

We thank both reviewers for their insightful and well-articulated comments and for the time spent checking the derivations of the equations. We have revised the manuscript in response to their important points, and to refocus the work. Changes to the document are highlighted in blue text for ease of reference, and the most important changes are discussed below. Many of the comments were made by both reviewers so we have not separated our responses.

The most important criticism from both reviewers appears to be that the final blade-element thrust equation that we derived:

$$\frac{1}{\rho} \frac{dT}{dx} = 2 \int_0^{2\pi} (w^2 + \lambda wx)x d\theta \quad (1)$$

is not novel. This was an oversight on our part, and we have added a reference to Glauert's (1935) work, as well as to Sørensen's (2016) book. We have also modified the title of the paper to reflect this recognition and have added three figures to help clarify the presentation. With that point clarified, the remaining contributions of the work deserve additional emphasis.

Our work addresses three key assumptions of blade-element theory through the lenses of impulse theory and helical vortex theory: (i) the assumption that radial velocity is negligible at the rotor, or at least that it has little effect on blade forces, (ii) that the Kutta-Joukowski (KJ) equation is applicable to individual blade elements, and (iii) that pressure on the lateral control surfaces does not contribute to blade-element thrust (commonly referred to as the assumption of blade-element independence). These points are now listed explicitly in the first paragraph of the introduction.

The second reviewer has noted that the streamlines and vortex lines need not be aligned in the rotating frame, even for a rigid wake. This is true when diffusion is present, when the circulation of fluid elements in the wake may change over time. However, we have treated the wake as a collection of trailing helicoidal sheets, and it is assumed that Kelvin's theorem holds for individual fluid elements. As such, the vortex lines and streamlines *must* coincide, or else the structure of the wake would appear to deform in the rotating frame (i.e. the wake would not be rigid). A note to this effect is given after equation (12). The insight regarding radial velocity is unique to the present derivation. We have deduced that the radial velocity must be of similar magnitude to the axial induction factor over at least part of the rotor plane, and that this most likely happens in the vicinity of the tip. This agrees with observations by Madsen et al. (2010) and Sørensen (2016, see Figure 3.2). For loaded rotors, the assumption of negligible radial velocity in the power producing region near the blade tips is unjustified, and further work to account for its effect on blade forces is warranted. The discussion of radial velocity is now in section 4.1, as the discussion has been reorganized into subsections.

In response to the first referee comments we have given an expanded discussion on the relationship between the KJ equation (1) and the conventional form of the momentum equation in section 4.1. This also addresses the issue of blade-element independence. Equation (1) above applies to individual blade elements irrespective of the loading on the rest of the blade. Specifically, we have proved in the main text and what is now Appendix C, that the pressure acting on the expanding flow through the blades does not contribute to the axial momentum balance. Of course, blade-element independence as conventionally understood pertains to axial momentum theory (AMT). By taking equation (1) for thrust and showing the conditions under which it collapses to the classical equation from AMT, we have shown the conditions under which that equation is also independent of loading on neighbouring elements. The

following conditions turn out to be sufficient for blade-element independence: negligible viscous drag on the blades; high number of blades and/or high tip speed ratio; and constant vortex pitch across the wake. This conclusion is not unique to the method of derivation equation (1), but rather follows from it and from symmetry arguments from helical vortex theory.

In the original manuscript, we briefly demonstrated that equation (1) is a generalization of the KJ equations for blade elements. This discussion has been expanded in section 4.2, and a figure has been added to show how Stokes' theorem has been invoked.

Now that we have removed claims of novelty regarding equation (1) above, we recognize that the presented derivation is more complicated than strictly necessary to support our conclusions. One motivation to keep the full derivation is for future work. In particular, we are interested in modelling unsteady flows, and for this purpose the unsteady impulse term may prove useful. The shorter of the two derivations of equation (1) has been retained in the body of the work (using a stationary frame of reference), and the other (in the rotating frame) has been moved to Appendix A. We have also added a short analysis of the relationship between thrust and torque, showing that the usual assumption that extracted power equals the thrust multiplied by the velocity through the rotor, is incorrect at low tip speed ratio (TSR) but becomes approximately correct at high TSR (see section 4.3 in the revised manuscript).

Appendix B has been added to show the derivation of equation (1) above using the unsteady Bernoulli equation, upon which previous derivations are based. Its inclusion allows the reader to see that the same assumptions are required to get to (1) whether one uses the Bernoulli equation or impulse theory. In our impulse-based derivation, however, Equation (11) in the revised manuscript is a more general expression which avoids the thin-wake assumption, but this cannot be avoided using the Bernoulli equation. We have presented this as an advantage of the present derivation in the text below Equation (28) in the revised manuscript. Appendix C is the same as the appendix in the original manuscript, with only a couple minor wording changes.

Figures have been added to make the analysis easier to follow, depicting the volumes and surfaces under consideration. Also, in response to a request for clarification from another colleague, we have added a brief introduction to the concept of impulse in the second paragraph of the introduction. This small change should make the paper more accessible to a wider range of wind energy researchers and practitioners.

We hope that our revisions are to the satisfaction of the reviewers, and we look forward to any further comments they have.