

Review of WES-2019-93 V3

An impulse-based derivation of the Kutta-Joukowski equation for wind turbine thrust

by

Eric J. Limacher and David H. Wood

General comments:

The manuscript (now in its 3rd version) has been reformulated and extended considerably.

I agree on publishing it if my recommendations from below are included.

Thank you, once again, for your useful feedback. We have responded to each of your comments below, and the changes in the revised manuscript are in red text for ease of reference.

Specific Remarks

Page 1, line 18:

You mentioned “all texts on wind turbine aerodynamics”. This is not completely true. To be update, please add:

S. Schmitz, Aerodynamics of Wind Turbines, Wiley (2018)

A.P. Schaffarczyk, Introduction to Wind Turbine Aerodynamics, 2nd Ed. SpringerNature (2020)

Thank you for suggesting these recent citations. They have been included in the first paragraph of the introduction. We have also changed “all texts” to “many texts.”

Page 8, lines 165 to 187.

Pseudo-equations using “ \approx ” should not be present in a scientific paper. Again: Eq (24) is an integral over two functions - to draw any conclusion about the local behaviour demands mathematical assumption in which regularity class the functions $v^2(r)$ and $a^2(r)$ are embedded. Any decisive conclusion can only be drawn from the differential equation. See G. Gallavotti, Foundations of Fluid Dynamics, section 2.4. In particular a possible (edge-)singularity at $x = 1$ may spoil the argument.

You are correct to point out that our arguments depend on the assumption that v^2 and a^2 are at least C^0 -continuous. We have now made that assumption explicit in the modified paragraph on pg. 8. With that made clear, we can definitively argue that $v^2 = a^2$ somewhere on S_U , once we note that $v=0$ at $x=0$ by symmetry while $a \geq 0$ at $x=0$. We then point the reader to three studies employing actuator disk simulations that show $v=a$ occurs near the rotor tip for a wide range of thrust coefficients.

We have likewise altered the corresponding claims in the abstract and conclusion. We claim only to show that the magnitudes of v and a are equal somewhere on the plane containing the rotor, which previous studies have shown to occur near the rotor tip over a wide range of thrust coefficients.

Page 2 Eqs. (1) to (2):

If I insert $u_{\theta} = -2w$ into Eq. (1) I feel that a factor of 2 is missing in the w^2 term of Eq. (2).

Please correct this typo, if it is one.

Yes, this was a typo. Thank you for catching it. It has been corrected, and Equation (2) is now consistent with Equation (25).

Page 12, line 261

I do not understand why $\partial \phi / \partial \theta$ equals $x u_{\theta}$. Please explain.

In the unsteady Bernoulli equation, ϕ is the scalar potential, related to the velocity vector according to $\mathbf{U} = \nabla\phi$. The azimuthal component of velocity is thus $u_\theta = \frac{1}{x} \frac{\partial\phi}{\partial\theta}$. A note to this effect has been added after Equation (A2).