Please find below our review notes

Page/Line	Reviewer (Paul van der Laan) Comment	Changes Made
Title and General	Interesting Article but a model verification and validation is missing.	The model has been comprehensively validated against data from over 20 wind farms. Some of the validations are available here: https://proplanen.info/wakeblaster, and more will following in due course. Including the validations in the paper would have made it too long, because we would have needed to include methodology etc. as well as the results. The paper focuses on providing a clear and concise presentation of the theoretical background for the model, and the initial verification of its performance. A model verification is included, as indicated by the paper's title. The verification has been expanded in the paper, and references to the validation have been added (see below).
Page 1, abstract	1. In the abstract you mention:The WakeBlaster model is verified, calibrated and validated using a large volume of data from multiple onshore and offshore 10 wind farms. I cannot find a reference to this work and it is also not included in the present work. (...) In addition, please note that the abstract should include a motivation, a short summary of the work, and the main results and conclusions,so it cannot contain conclusions based on previous work.	In a different session at the WESC, the authors presented validation results from one of the wind farms (Verification and Validation of the Waked Flow of a Large Wind Farm), but a paper was not submitted. The presentation has now been uploaded to the WESC recommended repository, and a reference has been included in the paper. Additional references to further validation work and an external validation (blind test on 5 offshore wind farms) are now included. 1.1 Page, Line 55. References made in the text to validation were inconsistent with the paper's title, content and the primary purpose of the paper and so these have been removed.
Limitations	2. How are the results post processed when the annual energy production is evaluated? For example, do you include a Gaussian filter to represent wind direction uncertainty? (As introduced by Gaumond et al. (2014) and applied for RANS in van der Laan et al. (2015a)).	The model presented does not include annual energy yield calculations; it is only a flow case calculator. The integration of flow cases to calculate the energy yield is done on the client-side, and therefore outside the scope of this paper. When using the model in validation studies we have (where appropriate) implemented a Gaussian filter to account for wind direction uncertainty, but this is also outside of the scope of this paper.
Competing Interests	3. Since the authors are both employees at ProPlanEn, a commercial entity that is selling the presented model, it would make sense to mention this in the Section Competing interests.	The affiliation is provided in the list of authors and the funding details are provided in the acknowledgements. However, given that the reviewer comments on this, which in itself can be taken as evidence, that a potential conflict of interest could be perceived by others. It is agreed that any such perception is to be avoided. The relationships of the authors to ProPlanEn Ltd were therefore added to the competing interest section.
Limitations	4. Can WakeBlaster handle (complex) terrain? If not, I would mention that the model can only used for wind farms in flat terrain and offshore conditions.	WakeBlaster relies on the underlying flow model, and makes use of the speed-ups induced by terrain and/or roughness. This is equivalent to current industry models, but it does not go beyond that. This has been added to the limitations section, as suggested.
Pages 2-3, Lines 58-60:	5. I do not understand what you mean by In order to account for the unsteady terms, it uses eddy viscosity turbulence closure, where the eddy viscosity is calculated from the combined wake and ambient wind speed shear profiles. I guess you mean turbulent fluctuations instead of unsteady terms. Unsteady terms can be handled by including a transient term, as can be done in Unsteady RANS (URANS).	It is agreed that the sentence was confusing; this has been corrected.
	6. The reference to Abramovich (1963) is not very accessible. In addition, eq. (2) of the article is the boundary layer equation without viscous effects and I do not understand how this equation can be extended to three dimensions because the original boundary layer equation describes a streamwise U and vertical velocity, which in your coordinate system is W not V . I would suggest to start Section 2.1 with the 3D RANS equations including external forces ...	Abramovich 1963 (MIT Press) is a classical textbook on the topic of turbulent jets, which is still in print and available from any university library by inter-library loan, or from Amazon. It is geared to experiments and engineering applications, and is well known. It is considered preferable to to provide a reference to reference to providing a lengthy derivation from first principles.
	6. (...) in order to arrive at eq. (5) of the article we need to assume that $\partial V/\partial x$ and $\partial W/\partial x$ are zero, which is not mentioned in the article. The assumptions (a) and (c) are neither mentioned. Assumptions: a) The momentum equations for v and w are ignored. d) The gradient of the normal Reynolds stress in the streamwise direction is zero.	The assumptions have been revised. Changes have also been made to section 3.1, to clarify that we are looking at a free jet and modelling the momentum in the flow direction.
	6. (...) We assume the eddy viscosity to be variable.	The eddy viscosity changes downstream, throughout the domain, as the wake decays. It is assumed that the eddy viscosity is constant across the wake. No attempt is made to model the fine structure in the near wake.
	7. You mention that the near wake stream-wise velocity profile is prescribed for each wind turbine. How do you determine the end of the near wake and how does it vary with the wind turbine thrust coefficient and atmospheric conditions as turbulence intensity and stability? Do I understand correctly that eq. (7) is used to determine the initial magnitude of the centerline wake deficit at a defined downstream location? In addition, eq. (7) is derived from wind tunnel measurements where the turbulence length scales and Reynolds-number are very different from utility scale turbines, so would that mean eq. (7) needs to be recalibrated?	Yes, the fundamental work by Ainslie is used for the initial centerline wake deficit. Wq. (7) defines the wake deficit at a fixed downstream distance of 2D. This approximation has proven reliable over decades of use in various engineering codes. It is derived from a mix of field experiments (MW turbines) and wind tunnel experiments. Some of the data used to derive (7) is problematic and a re-calibration has been discussed. However given that the application of this equation has a long history in the industry. and that there is an absence of sufficient full-scale data to re-calibrate the function, use of the equation remains a reasonable choice.

	7. (...) Finally, I was wondering why you are not modeling the wind turbine thrust force as an external force in the streamwise momentum equation instead of prescribing a velocity deficit profile in the near wake.	For a detailed model of the near wake, one would need to set the radial distribution of the axial force, and use a model for the extraction of momentum in the induction zone and near wake. This requires the flow expansion to be modelled, including the radial and tangential flow components and the elliptic effects. The model would increase in complexity, beyond what can be validated. It would also become computationally which is unnecessary for practical application. The approach introduced by Lissaman and Ainslie is ingenious, in that it avoids (with acceptable loss of accuracy) the complexity of modelling the near wake. Ongoing efforts to re-calibrate the function (7) are, yet, inconclusive.
Section 2.4	8. The eddy-viscosity is no longer a constant, as assumed in eq. (5) of the article, which is inconsistent. Please motivate and clarify.	The eddy viscosity changes are at a slow time/length scale, over the domain and while the wake develops. In the near wake there are strong radial changes and blade tip vortices. Unlike in the wind tunnel, in a field experiment these small scale turbulence decays quickly. Outside of the near wake, the eddy viscosity changes more slowly. Turbulent eddies have the scale of the rotor diameter and can be approximated as constant over the length scale. Over longer time/length scale eddy-viscosity is not constant, as it is a property of the flow rather than the fluid, and it changes in all directions over the domain. We have added a sentence in section 2.1, to clarify this approximation.
Page 6 Line 158	9. There are a number of undefined parameters and constants. What are the values of ??? (Page 6, line 158), ??? (eq. 13), ??? (eq. 14), ??? (eq. 14)?	All parameters are now defined.
	10. Should k be κ in eq. (14)?	The naming of k is correct. It refers to the eddy viscosity calibration constant (section 2.4, bullet point 4). However, I do note that it could be easily redefined by substituting $\phi' = \frac{\phi}{k}$.
Section 2.4.1	11. . I suspect that the eddy-viscosity lag model could be replaced or simplified using a length scale limiter in the form of an fP function (...) that only has one constant to calibrate, see for example van der Laan et al. (2015b) or van der Laan and Andersen (2018).	We agree that the fp approach is similar and that it could be useful to compare them at some point in the future. The 'fixed' eddy viscosity lag model also has only one calibration parameter.
Section 3.1	12. : Please motivate the chosen grid resolution of D/10, where D represent the rotor diameter, using a grid refinement study. In addition, a domain height of 3D seems very low to me, please show that this domain height does not affect the solution. I normally use 25D for 3D elliptic RANS simulations of wind farms. What are the other dimensions of the 3D flow domain? Do you use stretching of cells in order to reduce the total number of cells or is the domain discretized uniformly?	The resolution is balanced with processing speed. It must be high enough to represent wakes (in the order of magnitude of the rotor diameter) and of sufficient detail for modelling partial wakes, wake interaction, and wake boundary layer interaction. The grid is a uniform rectangular structured grid with dx=dy=dz. The simple structure simplifies the model implementation and avoids numerical challenges at grid transitions. Additional vertical layers are not required for numerical stability, because we are using a downstream marching solution. Comparing the domain with an elliptical solver is not really valid - it is not surprising to us that an elliptical solver would need a larger domain. More information on the grid has been added in section 2.3.1.
Section 3.1:	13. You mention that a flow case of Horns Rev I takes 5 s. (Please briefly introduce the Horns Rev I wind farm here). That would mean an annual energy production calculation of 22 wind speeds and 360 wind directions would take 11 hours on a single CPU. This can be made parallel as you mention (using a few hundred cores). However, WakeBlaster should provide more accurate results compared to an engineering wake model that can calculate the AEP in about 1 s on a single core in order to make sense to run. Therefore, I would suggest to both validate WakeBlaster with wind farm measurements and compare the performance with one or two engineering wake models in the present work.	Yes, the AEP calculation consists typically of 2,500-10,000 flow cases, or 55,000 if you calculate the AEP for a representative year, at a time-step of 10 minutes. In the third party OWA blind tests, WakeBlaster delivered results with a more detailed structure and a reduced standard deviation. This blind test can also be interpreted as a practice test, where we were able to show that it is realistic to use CFD in an iterative process. The description of Horns Rev was added here, and references to both our own validations and third party offshore validation were added to section 1.1.
Section 3.2	14. You could use this wind farm as both a verification (grid study) and validation case. Presenting a show case is not enough for a scientific document.	The verification has been expanded and references to validation have been added.
Section 4:	15. You could add that only flat terrain is considered. In addition, I do not agree that meandering of the ambient wind direction is a limitation of the model because its effect on wake mixing can be modeled by either changing the eddy-viscosity or by running several wind direction cases and average them using a Gaussian filter, see for example van der Laan et al. (2015a), wich is based on the work of Gaumond et al. (2014).	The limitation referring to meandering has been removed, and limitations with respect to terrain have been added.
Conclusion	16. The conclusions are not based on the results of the present article: (...)	Agree, fixed (reference to validation removed)
Page 2, Line 45.	1. You write here: Models of this group are, in principle, also capable of solving the upstream effects of wind turbines. You are right about the upstream effects, however, it is also the interaction of the wind turbine wakes and wind turbine induction zones, which represents the elliptic nature of these models.	The text in section 1.1 was expanded accordingly.
Reference	My last name is miss-spelled in the corresponding reference (Laan should be van der Laan)	Apologies, corrected.
Page 1, Line 25:	3. There is a typo in a citation: citetSchlez2009.	Corrected
Section 1:	4. You mention parabolic and elliptic solvers. While I am aware of the meaning of these terms, it would be useful to explain them in order to reach a broader audience. For example, you could mention that a parabolic solver does not need to iterate numerically and information of the flow is only transported with the flow direction, while elliptic solvers have to iterate to solve the equations and information is transported in all directions.	Text in section 1.1 was expanded accordingly.
Eq. (1):	5. There is an additional plus sign that can be removed.	Corrected
		Thank you for the review.

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Page/Line	Anonymous Reviewer 2	Changes Made
Title	“Verification” can be misleading since the paper does not contain any comparison with experimental data. Consider adding a plot including the validation in Lillgrund for a flow case in the results chapter. In that case, consider change ‘Verification’ by ‘validation’	"Verification" is not misleading in this instance. "Verification" is an evaluation of how a product meets its specification and expectations, which is covered in chapter 3. "Validation" would be a comparison with experimental data, and it is not presented in this paper, in order to keep it concise.
Abstract, Line 10	add reference to any report including validation	References to model validations were added to section 1.1.
Page 1 – line 16:	describe further ‘single turbine wake model’, what implies	The description has been expanded.
Page 2 – line 37	this sentence is hard to understand. Better use: “… and later Crespo et. al. (1994) developed an extension for wind farms called UPMPARK”	Done.
Page 2 – line 54	change ‘verification’ by validation. Verification implies ensures that a model is working properly (equations solved as expected, no bugs), validation implies agreement with experimental data (reality physics)	Verification was intentionally chosen, as the paper in its current form does not include a validation. Inconsistent references to validation have been removed throughout the text.
Page 2 – line 57	suggestion: Change title of chapter 2 by “Theoretical background”?	Done.
Page 3 – line 59	unsteady terms or fluctuation term of velocity vector?	This has been rephrased and clarified in the text.
Page 3 – line 63:	Avoid “We”, use instead: “Cartesian 3D vectors are used for displacement…”	Changed in the four relevant instances lines (49, 69, 166, 303) in the text.
Page 3 – line 65:	use streamwise and transversal components, instead of mean and lateral	"Mean" and "lateral" have been replaced by "streamwise", "horizontal" and "vertical". Transversal has been used to address both horizontal and vertical directions.
Page 3 – line 70:	add mass conservation equation as well at this point	Done.
Page 3 – line 74	at the pressure assumption, needs “=0” at the end	Done.
Page 4	avoid mass conservation equation here if listed in 2.1	Done.
Page 4 – line 89	add a new chapter here on Grid resolution and boundary conditions, there are no references except in chapter 3.1 which can be here. (...)	Added to 2.3.
Page 4 – line 89	(...) Also justify here why a rotor disk is composed by 80 cells as specified in the abstract, should not this value depend on rotor area?	The high resolution is required to capture the wake profile for full and partial wakes with sufficient accuracy. The default model resolution is 0.1 turbine diameters, which makes it independent of the rotor area. At this resolution the rotor is covered by an average of $10*10*\frac{\pi}{4}/\approx 80$ points. However, this number varies from rotor to rotor, as the grid is not snapped to any individual turbine rotor. The number has been deleted in the revised text.
Page 4 – line 101:	specify the distance at which the near wake is placed (where the momentum deficit is injected)	Done.
Page 4 – line 103	using alfa for turbulence intensity can be misleading (same sign to refer to shear). To avoid mistakes, use TI instead	Done - now using I for turbulence intensity, and the left hand side of the equation has been corrected.
Page 5 – line 117	use transversal instead of lateral	Done.
Page 5 – line 128	remove ‘a’	Done.
Page 6 ch 2.4	general comment to chapter 2.4: since turbulence viscosity estimation (and consequently WakeBlaster) depends on those 5 parameters, a general recommendation or comment should be included (if feasible) about their range values, do they depend on the wind farm layout or scenario? (very close turbines, interaction of wakes with ground, etc.)	The description of the parameters has been expanded. They are independent of the wind farm layout and scenario.
Page 6 – line 158:	scalar velocity” to be changed by “turbulence viscosity”	This sentence has been corrected and rephrased.
Page 6 – line 162:	“…is solely based on..”	Done.
Page 8 – line 199:	this sentence is hard to understand, please re-write	Sentence was rewritten.
Page 9 – line 209	Chapter 3 should be dedicated to apply WakeBlaster to a particular flow case in a particular wind farm (Lillgrund). Please include a first sub section on describing Lillgrund wind farm in detail (layout, rotor diameter, etc.) and also include another section (if data were available) comparing wakeblaster and experimental efficiency values in a particular flow case.	Lillgrund wind farm details added and renamed Verification for consistency throughout the text.
Page 10 – line 211	Chapter on computational performance should be included on the numerical solution. This is not something inherent to the Lillgrund simulation	Chapter 3 has been restructured into two sections. Section 3.1 covers computational performance verification, and Section 2.3 is dedicated to implementation of the numerical solution.
Page 10 – line 231:	specify if the case corresponds to neutral atmosphere	Done.
Page 12 – line 250:	make some reference to limitations on complex terrain. Additionally, it could be mentioned the possibility to include RSF or WRG files in order to take into account the effect of orography on the free stream flow	Done.
		Thank you for the review.

Theory and Verification of a new 3D RANS Wake Model

Philip Bradstock[1,2] and Wolfgang Schlez[1]

[1]ProPlanEn Ltd, 71-75 Shelton Street, London, WC2H 9JQ, United Kingdom
[2]Bitbloom Ltd, Desklodge, 1 Temple Way, Bristol, BS2 0BY, United Kingdom

Correspondence: Wolfgang Schlez (wolfgang.schlez@proplanen.com)

Abstract. This paper details the background to the WakeBlaster model: a purpose built, parabolic three-dimensional RANS solver, developed by ProPlanEn. WakeBlaster is a field model, rather than a single turbine model; it therefore eliminates the need for an empirical wake superposition model. It belongs to a class of very fast (a few core seconds, per flow case) mid-fidelity models, which are designed for industrial application in wind farm design, operation and control.

The domain is a three-dimensional structured grid, ~~with approximately 80 nodes covering the rotor disk~~a node spacing of a tenth of a rotor diameter, by default. WakeBlaster uses eddy viscosity turbulence closure, which is parameterized by the local shear, time-lagged turbulence development, and stability corrections for ambient shear and turbulence decay. The model prescribes a profile at the end of the near-wake, and the spatial variation of ambient flow, by using output from an external flow model.

~~The WakeBlaster model is verified, calibrated and validated using a large volume of data from multiple onshore and offshore wind farms. This paper presents example simulations for one offshore wind farm.~~

1 Introduction

In wind farms, wind turbines located downstream of other turbines will experience wake losses. Wind farm development and assessment processes require multiple iterations of configurations, as well as fast project turnaround.

A good understanding of how wake loss works can give a company the competitive edge, while an unexpected systematic performance loss can eliminate the expected profit from a project, or even from an entire project pipeline. Given the importance of wake losses, it may appear contradictory that many in the industry still use analytical single turbine wake models. Using single turbine wake models means that the wake from each turbine is propagated independently, wake expansion is not impacted by neighbouring wakes, and multiple wake deficits are superimposed using an empirical wake superposition model. Single wake models are based on an approach suggested 40 years ago, by Lissaman (1979) and Lissaman et al. (1982), who transferred the work of Abramovich (1963) on free jets to wind turbine wakes. Jensen (1983) presented what is still the most prominent model in this category. Other prominent models of this type include numerical solutions, by Ainslie (1988) and Ott (2011). More recent analytical models include that of Ishihara and Qian (2018).

The longevity of the single wake model approach also speaks for the quality and practical usefulness of these early models. However, in order to provide accuracy for the full range of wind farms (e.g. large wind farms, closely cross-spaced farms, low hub height wind farms, wind farms with stable conditions, or offshore wind farms), an increasing number of empirical corrections had to be made, and parameters added, informed by new experimental data from wind farms, scale experiments, or higher fidelity models - see, for example, Liddell et al. (2005), Schlez et al. (2006), ~~citetSchlez2009~~Schlez et al. (2009), and Beaucage et al. (2012). A range of analytical single wake models and superposition methods are reviewed by Porté-Agel et al. (2019).

The increased computational power and scalability available today allows higher fidelity wake models to be used in the iterative process of wind farm design. These models widen the operational envelope, include more physics, and reduce model uncertainties in non-standard situations. ~~We present the~~ The theory behind one such model is presented in this paper: a 3D RANS (Reynolds Averaged Navier-Stokes) wind farm wake model, WakeBlaster.

1.1 Related Work

In order to gain a more detailed understanding of wake losses in a wind energy research context, two groups of 3D-RANS codes have been developed. The models are referred to as 'field models', to distinguish them from the single turbine models by Crespo et al. (1999).

The first group of 3D-RANS codes are parabolic solvers, using the thin shear layer approximation, see Ferziger et al. (1997). Parabolic solvers assume a dominant flow direction and information is transported only downstream. Crespo et al. developed UPMWAKE at UPM (Universidad Polytechnica de Madrid), and later Crespo et al. (1994) ~~based a parabolic model of the flow field inside a wind farm~~developed an extension for wind farms, called UPMPARK~~, on it~~. A number of further variants have been developed and reviewed by Vermeer et al. (2003). One branch was continued by TNO/ECN (The Energy Research Center of the Netherlands), and it resulted in the WakeFarm presented by Schepers (2003), and FarmFlow model presented in Eecen et al. (2011). Renewed interest in mid-fidelity models has recently led to the independent development of several new models in this group, like those presented by Trabucchi et al. (2017) and Martinez-Tossas (2019).

The second group of 3D-RANS field models, the elliptic solvers, is more widespread. Elliptic solvers are ~~used across other industries to parabolic solvers, and they are~~ generally more powerful, ~~but~~ and they iterate equations numerically, in order to allow information to be transported in all directions; this makes them more expensive computationally (by several orders of magnitude)~~more expensive~~. These models use a k-ϵ or k-ω turbulence closure, describing the generation and dissipation of turbulent kinetic energy. Models ~~of~~ in this group are ~~, in principle,~~ (in principle) also capable of solving ~~the upstream effectsof~~ upstream effects, such as the interaction of wakes in the induction zone, and the near wake of wind turbines. Some models are based on general purpose flow solvers, whereas others are in-house developments - examples can be found in ~~the~~ publications by Crespo et al. (1988); Prospathopoulos et al. (2010); Barthelmie et al. (2011); van der Laan et al. (2017); Michelsen (1994).

The WakeBlaster model developed by ProPlanEn by Schlez et al. (2017b) belongs to the parabolic solver group. A parabolic solution offers a good balance between improved accuracy, additional detail, and computational costs. The target of the new model is to improve the accuracy of wind farm loss modelling. Two specific aims are to address the interaction between wakes, as well as the interaction between wakes and the atmospheric boundary layer for different levels of atmospheric stability. Special attention was paid to the validation of the model, using data from a wide range of wind farms and atmospheric conditions, which has been reported by the authors in Schlez et al. (2017a, 2018, 2019); Bradstock et al. (2018); Braunheim et al. (2018), and independently evaluated and compared to engineering models in a blind test for offshore wind farms, by Sanz et al. (2019) .

The fundamental equations and assumptions for this solver are shown in the following Section 2. Section 3 presents as example the verification of the model for an offshore wind farm and the results of verifying the computational performance. Section 4 discusses model limitations, followed by the conclusions in Section 5.

2 TheoryTheoretical Background

The WakeBlaster wind farm simulator is based on a Reynolds-Averaged Navier-Stokes (RANS) set of equations, which is used to solve the propagation of wake dissipation through the farm domain, in Cartesian 3D coordinates. In order to account for the unsteady termsfluctuation term of the velocity vector, it uses eddy viscosity turbulence closure, where the eddy viscosity is calculated from the combined wake and ambient wind speed shear profiles.

2.1 RANS Equations

The wake model uses RANS equations for momentum conservation, and mass flow conservation to calculate the three components of wind velocity in the axial, lateral and vertical directions. We use Cartesian 3D vectors are used for displacement $\vec{\mathbf{x}}$ and wind speed relative to ambient $\vec{\mathbf{u}}$: $\vec{\mathbf{x}} = \left[x, y, z\right] \quad \vec{\mathbf{u}} = \left[u, v, w\right]$, where the first element of the vectors (x) is along the mean wind directionthe streamwise component, the second element (y) is horizontal and perpendicular to (x), and the third element (z) is vertical (starting from the ground up) and makes up a right-hand coordinate system.

The Reynolds averaged momentum and mass conservation equation can be expressed for an incompressible flow in two dimensions, for either a free jet or a wake submerged in an incompressible fluid, as given by Abramovich (1963):

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial \overline{u'u'}}{\partial x}+\frac{\partial (uv)}{\partial y}+\frac{\partial \overline{u'v'}}{\partial y}=\mu\nu\frac{\partial^2 u}{\partial y^2}+-\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (1)$$

representing the momentum in the flow direction, where u', v' and w' denote fluctuations from mean values., and ν the viscosity and ρ the density of the fluid. The corresponding continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

The momentum equations in transversal directions are not considered in the description of a free jet or wake present beyond the near wake of a wind turbine.

2.2 Simplifying Assumptions

The following simplifying assumptions are applied by Abramovich for a stationary free wake, expanding into an infinite region:

Viscosity The effect of molecular viscosity is small $\mu\frac{\partial^2 u}{\partial y^2}=0$ $\nu\frac{\partial^2 u}{\partial y^2}=0$ compared to the turbulent viscosity

Pressure Flow pressure gradients can be neglected $\frac{1}{\rho}\frac{\partial p}{\partial x}$ in most cases $\frac{1}{\rho}\frac{\partial p}{\partial x} = 0$

Stationary The flow is stationary with respect to the mean velocities $\frac{\partial u}{\partial t} = 0$

Thin shear layer approximation Fluctuations along the flow change much slower than in the transversal direction $\frac{\partial \overline{u'u'}}{\partial x} = 0$

After substituting the continuity equation and applying the simplifying assumptions Abramovich (1963) obtains:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial \overline{u'v'}}{\partial y} = 0 \tag{3}$$

or expanded to three dimensions:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} = 0 \tag{4}$$

and using the eddy viscosity turbulence closureUsing the Boussinesq eddy viscosity assumption, the stress components $\overline{u'v'}$ and $\overline{u'w'}$ are expressed as:

$$-\frac{\partial \overline{u'v'}}{\partial y}\overline{u'v'} = \frac{\partial^2 u}{\partial y^2} - \frac{\partial \overline{u'w'}}{\partial z}\epsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \approx -\epsilon\frac{\partial u}{\partial y} \qquad \overline{u'w'} = \frac{\partial^2 u}{\partial z^2}-\epsilon\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \approx -\epsilon\frac{\partial u}{\partial z} \tag{5}$$

where $\bar{\epsilon}$ denotes the eddy viscosity, which is considered to be scalar and isotropic. This leads to ϵ denotes the isotropic eddy viscosity. The streamwise variation in transversal velocities ($\frac{\partial v}{\partial x}$ and $\frac{\partial w}{\partial x}$) is small compared to the transversal variation of streamwise velocity ($\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$). The spatial variation in eddy viscosity can be neglected in first approximation and is therefore approximated as a constant, the governing momentum conservation equation can now be written as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \epsilon\frac{\partial^2 u}{\partial y^2} - \epsilon\frac{\partial^2 u}{\partial z^2} = 0 \tag{6}$$

while maintaining continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

2.3 Numerical Solution

The ambient wind field is determined by an external flow model, and it determines the inflow conditions and spatial variations over a site. The turbine is represented by its hub height, diameter and other readily available and measured characteristics.

2.3.1 ~~Wind Turbine Momentum Extraction~~Model Domain

The waked wind field is set up by creating a two-dimensional flow plane, which forms a cross-section along the y and z axes of the velocity vector $\overline{u}$. The flow plane is ~~bounded by the ground, at $z = 0$, and is large enough above and to the side of the resident wind turbines to ensure that boundary conditions have no more than a negligible impact on wake dissipation. The flow plane is~~ propagated downstream along the x coordinate and ~~, at each point along the x coordinate~~ when it passes a turbine, a wake is injected into the flow plane.

The grid spacing is set by default to a tenth of rotor diameter. In the vertical direction, the grid starts at the ground $z = 0$ and reaches up to a default height of three rotor diameters or 31 grid layers. In the horizontal direction, the grid is expanded, as required, to enclose each wake injected into the flow plane with an additional four rotor diameters to the side to allow for wake expansion.

2.3.2 Wind Turbine Momentum Extraction

Axial-momentum theory prescribes pressure building up in the induction zone upstream of any wind turbine or wind farm, and pressure recovery in the near-wake downstream of the rotor. The momentum that each of the turbines extracts in the process is the wind speed dependent thrust coefficient, as a function of the idealised incident wind speed, U_{inc}, at each turbine location, without the presence of the turbine.

In the model, the momentum deficit is injected at the end of the near-wake ~~, for~~ (which is assumed to be at 2 diameters downstream of the rotor) of each turbine, and it is distributed over an expanded rotor area, using the blunt bell-shaped wind speed deficit profile from Lissaman et al. (1982). The centre-line wind ~~speed U_{cent}~~deficit relative to incident wind speed D_m, experimentally determined by Ainslie (1988) at a downstream distance of 2 diameters is used as a function of inflow turbulence ~~α~~ I_{inc} and thrust coefficient c_t.

$$U_{cent} D_m = U_{inc}(c_t - 0.05 - (\left(16c_t - 0.5\right)\frac{\alpha}{10}) \frac{I_{inc}}{10} \qquad (7)$$

The radial width of the profile is then derived by ensuring momentum conservation with regard to the thrust coefficient of the turbine.

2.3.3 Flow Plane Propagation

The flow plane is propagated according to equation 6 using the alternating direction implicit (ADI) method described by Peaceman and Rachford (1955); von Rosenberg (1983), where it is alternately solved in the xy and xz planes, incrementing the x (downstream) coordinate by half a propagation step between each solving plane, so that both planes are solved once per step. By solving for each row or column in the flow plane, and by employing the central difference method, the problem can be set up numerically in a tridiagonal matrix equation, which can then be solved efficiently for the axial velocity, u, by the Thomas algorithm Thomas (1949), described for example in Burden and Faires (2001). In 3D Cartesian coordinates the tridiagonal equation must be solved for every row or column of the flow plane, depending on which direction ~~we are solving for~~a solution is obtained. Dirichlet boundary conditions are used by enforcing $u = 1$ in the extremities of the flow plane.

At each half-step of the solving process, the ~~lateral~~ horizontal and vertical velocities, v and w respectively, are calculated for all points in the flow plane according to 2. For any given step there are two unknowns in this equation, v and w, and therefore it cannot be solved analytically in a single step. Instead, the unknowns are calculated numerically, by calculating each individually, and iterating until their values converge. By rearranging equation 2, v and w can be expressed individually for a parabolic flow:

$$v = -\int \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} dy \quad ; \quad w = -\int \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} dz \tag{8}$$

In practice, due to the assumption of incompressibility, this formulation will lead to a local velocity shear, resulting in non-zero lateral and vertical velocities that are infinitely far from the source of shear. In reality this would not be the case, due to the compressibility of air. Therefore, in order to account for the effect of compressibility, a spatial damping term is introduced, so that v and w tend to zero at $y = -\infty$, $y = \infty$ and $z = \infty$:

$$v = -\int \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \gamma v \right) dy \quad ; \quad w = -\int \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \gamma w \right) dz \tag{9}$$

where γ is a user-configurable positive constant that determines the strength of lateral and vertical velocity damping. As these integrals are indefinite, boundary conditions must be assigned. In the vertical direction, it is ~~a~~ given that vertical velocity at ground level is zero, as mass flow cannot pass into or out of the ground. Therefore, the condition $w_{z=0} = 0$ is applied, leading to:

$$w(z) = -\int_0^z \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \gamma w \right) dz' \tag{10}$$

In the lateral direction, the physical boundary conditions are that $v_{y=-\infty} = v_{y=\infty} = 0$, because the wind farm wakes cannot induce lateral velocity far from the farm. However, for numerical purposes, the size of the flow plane is constrained, and it cannot be guaranteed that the velocity will reach zero on both sides of the flow plane. Therefore, the lateral velocity is integrated in each direction, starting from zero, and the mean of the two is taken. This is expressed as:

$$v(y) = -\frac{1}{2} \int_{y_{min}}^{y} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \gamma v \right) dy' + \frac{1}{2} \int_{y_{max}}^{y} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \gamma v \right) dy' \tag{11}$$

where y_{min} and y_{max} are the lateral location of the edge of the flow plane.

2.4 Eddy Viscosity Calculation

The key term controlling the rate of wake dissipation is eddy viscosity. Eddy viscosity has dimensions of length squared over time, and it can be represented by multiplying a length scale of the shear layer by a velocity scale of the flow field.

WakeBlaster calculates eddy viscosity from the shear profile of axial velocity in the yz plane. In order to do this, it creates a combined flow plane of the ambient wind speed, U_{amb}, multiplied by the solved wake flow plane, u, which is relative to ambient wind speed. In neutral atmospheric conditions, the ambient wind speed is calculated as a logarithmic function of height above ground:

$$U_{amb}(z) = \frac{u^*}{\kappa} \ln \frac{z}{z_0} \tag{12}$$

where u^* is the friction velocity, taken to be 2.5 times the value of standard deviation of the axial wind velocity, κ is the von-Karman constant (value = 0.4) and z_0 is the roughness length. The unknown parameters are determined from inputs to the simulation, such as wind speed and turbulence intensity at a particular height (usually the hub height of one of the turbines). The eddy viscosity is then calculated for every point in the flow plane, using the following process:

1. Create a combined flow plane by multiplying the ambient surface layer wind speed profile by the waked flow plane velocity u.

2. For each point, identify the local minimum and maximum velocity. For a point located at (y, z), local is determined as the range $[y - \eta z, y + \eta z]$ and $[(1-\eta)z, (1+\eta)z]$, in the lateral and vertical directions respectively, where η is a configurable constant which meets the criterion $0 < \eta < 1$.

3. In each of the two directions, the component of eddy viscosity is calculated as $\epsilon_i = \Delta u_i \Lambda_i$, where Δu_i is the difference between minimum and maximum velocity and Λ_i is the distance between the maximum and minimum points. This process is shown in figure 1.

4. The overall ~~scalar velocity~~ eddy viscosity is the calculated as $\bar{\epsilon} = k\sqrt{\epsilon_y^2 + \epsilon_z^2}$, where k is a ~~configurable calibration constant~~postive calibration constant, which although configurable is considered to be independent of wind farm size and layout.

For a logarithmic wind speed profile in the vertical direction with no lateral variation, this method leads to an eddy viscosity that is proportional to the height above ground.

2.4.1 Eddy Viscosity Lag

The eddy viscosity, as so far described in section 2.4, is solely based on the wind shear profile. However, no newly created shear profile instantly generates turbulence, and therefore eddy viscosity - in reality, there is a lag between the change in a shear profile and its effect upon eddy viscosity and wake dissipation. In WakeBlaster this lag is formulated in terms of downstream distance, and it has two distinct models.

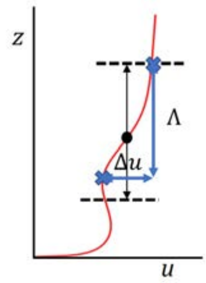

Figure 1. Calculation of the vertical component of eddy viscosity by finding the points of minimum and maximum velocity within a given height range.

The ‘fixed’ model obeys a first order lag equation:

$$\ell\Lambda\frac{d\epsilon}{dx}+\epsilon=\overline{\epsilon} \tag{13}$$

where ϵ is the lagged eddy viscosity, Λ is the length-scale defined in the previous section, and ℓ a ~~configurable~~ positive constant defining the lag length relative to the length scale and is considered to be independent of wind farm size or layout.

The ‘turbulence dependent’ model gives a larger lag distance when the eddy viscosity and turbulence are low, and it obeys the following equation:

$$\frac{\Lambda}{\phi\frac{\epsilon}{\kappa z}+\frac{\Lambda}{\lambda_{max}}}\frac{d\epsilon}{dx}+\epsilon=\overline{\epsilon} \tag{14}$$

where ϕ is a ~~configurable~~ positive parameter that determines the strength of turbulence on the lag length, and λ_{max} is also a ~~configurable~~ positive parameter that corresponds to the lag length when turbulence is zero. Both parameters are calibrated against extensive wind farm observational data and are considered to be independent of wind farm size and layout.

2.4.2 Atmospheric Stability

When simulating atmospheric conditions that are not neutral, the calculation of eddy viscosity is modified. This modification uses the Monin-Obukhov length, L, and the concept of non-dimensional wind shear, ϕ_m, which is defined by Businger (1971), as:

$$\phi_m=\frac{\kappa z}{u*}\frac{\partial U}{\partial z} \tag{15}$$

Furthermore, according to Businger (1966), the non-dimensional wind shear is empirically approximated as what tends to be known as the Businger-Dyer relationship:

$$\phi_m = \begin{cases} 1+5\zeta & \text{stable } (L>0) \\ 1 & \text{neutral } (L \text{ undefined}) \\ (1-16\zeta)^{-\frac{1}{4}} & \text{unstable } (L<0) \end{cases} \tag{16}$$

where $\zeta = \frac{z}{L}$. The ambient wind speed shear profile is then modified by introducing ψ_m:

$$U_{amb}(z) = \frac{u*}{\kappa} ln\left(\frac{z}{z_0} + \psi_m(\zeta)\right) \tag{17}$$

where:

$$\psi_m = \int_{\zeta_0}^{\zeta} [1-\phi_m] d\zeta \tag{18}$$

where $\zeta_0 = \frac{z_0}{L}$. Furthermore, the vertical component of the eddy viscosity, ϵ_z, is also modified by the non-dimensional wind shear:

$$\epsilon_z = \frac{\Delta u_z \Lambda_z}{\phi_m} \tag{19}$$

The horizontal component of eddy viscosity ϵ_y, is left unmodified.

2.5 Wind Turbine Power Calculation

WakeBlaster calculates the power output using power curve input from the user. In order to calculate accurate power, corresponding to the variant wind speed across the rotor, a rotor equivalent wind speed (U_{rot}) is calculated. This is done by first calculating the combined ambient and wake axial velocity ($U = U_{amb}u$ at the rotor plane), and then integrating across the rotor disk area:

$$U_{rot} = \sqrt[n]{\int_A U^n dA} \tag{20}$$

where n is an integer. A popular approach is to use $n = 3$ as suggested in IEC61400-12-1 (2017), based on the principle that power the power available in the wind is proportional to the cube of the wind speedas suggested in IEC61400-12-1 (2017). However, WakeBlaster uses a value of $n = 1$ by default , i.e. a linear average, with validation having shown this to lead to

~~a more accurate prediction of power output~~as turbines will not be able to realise the full potential of a sheared inflow over the rotor. As this method is performed on the combined ambient and wake axial velocity, the effects of wind shear on power production are implicitly included whenever the severity of the wind shear depends on the turbulence and atmospheric stability of the flow case.

A general directional variability of the wind within each flow case is included in a standard power curve. A rotor yaw angle can be set per turbine, to consider in the power calculation a known average directional misalignment with the rotor plane. A model to modify the power curve for site specific directional variability over the rotor, for example changes with height or for specific meteorological conditions, is not included in the model.

WakeBlaster uses IEC methods in IEC61400-12-1 (2017) to adjust the power curve for air density and turbulence intensity. The rotor equivalent turbulence intensity is also calculated using the integral method above, but instead using a value of $n = 2$.

~~Layout of the Lillgrund wind farm. The turbine rotor diameter is 93 m with a hub height of 68 m. The turbine spacing is approximately 4.3 rotor diameters along the South-West to North-East rows and 3.3 diameters along the South-East to North-West rows.~~

3 ~~Simulation Results~~Verification

In this section~~we verify the computational~~, the grid dependence and sensitivity is analysed and an estimate of the numerical uncertainty is thereby provided. Computational performance for large wind farms ~~, and then show the model predictions for an offshore wind farm.~~ is verified, and offshore wind farm model predictions are inspected graphically, for plausibility.

3.1 Grid Dependence and Sensitivity

A structured grid, in terrain following coordinates, is used in the model. The grid resolution is scaled with a length scale that characterises the specific flow - the rotor diameter. The grid is equally spaced in all directions, and no stretching, compression, or nesting is applied to any part of the domain. This minimalist design is computationally efficient and it avoids potential numerical errors - for example, at grid interfaces which do not match.

The solver is designed for a single purpose: modelling the impact of wind turbines on the underlying flow and consequential wind farm wake losses. A wind turbine's wake scales with its rotor diameter and its height above ground. In order to match the dominant scale in the flow for each wind farm, the grid resolution is fixed at 0.1 diameters, and it thus scales with the rotor diameter.

By analysing the sensitivity of model results to changes in grid resolution, it is verified that the results are not sensitive to the grid resolution over the expected range of application. Challenges could arise, for example, when using an average resolution in wind farms with mixed turbine diameters and turbines mounted at low hub heights. In an annual energy calculation, the overall wake loss is composed of several thousand individual flow cases. Wake loss model errors are commonly estimated to be in the range of 10-20 percent, relative to the average annual wake loss. Numerical errors should be an order of magnitude

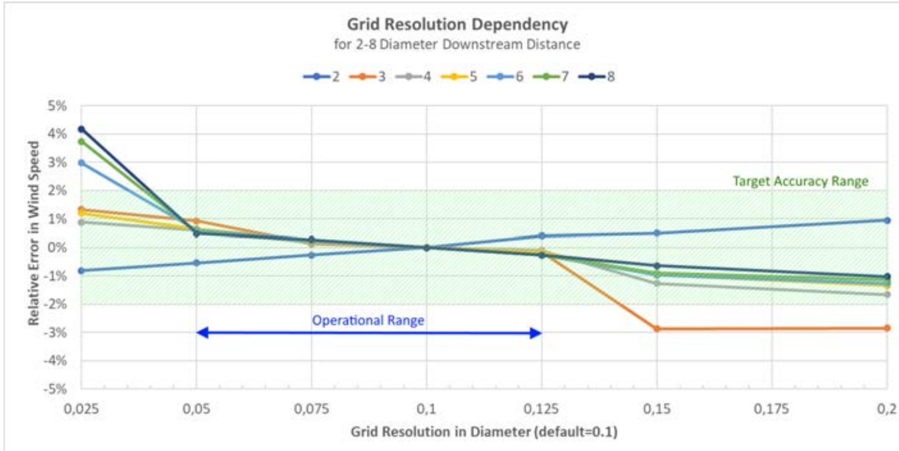

Figure 2. Grid dependency for a pair of wind turbines at downstream distances of 2-8 diameters. The wind speed of the flow case is 8 m/s (strong wake effects), at neutral stability and 9.5% turbulence. The range of acceptable numerical accuracy is highlighted. The numerical error increases outside of the operational range of 0.05 to 0.125 diameters, with a default value of 0.1 diameters.

lower. Ignoring error compensation between flow cases, an error of 1-2 percent - relative to the wind speed difference for an individual flow case - is acceptable.

The sensitivity in a flow case with strong wakes was tested, and the results are presented in 2. The relative error for an operational range of 0.05 to 0.125 diameters is below 2 %. The accuracy of the numerical model decreases when the resolution is lower, because the model can no longer resolve the structure of the flow sufficiently. Numerical accuracy decreases when the resolution is outside of the operational range, because of numerical instabilities. The current choice of grid resolution (0.1 diameters) represents a reasonable compromise between computational efficiency and accuracy.

The grid resolution in the model scales automatically with the rotor diameter. The default resolution is set within the operational range for low dependence on grid resolution. Neither the grid nor the resolution are variables that should, under normal circumstances, be adjusted by any user. The grid sensitivity study indicates that the model implementation accuracy is fit for purpose.

3.2 Computational Performance

WakeBlaster is a ~~medium fidelity~~ medium-fidelity tool, which is typically capable of running each flow case in a few seconds, on the single core of a modern processor. With the default settings (a flow plane resolution of 0.1 rotor diameters ~~,~~ and a domain height of three diameters), the time (in seconds) to run a single flow case (T_{fc}) is (on an Intel i5 8^{th} generation processor) approximately:

$$T_{fc} \approx 0.0015 \frac{A}{D^2} \text{ s} \tag{21}$$

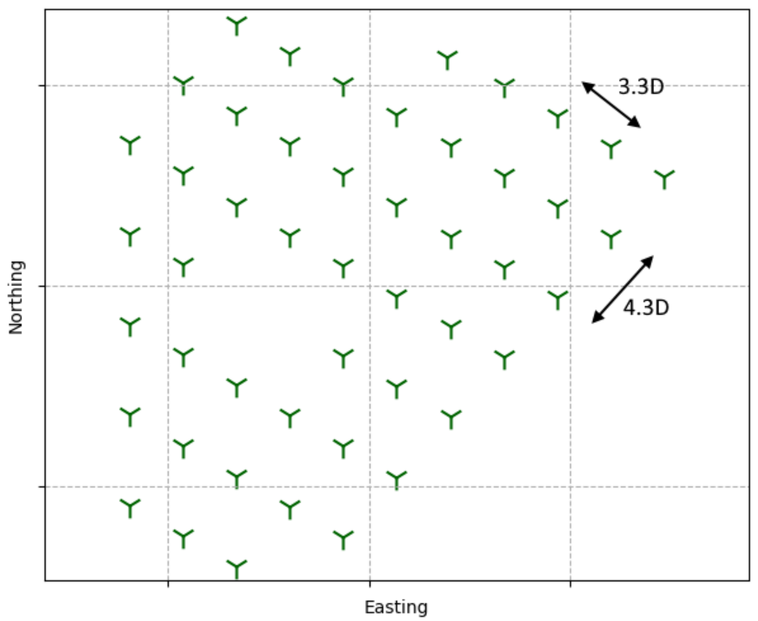

Figure 3. Layout of the Lillgrund wind farm. The turbine rotor diameter is 93 m with a hub height of 68 m. The turbine spacing is approximately 4.3 rotor diameters, along the South-West to North-East rows, and 3.3 diameters along the South-East to North-West rows.

where A is the area of the wind farm and D is the rotor diameter. The T_{fc} is proportional to the area of the wind farm, and (at equal turbine density) ~~proportional~~ to the number of wind turbines in the wind farm. However, the exact time will depend on the wind farm's layout, the wind direction~~and the processor architecture~~ , and the architecture of the processor. The T_{fc} is also proportional to the cube of the flow plane resolution, although results do not show any significant improvement in accuracy when the resolution is increased.

For example, a typical flow case for Horns Rev - a wind farm with 80 turbines arranged in a grid, with inter-turbine spacing of 7 diameters - runs in about 5 s~~, and the largest farms in the North Sea under planning around a minute~~. Unless hysteresis effects are included in a time series simulation, each required flow case remains independent from the others, allowing many flow cases to be run in parallel. As WakeBlaster is hosted on the cloud, this allows a high level of parallelisation across ~~dozens~~ tens or hundreds of processors, meaning that an energy assessment consisting of (for example) 2,000 flow cases can be completed in a matter of minutes, even for large wind farms.

3.3 ~~Waked Flow Visualisation~~

~~As~~

3.3 Visual Verification

Using a three-dimensional wake model, it is possible to create plots of the three-dimensional waked flow field for the complete wind farm, for a particular flow case. ~~In this article ,~~ This article presents a visualisation of a single flow case from the Lillgrund wind farm, located in the Øresund Strait, between Sweden and Denmark~~, is presented~~. The Lillgrund wind farm presents a good case study, because the small spacing between turbines (3.3 and 4.3 rotor diameters, along the two principal rows) leads to large wake effects. The layout is shown in figure 3. The turbines have a rotor diameter of 93 m and a hub height of 68 m above mean sea level.

Three cross-sectional slices in the xy, xz and yz planes, for a flow case of 8 $\mathrm{m/s}$ wind speed, 270 deg wind direction~~and~~, 6 % turbulence intensity, and neutral atmospheric conditions, are presented in figure 4.

These simulations indicate that there is significant interaction between ~~the~~ wakes originating from ~~each turbine~~individual turbines, and this supports the assumption that the wakes cannot be modelled independently. The wake interaction leads to a complex ~~wind farm wake shape at the downstream end of the~~ wake shape downstream of the wind farm. The low hub height of the wind turbines (68 m), relative to their rotor diameter (93 m), results in significant ground-wake interaction effects. ~~Due to the fact that~~ As ambient mixing from below is limited, single turbine wakes become asymmetrical in shape, and the point of greatest deficit drifts downwards to below hub height.

4 Limitations

The code is a mid-fidelity code designed to be fast and capable of simulating projects with several thousand turbines, working with limited amount of readily available input data and be used in an iterative design process. This limits the level of detail that can be included in the sub-models.

- No direct interaction between the turbines and no description of the axial pressure gradient are included in the model. The induction zones directly upstream and downstream (near wake) of turbines can overlap and interact. This may lead to changes in turbine performance and turbine characteristics and no attempt has been made to quantify such effects.

- A basic representation of the the ambient flow is used as input to the model. The wake is modelled as a perturbation of the underlying flow. No attempt has been made to model a two-way interaction with the atmospheric boundary layer.

- The model ~~is designed for stationary flow cases. No detailed models representing dynamic changes in wind direction, wind speed or turbine control were included.~~ uses the directional speedups predicted by a suitable flow model (for example in a RSF/WRG format) to account for spatial variation of the wind resource, for example due to orography, or roughness. Further complex terrain effects, like flow separation, are not considered.

- The ambient wind direction is assumed to be constant throughout the wind farm. Therefore in curved flows (due to terrain or due to meteorological factors), downstream wake locations may not be accurate.

The WakeBlaster model undergoes continuous, data driven improvement, and refined models will be added successively.

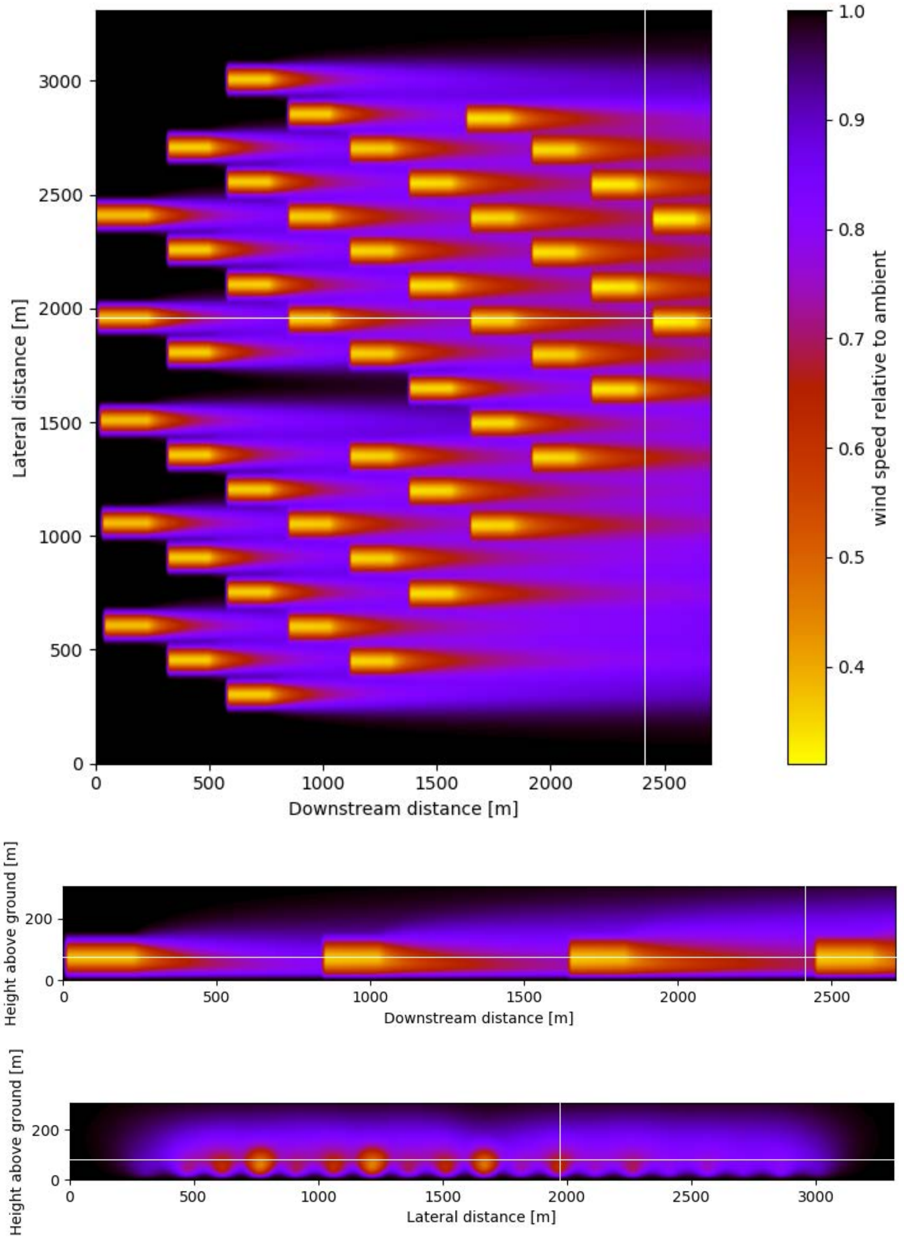

Figure 4. Plots of the axial velocity in the wind farm relative to ambient wind speed for a flow case of 8 m/s with the wind from due West. From top to bottom: xy (birds-eye) slice at hub height; xz (side-on); yz (front-on). The white lines show the corresponding planes of the other plots. The xy plot is taken at the turbine hub-height above sea-level - 68 m.

5 Conclusions

This is the first publication to present the theoretical background of WakeBlaster in some detail. WakeBlaster is a recently developed 3D-RANS solver that is specialised to simulate the waked flow field in wind farms. The characteristics of this model show the desired performance balance between speed and realistically achievable accuracy.

~~The model has been validated against performance data from offshore and onshore wind farms. Further detailed validation will focus on the specific 3D elements of the model, such as wake superposition and wake boundary layer interaction.~~

Code availability. WakeBlaster calculations are provided as a cloud service and designed for integration in other software packages. WakeBlaster is available from ProPlanEn directly (www.wakeblaster.net) and through third party implementations. WakeBlaster has been integrated in EMD's WindPro software and is scheduled for release in May 2020.

Author contributions. Philip Bradstock: formal analysis, investigation, methodology, software development, data curation, verification, visualisation, writing; Wolfgang Schlez: conceptualisation, funding acquisition, project administration, resources, investigation, supervision, methodology, writing

Competing interests. WakeBlaster is a commercial product of ProPlanEn Ltd. Wolfgang Schlez is the founder and sole share holder of ProPlanEn Ltd. Philip Bradstock was employed by ProPlanEn Ltd at the time of carrying out the model development. He is a director of Bitbloom Ltd. providing services to ProPlanEn Ltd.

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