Blade element momentum (BEM) models have been used for many years for the aerodynamic analysis of wind turbines and propellers. Contrary to the statement on line 21, they were not introduced by Glauert (1935, reference in manuscript), who did however, develop them in a form that has been used for wind turbine analysis for nearly a century. For example, Lock et al. (1925, reference below) gives a detailed account of the pre-Goldstein (1929) and pre-Glauert version of BEM.

Since the introduction of BEM, many second order corrections to it have been studied but these are not considered in the present manuscript. They include the effects of finite blade number, e.g. Clifton-Smith (2009), Wood et al. (2016), Schmitz & Maniaci (2017), Wimhurst & Willden (2017, 2018), the nonlinearity of the governing equations, Wood & Okulov (2017). Further, Limacher & Wood (2020) showed that the concerns of Goorjian (1972, reference in manuscript) over the role of the forces on the expanding streamtubes can be avoided easily. Therefore the claim in the manuscript to present a “state of the art” BEM analysis is not valid.

The manuscript concentrates on two aspects of BEM: the effect of wake expansion and an extension of the model of Bak et al. (2006) for rotational effects on the blade element forces. It is not demonstrated that these second order effects are more important than the others, but the reasonable agreement shown between the available measurements and the new model suggests that the contribution is worthwhile.

The axial induction factors, *a* for the near-wake, and *b* for the far-wake, are assumed to be independent of radius, in for example, Equations (3) and (4). This is a major simplification which is not justified. Constant *b* in the far-wake gives a “Joukowsky” wake comprising a concentrated hub vortex of strength $N\Gamma$ where Γ is the maximum bound circulation and *N* is the number of blades, and *N* helical vortices at the edge of the wake. Then the relation between pitch and far-wake velocity is easily determined from the Kawada-Hardin equations (Kawada, 1936; Hardin, 1982) as

$$v = 1 - \frac{N\Gamma}{2\pi h}$$

Where *h* is the vortex pitch, which contradicts Equation (12). For a Joukowsky wake, Wood (2007) rediscovered the result of McCutchen (1985) that the rotational velocity contribution to the energy equation is cancelled by the contribution of the radial pressure gradient, and so can be ignored, contrary to the statement of its importance made several times in the manuscript.

It is also unlikely that the strength of the trailing vortices is set only by the lift on a blade element. When the angular momentum equation, (6) in the manuscript, is balanced against the blade element forces, the element drag is involved, and, therefore, the rotational induction factor *a*’ is partly determined by the drag. Since *a*’ is also the normalized circumferential velocity induced by the trailing helical vortices, the circulation of those vortices is also partly determined by the element drag.

The manuscript is poorly written in places. For example, “plane” is often rendered as “plan” and there are many examples of poor expression which should be caught by a grammar checker.

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