LIDAR) can not resolve turbulence in such a small scale as a UAS is capable of. LIDARs usually cycle cone measurements that resemble averages over a huge volume compared to a UAS line measurement. However, the method of operation allows for long term measurements whereas UAS excels at in-situ small scale measurements. Appropriate literature will be added. The lack of references is also contributed to the lack of publications of remote sensing experiments that try to resolve small scale turbulence in wakes. The wakes itself can be measured by LIDAR but focus of this manuscript is the identification of tip vortices that only have been verified qualitatively by Subramanian et al.

- *The structure of the text is sometimes confusing and much information is given in the results and discussion and outlook section that should have been introduced before. I provide details in the specific comments.*
  Thank you for the feedback. We will try to improve the structure of the text. Also by implementing the specified suggestions.

- *A major problem of the manuscript is that all the analysis is only done for a single sample of a wake vortex pair. If possible at all I would strongly urge the authors to investigate if they can maybe use some other flights. Maybe even flight levels above or below hub height could still be valuable.*
  Additional vortex measurements are available. In those flight legs usually the vortex pairs are visible. However, an evaluation of the core radius and vortex strength is mainly not possible due to the distance of the UAS being larger than r_c. From all the available data only one flight leg showed two vortices were the evaluation method presented in the manuscript was feasible.
  The aim of this manuscript is to establish an evaluation method for MASC-3 to later examine future wake measurements and then be able to establish a statistic for near wake vortex behaviour (e.g. turbulence and stratification dependent).

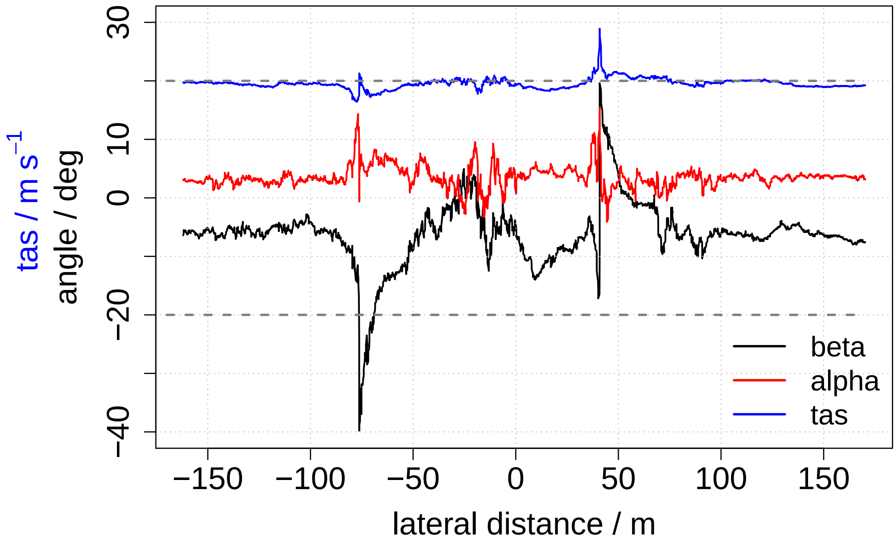

Figure 1: Wind angles and true air speed (TAS) of the 5-hole probe for the evaluated vortex measurements. Grey dashed line shows the calibration limit of -20 to 20 degrees. Overstepped angles are interpolated.

- *It is known that the estimation of the UAV attitude is a major source of error for the wind calculation. It is also known that navigation systems are less precise in dynamic flight manoeuvrers. I would therefore at least expect that attitude angles as well as airspeed and flow angles during the flight through the vortex are shown. The authors raise the issues themselves in the discussion, but I think it is necessary to include a proper analysis of this in the manuscript, including an estimation of uncertainty of the wind vector and thus the circulation.*
  The attitude angles of the UAS will be shown in an extra evaluation (e.g. subsection) also including an error estimation of the wind velocity and the measured circulation. See also Fig. 1 and Fig. 2 for a firs look.

- *I recommend some copy-editing to be done on English grammar and expressions.*
  We will look into that.

Specific comments

Abstract

The abstract should be rewritten with more statements about the hypotheses that were investigated in the study. A description of the results / conclusion is missing.
p.1,ll.1ff: wind converter should be either wind turbine or wind energy converter. Preferable: "rotor blade of a wind turbine". The whole first sentence is very hard to read and grammatically wrong. Starting with the relevance for numerical models in the abstract is misleading, because numerical models are not the topic of the paper.
p.1,l.6: what is the difference between atmospheric and meteorological quantities?
The abstract will be rewritten. All other annotations concerning the abstract have been implemented.

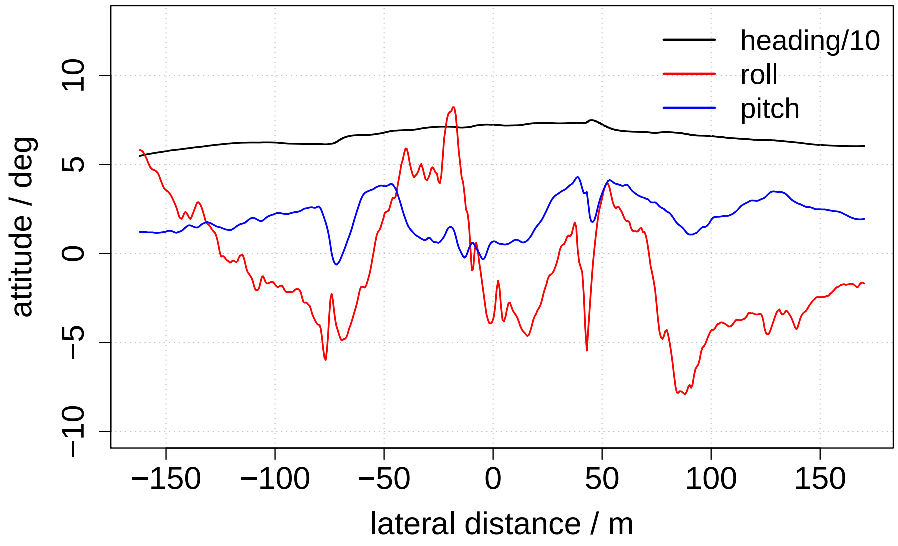

Figure 2: Attitude angles of the UAS for the vortex measurement flight. The entry and exit of the wake can be seen. The UAS remains on a stable flight path through the wake.

Introduction

p.2,ll.9-13: this paragraph should go into the description of the aircraft (Sect. 2.1)
p.2,ll.14-21: this paragraph could go in the experiment/site-description in Section 2
p.2,ll.22-31: here, a lot more literature should be referenced. Wake vortices are a major field of research and the state-of-the-art has not been evaluated at all.
All annotations have been addressed. Additional references and literature has been added.

Section 2

I miss a detailed description of the atmospheric conditions during the flight. Stratification, turbulence in the inflow, etc. which are important for the wake development are not given but should be available from the data.
The day the tip vortex measurement flights took place, only measurements at three different heights were made. The flight strategy was aiming on capturing tip vortices at hub height, and at the top and bottom of the wake. Since the distance to the nacelle is 0.25D or about 27 m the stratification of the atmosphere should not have a significant influence on the wake development. In an additional section a description of the atmosphere is now available, including turbulence.

A list of available flights and an explanation why only a single flight leg is analysed is missing.
A total of five flight legs at 0.25D are present. From these measurements only in one leg the criterion of $\Delta y < r_{\mathrm{c}}$ was met. Also a new subsection has been added 'Available data' where the data availability and the atmospheric condition are mentioned.

p.3,ll.3-8: Here, description of the aircraft is mixed with operational procedures. I feel like this should be separated.
Content has been separated.

p.3,l.11: Since RTK GPS is mentioned here and is not a standard feature of UAS in atmospheric research, some additional information would be appreciated: what kind of receiver is used (L1 or L1/L2 phase); is a local base station or RTCM-services used for correction data / what is the baseline? What

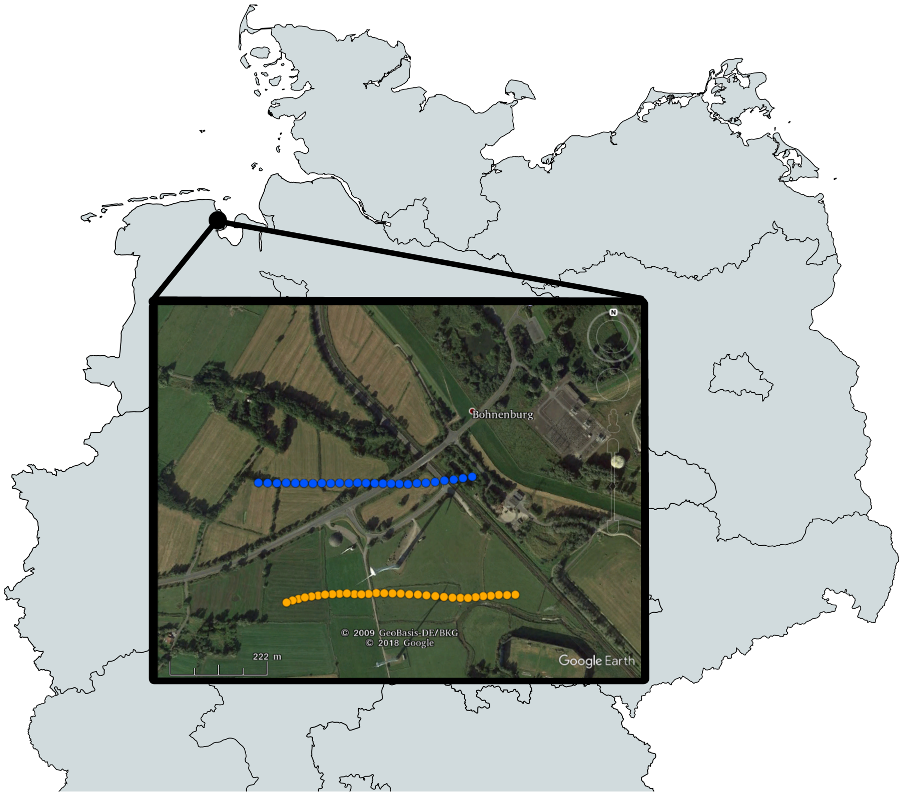

Figure 3: Location of the E-112 WEC in the north-west of Germany near the North Sea coast. MASC flight tracks in front (blue) and in the wake (orange) of the E-112 with northern main wind direction (5 degree north that day). On the Google Earth image the WEC is oriented toward south-easterly wind direction. Map created with mapchart.net

is the advantage of the very accurate flight path in atmospheric measurements?
Sorry, we made a mistake. RTK was not used during the flight and will not be mentioned in the manuscript any more.

Fig.3: I think a more schematic background (not Google Earth) with a better scale and legend would help
Google Earth map/image was replaced with a schematic map of Germany (cf. Fig. 3).

Section 3

Fig.6: It is unclear how TKE has been calculated. How large is the averaging window?
The TKE calculation serves a qualitative purpose. Therefore an averaging of 1 s (100 data points) has been used. The integral length scale was not calculated.

Fig.7: Nice figure, but watch out for which lines cross in front of or behind other lines to get the 3D visualization right. I think the red rectangle encloses the blue vortex, right? And "distance" should probably get a variable name or could even be left out.
First of all, thank you for the compliments. The line issue has been addressed. They should now support the viewer's perspective. 'Distance' has been removed. An updated figure will be found in the new manuscript (cf. Fig. 4).

p.9,l.5: Except that the vortices along the horizontal axis are not generated at the same time for the WEC.
In the specific line it is talked about the x-axis: Along the x axis, which is also the flight path of the UAS (pointing east with the main wind direction approximately 10° north), the vortices indeed show a little temporal delay. The first encountered vortex travelled a bit farther than the second vortex. This is now mentioned and could also explain the smaller core radius of the second vortex. We would like to

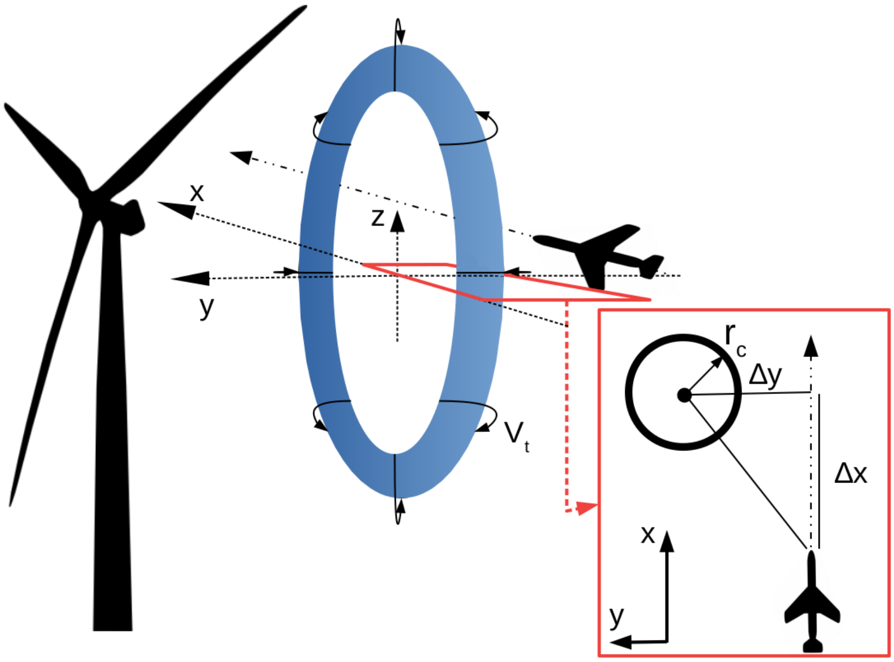

Figure 4: Simplified sketch of a vortex pair passed by the UAV to the right. In reality it would rather have a helical pattern than a ring shape. Velocities and axis according to meteorological standards, therefore axis and orientation according to the in-situ conditions. y axis pointing north, x axis pointing east. At hub height the w component (along z axis) vanishes. The red rectangle illustrates a top view of a tip vortex with distance Δy to the UAS.

argue for simplicity reasons that the flight path was perpendicular to the wake and therefore no huge 'ageing' differences occur.

Section 4

p.14,l.1: What is the vortex coordinate system and which angles are used for the rotation? This has not been introduced before.
Thank you for this comment. 'Vortex coordinate system' might really be the wrong expression here. What we were trying to say is that the 'horizontal wind data' have been rotated into the vortex rotational plane. So after this rotation the horizontal wind plane is parallel to the rotational plane of the tip vortex. The rotation was accomplished by rotating the x and y axis (u and v wind component respectively). This information will be added in the new version of the manuscript.

p.14, ll.26ff: this needs to be introduced and explained in more details in the methods section. Why can this correction not be done for other flight levels to increase the number of samples of tip vortices?
In principle it is possible to look at different altitudes. The flight strategy was to concentrate on measurements at hub height, since the probability to hit a tip vortex is the highest at this level. Also the introduced simplification of the vortex only rotating in two dimensions is mainly true at hub height. The rotation into the vortex rotational plane only makes sense, when the vortex has passed with the necessary criterion ($\Delta y < r_c$). Otherwise an evaluation of Γ and r_c is not possible.

Fig.16: I think this figure is not necessary.
This figure has been removed.

p.15, l.5: "rotated slightly" → what does that mean?
Done. The wind is rotated into the wake direction.

p.15, l.10ff: It is said that the wind speed deficit plays an important role for the tip vortex, but this is not discussed any further. Is BH even an appropriate model under these circumstances?

The short answer is 'yes'. The u component of the model is not affected by the deficit, especially since the data has been rotated into the wake direction. Also the peak position (on the x axis) for the v component in the BH model is also not affected. Only the slope and magnitude.

Fig.18: not sure if this figure is necessary
Will be dropped from the manuscript. It was simply thought to be a visualisation of the applied correction.

Section 5

p.19,l.7f: These are too strong statements for an experiment with a single sample

p.19,l.15ff: The issues that are raised here are not insignificant and call for some more analysis and quantification.
Attitude angles and true air speed variations will be analysed and the results addressed accordingly in the new manuscript.

Section 6

p.19,l.21: What is the equation by Sorensen et al. (2014) that is meant here?
Done. Eq. 20. Reference has been added.

p.19,l.23: "aggravates an evaluation" - what do you mean by that?
The evaluation is done mainly graphically. The second tip vortex is embedded in a relatively high level of turbulence (wake deficit, shear, etc.). The second tip vortex does not show a clear border to the undisturbed atmosphere as tip vortex 1 does. The reference velocity levels for the evaluation are therefore harder to extract from the measurements.

p.19,ll.27ff: The information about the 5-hole probe calibration range and why other flights could not be used should be given in Section 2 already. In section 2, it was said that the UAV operated with RTK GPS which is contradicted here. The subsection 'Data availability' has been added in Section 2 where we explain briefly why no other vortex examples are available. The RTK GPS mentioning will be stripped. We will link to a recent MASC-3 paper by Rautenberg et al. 2019.

Technical corrections

p.1,l.1ff: wind converter should be either wind turbine or wind energy converter. Preferable: "rotor blade of a wind turbine". The whole first sentence is very hard to read and grammatically wrong. The second sentence raises problems of numerical simulations that are not the topic of the paper.
p.1,l.6: what is the difference between atmospheric and meteorological quantities?
p.1,l.8: u,v,w, italic please
p.2,l.3: underestimate
p.2,l.4: "Another way" or "Another method"
p.8,l.7: "several analytical models are available"!?
Fig.8: x-axis should be labelled r
p.11,l.18: L is not proportional to the velocity ratio → The velocity ratio is a function of L.
p.14,l.10f: "In Fig.14 shown as a solid purple line." What is?
p.14,l.21: "clear and sharp jump" - strange expression
p.14,l.22: Fig. 16 appears before Fig. 15 in the text.
Fig.17: What is u and what is v should be mentioned in the caption. the same line style should be used for the same analysis method, i.e. dashed line for simple BH, and dotted for corrected version for example.
All corrections have been adopted and implemented in the new manuscript.

p.1,l.18: "their individual capacity and diversity" (what do you mean by diversity?)

Here we wanted to point out that there are different WEC designs for different terrain (complex vs. homogeneous) with all their challenges (high wind speeds, high turbulence and increased stress on blade structures etc.) The paragraph has been rewritten.

Author's response to Referee #2

May 21, 2019

Thank you for the detailed review of the manuscript. In the following I will comment on each point. The referee's comments will be repeated in blue italic before the answer.

General comments

I agree with the comments from RC1, so I will focus on some of the other things I noted. I have written down some general comments below, followed by a list of specific comments. As it is, I found the article hard to follow, which is unfortunate since the results are interesting. The authors should focus on guiding the reader through their thoughts and results, in a clear and concise way, reducing the length of the paper. This will require a major revision of the paper. I also believe the authors should spend an important amount on time correcting the language of the article. I've highlighted a couple of paragraph and sentences that

- *I also believe the authors should spend an important amount on time correcting the language of the article. I've highlighted a couple of paragraph and sentences that could be improved up to page 11, after which I stopped mentioning these. The authors should still work on the text after page 11.*
  We will look into that.

- *Regarding the title, I would probably not support using the word 'first' in the title since it brings a competitive touch to it that is unnecessary in my opinion. Also in light of the following publication, it may unfortunately not be justified to claim this 'first' attribute: F. Carbajo Fuertes et al - 2019 - 'Multirotor UAV based platform for the measurement of atmospheric turbulence: validation and signature detect ion of tip vortices of wind turbine blades.'. The author may also consider the studies from, Kocer et al. 2011 and Reuder and Jonassen 2012 cited in the above reference. (Please note that I'm not an author of any of these papers). Yet, I leave this up to the authors and the editor to decide whether to change the title.*
  Thank you for your valuable input there. The paper by Carbajo Fuertes was put online on March 28 2019. So there was no possibility to the authors to have known from these results.
  Following your suggestion I had a look at Fuertes 2019. The results look good. It is nice to see that two different approaches and measurement systems come to a similar result and output. Fuertes also derives a circulation of 50 m^2/s from his measurements. Given that he measured at a different wind turbine, the circulation strength is of the same order of magnitude. However, I could not find any information about the vortex fit he is using for his measurements as well as a detailed information about the circulation calculation.

- *I would personally prefer the equations to be closer to the text. As it is now, the equations are usually floating at the end of the paragraph which can make the discussions hard to follow.*
  I will consider this point to be mainly to be an editorial problem. I will see, to put equations closer to the text. However, in the later two-column layout everything will change again. So I might need input from the editor at this point.

- *It appears that the method presented can be attributed to Fischenberg, and the author may need to be clearer when highlighting if something is new or unique in their approach compared to what was already published (apart from the measurement campaign). The model using a regularized vortex cannot really be seen as a new contribution or method. The experimental data though are of high value.*
  It is correct that the method and model is not new. But as we state that the model has its origin in aviation, it is new to be used to describe wind turbine wake vortices properties. We added a paragraph to highlight this new approach in WEC wake studying.
  Also the described method illustrates how to obtain results from in-situ line measurements of a UAS, including systematic measurement uncertainties like skewed/canted vortex hoses etc.

- *The amount of measurements appear unclear, and some statistical analysis and information about the ensemble of results available could be valuable. Reading the article, it seems that only one vortex was analysed. Further data with different distances downstream should be incorporated since according to section 2 more data was acquired. Statistical tools should also be used to mention the uncertainty on the fitted parameters and to quantify the error between the model and the measurements.*
  A new subsection '2.3 Available data' has been added. We also state why only one pair of vortices could be used for an evaluation. A simple vortex measurement in the wake is not sufficient, also the criterion $\Delta y < r_c$ must be met which is a rare condition. We comment on this in the 'specific comment' section below in more detail.

- *The figures are usually clear. The authors could yet reduce the number of figures, particularly in the first 10 pages, or by combining the measurements with the fitted model in figures 10-17.*
  We will take out some figures from the manuscript. But also some new ones will be added. Mainly to do the uncertainty evaluation (wind angles, true air speed, etc.). We will also try if some figures could be combined. But that might add to the readers confusion.

- *I hope my numerous comments will not discourage the authors, and I strongly encourage them to further work on this paper. As I mentioned earlier, the article has some great potential, it just needs some additional work to reach the point of publication.*
  Highly appreciated emphasis. Thank you!

Specific comments

Abstract

The statement in the abstract 'the BH model can be used to describe wake vortices' is probably too strong and would need to be moderated since this simpler model is not capturing all the dynamics. I will comment more on this later.
We will add the simplifications that have to be made. The BH model is also only a tool to describe the vortex geometry in a 2D cut. This point might need additional stressing.

Introduction

I would think that bringing the context of Germany appears too specific, since the wind energy sector is growing in other countries.
This is indeed true. The introduction has been 'internationalised'.

p1 l21: "In research..." this sentence and the following two are hard to read and could be reformulated
The abstract as well as the introduction were rewritten.

p2 l8: The scaling problem of wind tunnel measurements could be mentioned here
Thank you for the hint. It will be mentioned.

p2 l10: make sure the acronym for UAS (and other acronyms) is made explicit in the introduction
Has been implemented.

p2 l15: Could you mention the arguments for the offshore comparison. It probably relies on arguments on the boundary layer when the flow comes from the shore, but wave exciting the turbines and the surface roughness may be different.
Yes, there is definitely an influence through the surface roughness compared to the sea. It can also be seen in Fig. 5 that the horizontal wind in a certain altitude experiences much more fluctuations. So there is an internal boundary (surface) layer that shows increased turbulence with a more stable marine layer on top.
This was only mentioned since for the briefly presented HeliOW project it would be desirable to fly and measure behind an actual off-shore wind turbine. But for several practical reasons this was simply not feasible.
The influence of turbulence is also a good point for future measurements to get a large amount of vortex measurements to do some statistics.

p2 l18: "The project aims for safe helicopter flight paths in off-shore wind energy parks" needs reformulation
Will be changed to "The goal of the project is also to determine safe helicopter flight paths in off-shore wind energy parks".

*p2 l19, l21 *University of* Munich, University *of* Stuttgart.*
Has been changed accordingly.

p2 l23: the wind turbine also generates strong coherent vorticies, can these be considered turbulence?
Coherent (in this case artificial turbulent) structures are not turbulence covered by Kolmogorov 1941. The coherent vortices might be considered as a large scale anisotrop turbulence. The surrounding atmosphere in the best case scenario can be considered as isotrop. In the far wake these coherent structures decay and add to the isotrop turbulence.

p2 l30: could you highlight more the difference between the study from Subramanian and yours?
Subramanian et al.detected tip vortices by a pressure signal along a longitudinal flight path. The vortices were also not quantified (vortex strength, core radius). We will add some more details to the manuscript and highlight the main differences.

Section 2

p3 l7: Wind speed and directions could have changed during this 15min period, do you have access to measurements to support this assumption?
Unfortunately, we didn't have access to highly resolved SCADA data for the measurement time which could have given us precise information about wind speed and direction changes. Moreover, we were flying about 27 m behind a 114 m diameter WEC. If the wind direction would have changed significantly we might have crashed into the rotor blades. For a single vortex measurement it is also not important, if the wind direction varies a bit. The wind velocity component rotation later in the manuscript was done for the single measurement. The mean wind direction is only important to set up the flight trajectory for the UAS.

Section 3

p5 l11: You could mention that the ring vortex is an approximation of the wake vorticity at high tip-speed ratio.
Thank you for the input. We will mention that.

p5 l14: This line can be reformulated to mention that this result is true under the vortex ring assumption.
The line has been modified.

p6 Figure 4: "recreation" may not be the correct word in the caption, maybe "model", or "reproduction" would be more accurate?
Done.

Figure 7: Instead of using north/east for the axis wouldn't it be easier in that case to use an orientation in the frame of the turbine, with y pointing upstream against the main wind direction? You could then remove the sentence at the end of page 6.
This was actual the intent when planing the flight path at the field campaign. The wind was slowly turning. So Figure 7 shows the actual flight path (direction) of the UAS. So y pointing north is almost into the main wind direction. But at the time of the measurement the wind direction was already a bit off. Also the post-processing software of the UAS data always writes the wind velocities according to the meteorological standards u pointing east, v pointing north and w pointing upwards. So the data in the end always has to be rotated into the main wind direction. And this is necessary to later make $\overline{u'} = 0$ in the wake.

p7 line1-12: The potential flow assumption probably appears too early in this paragraph and the paragraph could be reformulated. The definition of circulation as function of the vorticity is independent of this assumption. Equation (3) only uses an axi-symmetric assumption. It is yet true that the circulation of a vortex makes more sense in inviscid flows where the vorticity is condensed to confined singular regions.
We will look into a possible reformulation and implementation of the introduction of potential flow.

p10 l1-4: This paragraph needs should be reformulated, the language improved.
The paragraph has been rewritten.

p10 l9-15 and p11 l1-8: While reading the text I was confused since figure 9b didn't appear to be mentioned. The explanation could be improved by clearly explaining both figures and both scenario, before mentioning figure 10. Alternatively, you could tell the reader to focus only on figure 9a for now and figure 9b will be explained later. Also, equations 8-10 could be introduced first before drawing the conclusions that there is no unique solution. Further, the way the equations are introduced can be improved by telling to the reader what is coming, e.g. "The velocities at point 1 and 2 are...". Right now, they appear in the text announced.
Figure 9b is mentioned on page 11 l7. I agree that the two different cases of passing a vortex in the UAS measurements needs additional addressing to not lose the reader. The paragraphs have been modified.

p11 l9: "This double peak can be lead back to passing the maximum tangential velocity at $r = r_c$ at position 1 and 2". The sentence may need reformulation
Changed to: This double peak is caused by passing the maximum tangential velocity at $r = r_c$ at position 1 and 2".

p11 l12-15: Similar to the previous remark, the equations can be introduced before drawing the conclusions.
We think this comes down to personal preference. We would like to explain what's happening when passing the vortex at $\Delta y < r_\mathrm{c}$ and follow up with what that means for the previously introduces equations.

p11: "As shown above the presence and identification of a vortex (or a pair of vortices) is measurable". The language needs to be reformulated (one cannot measure an identification). Also, the previous section seemed to show that in some cases the determination was not possible, which would imply that the identification is not always possible.
Fair point. "As shown above the presence is measurable and a later identification of a vortex (or a pair of vortices) possible when the $\Delta y < r_\mathrm{c}$ is met."

Section 4

p13 l8: It would help the reader to provide some information about the measurement campaigns (how many samples were selected, what were the mean conditions), and some introduction about the samples you selected (and why you selected them) to present in the paper.
This issue will be addressed with the additional subsection about data availability.

p14 l26: It is not clear why you mention the skewed vortex at this stage. You may need to guide the reader. Also, the definition of the skewed vortex on figure 15 appears unclear. From the figure it seems that the vortex is simply rotated. I'm not sure this qualifies as a skewed vortex. Also the 2D cut appears to have some 3D aspect to it, which can be confusing. Introducing a coordinate system on the 3D vortex on the 2D cut can help the understanding.
Maybe 'canted' or 'oblique' vortex is the more precise expression. The additional 3D features are meant to help the reader. That is why the blue is more transparent. It could be removed, if it confuses more than it helps?!
We have to introduce the oblique vortex hose, since the real (measurement) world is different from the analytical approximation (ring vortex). We will try to make that more clear.

p14 l30: I am wondering if "analytical reconstruction" is the proper term and how this "reconstruction" is different from the previous section. The parameters you extracted from the measurements were fitted to an analytical model. The "reconstructed" vortex is this analytically modelled vortex, and it is intrinsically part of the results you presented in table 2. If I understand this correctly, it could make sense to have the modelled vortex directly on the figures 13-15, as "fitted vortices".
In Section 3 we want to present the theory/method how to "reconstruct" the vortex from GPS and pressure data. In section 4 the individual fit is calculated. One might consider to just remove this subsection, e.g. Fuertes 2019 never introduces any vortex model of the WEC and just "fitted" the data. As for the second point, in fact the GPS positioning standard deviation is in the range of the vortex core radius. So the double peak is not visible and overall the accuracy is not up to the in-situ, in-line pressure measurement. Thus, to keep an step-by-step approach and to use the analytical model, as a proof of concept, we prefer to keep it separate from the measurement.

p15 l9: What is meant by "artificially induced drop" of velocity? Does this refer to drop of velocity in the turbine wake? If this is so, the drop in velocity should be a function of the thrust coefficient at the rotor, and a value of 65% may not be comparable to other measurements unless they are at the same operating conditions.
You are correct, the statement has been changed. The actual deficit in percent or m/s is only important for the correction of the modelled v component.

Section 5

p18 eq20: You may have to introduce all symbols closer to the equation, even if these are obvious.
We will try to do so.

p19 l1-4: It appears surprising that the authors do not have more information about the turbine (thrust curve, pitch curve). Earlier in the text, it was mentioned that a model of the turbine was done. These quantities can then be obtained from a Blade Element Momentum code.
The argument here may simply be that most turbine have a pitch angle around +/- 1 degree below rated, and in the absence of data, you picked 0 degree. It is also not clear where the pitch angle enters in the equation. Most likely an argument of the thrust coefficient, but usually you'll have a thrust coefficient vs wind speed curve available for that turbine.
The engineering sector of wind energy converters is a highly competitive market. Although the University of Stuttgart has a generic recreation of the wind turbine, it is reserved for wake computations. The resulting aerodynamic properties of the turbine are however strongly confidential. That's why those properties, namely here the thrust coefficient, had to be estimated. As stated on p. 19 l1, we approximated the thrust coefficient vs. wind speed with an NREL 5 MW turbine. Since the E112 and NREL 5MW wind turbines both have similar rotor diameters and produce a similar amount of electricity, the approximation was judged reasonable.

p19 l10-11: These sentences need to be moderated. First, it appears that the study was only done on one vortex at a given operating conditions and a more quantitative analysis would be required. Second, it appears wrong to state that the wake of a turbine is described by two vortices. You could clarify your discussions based on the following considerations. The fact that two vortices are crossed on the trajectory of the drone is due to the likelihood of crossing the tip-vortices from different blades. This likelihood increases as the number of blades or the tip-speed ratio increases. When it is such, the wake

vorticity surface can be approximated to a vortex cylinder, in which case any trajectory of the drone will indeed cross the tip-vorticity surface twice. This cylindrical surface does not resemble two vortices spinning in opposite direction and the wake dynamics cannot be described by assuming that it consists of two vortices. What the author probably means is that the velocity field across a tip-vortex (or a tip-vorticity surface) resembles the one of a regularized point vortex. This analogy (which is natural given the different analytical vortex wake models of wind turbines) cannot be used to "describe" the wake, but it can be used to "estimate" some of the wake properties, that is, the tip-vortex core radius and circulation.
We agree. This paragraph needs to be more precise. We will implement your suggestions.

Section 6

p19 l21: The identification of one vortex strength does not appear to be enough to draw a conclusion, or the conclusion needs to be moderated. Also, it may not be necessary to attribute this equation to Sorensen et al. and instead it can be mentioned where this formula comes from: circulation for a rotor of constant thrust coefficient (This should also be mentioned earlier in the text p18 l1-6).
The conclusions will be readdressed and the statement moderated. Also we want to repeat at this point (and later in the new version of the conclusions) that the fact that we hit two (!) consecutive vortices in one flight leg at the crucial criterion ($\Delta y < r_c$) might also be considered lucky. That both measurements, after consideration of the non-ideal conditions (canted vortex hose) and therefore the rotation of the horizontal measurements into the vortex plane is giving two circulations strengths that match the theoretical calculation, needs an appropriate (strong) statement.