

The paper contains a comprehensive analysis employing the so-called 'impulse equation' to determine an equation for the thrust acting on a wind turbine. The background is a theory developed partly in a PhD dissertation of F. Noca (Caltec, 1997). The main idea is to get rid of using explicitly the pressure when determining the thrust in the axial momentum equation.

The paper inscribes itself in a long line of works dealing with the attempt of formulating a consistent derivation of the differential form of the general momentum equations for rotors. In fact, already Glauert, in his derivation of the general blade-element momentum theory, questioned the use of the standard axial momentum equation, and in 1972 Goorjian proved that the standard form of the axial momentum equation is not valid because it leads to a contradiction when combined with the other equations of the momentum theory. In the abstract of the present paper it stated that 'The new formulation gives a very simple, exact equation for blade element thrust which is the major contribution of this study. By removing the pressure partly through the kinetic energy contribution of the radial velocity, the new equation circumvents the long-standing concern over the role the pressure forces acting on the expanding annular streamtube intersecting each blade element.' However, I am not at all convinced that paper has solved the longstanding concern and that it adds anything new to the already known knowledge on the subject. In the following I will explain.

First, all equations seem to be consistent and I did not find any errors in their derivations. However, it also seems unnecessary complicated to start with a (very) general form of the impulse equation to end up with an expression that has been known for more than a century. Indeed, eq. (38), which, as stated in the conclusion, is being the most significant result of the present analysis, has been known and used by both Joukowsky in his pioneering work on propellers already in the start of the last century and by Glauert in his formulation of the blade-element momentum theory. However, they derived the expression simply by applying the Bernoulli equation in a rotating frame of reference along a streamline going through the rotor plane. This equation is fully consistent with energy conservation and is one of the main equations in the formulation of the Joukowsky propeller theory. In the work of Glauert, the equation corresponding to eq. (38) is written as

$$\frac{\Delta T}{\Delta A} = \Delta p = 2\rho w(x\Omega + w). \quad (1)$$

Independent of the way of derivation, this equation is (of course) both correct and consistent. However, the equation does not prove that there is no impact of the pressure on the lateral boundaries of the control volume and that the axial momentum theory can be used in its simple form. In its general differential form, the axial momentum equation reads

$$\Delta T = 2\rho a(1-a)\Delta A + \oint_{cs} p dA, \quad (2)$$

The problem stated by Glauert (and Goorjian) is if the pressure contribution can be ignored, i.e. if axial momentum theory can be written as,

$$\Delta T = 2\rho a(1-a)\Delta A, \quad (3)$$

that is ignoring the impact of the contribution of the pressure forces acting on the expanding streamtubes in the derivation. And here Goorjian was the first to prove that this simple form results in an inconsistency. So the problem is not solved, as the present work only shows that eq. (1) can be derived in different ways, but not that the pressure can be neglected in eq. (2).

My recommendation is that the derivations are maintained in the paper, as they constitute an alternative way of deriving eq. (1), but that the narrative and conclusion that this work solves the 'old' issue of the lack of an exact equation for blade element thrust is taken out of the paper and reformulated. The main problem is not how to relate the azimuthal velocity to the thrust, but how to include the axial velocity, and this problem is not solved in the paper.