



Figure 3. Estimated wind speed compared to rotor-averaged wind speed for a reference simulation.

by manufacturers. Any improvement on the methodology would be beneficial for the procedure of load estimation presented in this work.

3.2 Thrust estimation

We compute the estimated thrust, $T_{a,est}$, using Eq. (17) and the wind speed estimated in Sect. 3.1. In Fig. 4, we compare the estimated thrust value to the unsteady aerodynamic thrust from the simulation, $T_{a,ref}$. The values of $T_{a,tab}(\bar{u}_{ref}, \Omega_{ref}, \theta_{p,ref})$ are also shown in the figure.

We observe that the thrust signal is obtained with a mean relative error of 1.5 % over the range of operating conditions considered. The use of the estimated wind speed produces thrust values closer to the reference thrust than if \bar{u}_{ref} is used. In line with the discussions of Sect. 3.1, this supports the fact that the estimated wind speed provides an effective velocity that is consistent with the instantaneous state of the rotor but different from the rotor-averaged wind speed. However, it is also possible that compensating errors are at play, or that the thrust is less sensitive to changes in wind speed or drivetrain dynamics than the torque. Despite these open questions, we continue by assuming that the method provides thrust estimates with sufficient accuracy.

3.3 Reduced model of the mechanical system

In this section, we compare the 2-DOF mechanical model presented in Sect. 2.1 to the advanced OpenFAST model consisting of 16 DOFs. As mentioned in Sect. 2.1, we first improve the generalized force formulation acting on q_t . We adopt the notations from Fig. 2. The resulting force and moment at the tower top are written as \mathcal{F}_N and \mathcal{M}_N . The contribution of this load to the generalized force is $\mathbf{f}_N = \mathbf{B}_N \cdot [\mathbf{F}_N; \mathcal{M}_N]$, where, according to the virtual work principle, \mathbf{B}_N is the velocity transformation matrix that provides the velocity of point N as a function of other DOFs. Further details on this formalism are provided in Branlard (2019a). For the

single-tower DOFs considered, the \mathbf{B} matrix consists of the end values of the shape function deflection and slope (i.e., $\mathbf{B}_N = [\Phi_{t,1}(L_t), 0, 0, 0, v_1, 0]$, where L_t is the length of the tower, and v_1 is $\frac{d\Phi_{t,1}}{dz}(L_t)$). The shape functions are normalized at their extremity (i.e., $\Phi_{t,i}(L_t) = 1$) so that the generalized force is

$$\mathbf{f}_N = \mathcal{F}_{x,N} + v_1 \mathcal{M}_{y,N}. \quad (21)$$

We assumed that the main forces acting at the tower top are the aerodynamic thrust and the gravitational force from the rotor nacelle assembly (RNA) mass, M_{RNA} . We then obtain the loads as

$$\mathcal{F}_{x,N} = T_a \cos(\alpha_y + \theta_{\text{tilt}}), \quad \mathcal{M}_{y,\underline{E}} = T_a [x_{NR} \sin \theta_{\text{tilt}} + z_{NR} \cos \theta_{\text{tilt}}] + g M_{RNA} [x_{NG} \cos \alpha_y + z_{NG} \sin \alpha_y], \quad (22)$$

where, using Fig. 2, θ_{tilt} is the tilt angle of the nacelle; NR is the vector from the tower top to the rotor center, where the thrust is assumed to act; NG is the vector from the tower top to the RNA center of mass; g is the acceleration of gravity; and α_y is the y rotation of the tower top induced by the tower bending. For a single-tower mode, $\alpha_y(t)$ equals $q_t(t)v_1$. The linearization of Eqs. (21) and (22) for small values of q_t leads to

$$\mathbf{f}_N = q_t \left\{ -T_a v_1 \sin \theta_{\text{tilt}} + v_1^2 g M_{RNA} z_{NG} \right\} + (T_a \cos \theta_{\text{tilt}} + T_a v_1 [x_{NR} \sin \theta_{\text{tilt}} + z_{NR} \cos \theta_{\text{tilt}}] + v_1 g M_{RNA} z_{NG}), \quad (23)$$

where the term in parentheses is the main contribution, which justifies the use of T_a in Eq. (1); the term in curly brackets acts as a stiffness term. The presence of T_a in this term introduces an undesired coupling, and this term is kept on the right-hand side of Eq. (1). It is noted that the vertical force, $\mathcal{F}_{z,N}$, contributes to the softening of the tower. The main softening effect attributed to the RNA mass is included in the stiffness matrix, as described in Branlard (2019a). The contribution of the thrust to the softening, as well as an additional contribution of quadratic velocity forces to the generalized force, is neglected.

Please note the remarks at the end of the manuscript.