Response to comments of Dries Allaerts

- This paper aims to identify the optimal distribution of wind turbine set-points to mitigate flow blockage induced by atmospheric gravity waves and hence maximize wind-farm energy extraction. The authors simulate the response of the atmospheric flow to the wind-farm drag using a recently developed mid-fidelity model, and they introduce a corresponding optimization framework based on the continuous adjoint method. The results are promising and show that a non-uniform spatial distribution of wind turbine set points can increase the energy extraction of the farm by reducing the excitation of atmospheric gravity waves. I believe this paper is of interest to the wind energy community as it demonstrates the use of set-point optimization for wind farms and highlights the potential for new optimization and control strategies to cope with windfarm scale blockage.
  We would like to thank the referee for the kind words and the very constructive feedback in improving the quality of the paper.

- 1. Line 91: In the derivation of the three-layer model, no assumptions need to be made about the vertical component of the velocity. Rather, by averaging over the height of the respective layers, the horizontal momentum equations become independent of the vertical velocity.
  We have corrected our statement with the following sentences at P3-L90:

  "*The model equations are derived starting from the incompressible three-dimensional Reynolds-Averaged Navier-Stokes (RANS) equations for the ABL (Stull 1988). A depth-integration over the wind-farm and upper layer height is further computed, which removes the vertical velocity from the equations. Hence, the basic equation system is reduced to a set of only three equations: the continuity equation and the momentum equations in horizontal directions. Subsequently, the governing equations are linearized with respect to the background state variables, using some additional modelling assumptions for the turbulent stresses (see Allaerts and Meyers (2019) for more details).*"

- 2. Line 130: The relation between pressure and inversion layer displacement based on the complex stratification coefficient is not due to Gill 1982 (at least not the part concerning the atmospheric gravity waves). I think it is more appropriate to cite Smith 2010 instead.
  We agree on this. Hence, we have cited Smith 2010 instead of Gill 1982.

- 3. Eq. 17: Can you elaborate on the function of the complex stratification coefficient in the adjoint equations? That is, what do you mean with the negation of the arguments x and t. I assume this arrives from the partial integration and is similar to the sign reversal of the convective term, but it is not clear to me how I should interpret the current notation.
  The negation of the arguments x and t does not follow from partial integration. Rather, it follows from the following property. Given three functions $f, g, h \in L^1(\Omega)$, it can be shown

that

$$\int_\Omega \big[f(x) * g(x)\big]h(x)dx = \int_\Omega \int_{\Omega'} \big[f(x-x')g(x')dx'\big]h(x)dx$$
$$= \int_\Omega \int_{\Omega'} f(x-x')g(x')h(x)dx'dx$$
$$= \int_{\Omega'} \int_\Omega f(x-x')h(x)dx g(x')dx'$$
$$= \int_{\Omega'} \int_\Omega f(-(x'-x))h(x)dx g(x')dx'$$
$$= \int_\Omega \big[f(-x) * h(x)\big]g(x)dx.$$

Note that in the second passage we have changed the order of integration (Fubini's theorem). In the current application, this property allows us to write

$$-H_1 \int_0^T \iint_\Omega \Big[\mathcal{F}^{-1}(\hat{\Phi}) * \delta \boldsymbol{u}_1\Big] \cdot \nabla \Pi_1 d\boldsymbol{x}dt = -H_1 \int_0^T \iint_\Omega \Big[\mathcal{F}^{-1}(\hat{\Phi})(-\boldsymbol{x}, -t) * \nabla \Pi_1\Big] \cdot \delta \boldsymbol{u}_1 d\boldsymbol{x}dt.$$

To include this in the text, we have added the following sentence at P8-L213:

"*Note that the minus sign in the argument of* $\mathcal{F}^{-1}(\hat{\Phi})(-\boldsymbol{x}, -t)$ *is not a result from classical integration by parts, but arrives from applying Fubini's theorem to the convolution term in Eq. 12 and Eq. 13 (see Appendix A3 for details).*"

Moreover, we have also added the following lines at P24-L581:

"*Note that the minus sign in the argument of* $\mathcal{F}^{-1}(\hat{\Phi})(-\boldsymbol{x}, -t)$ *does not come from classical integration by parts. In fact, given three functions* $f, g, h \in L^1(\Omega)$, *it can be shown that*

$$\int_\Omega \big[f(x) * g(x)\big]h(x)dx = \int_\Omega \int_{\Omega'} \big[f(x-x')g(x')dx'\big]h(x)dx$$
$$= \int_{\Omega'} \int_\Omega f(-(x'-x))h(x)dx g(x')dx'$$
$$= \int_\Omega \big[f(-x) * h(x)\big]g(x)dx.$$

where in the second passage we have changed the order of integration (Fubini's theorem). This property allows us to write

$$-H_1 \int_0^T \iint_\Omega \Big[\mathcal{F}^{-1}(\hat{\Phi}) * \delta \boldsymbol{u}_1\Big] \cdot \nabla \Pi_1 \, d\boldsymbol{x}dt = -H_1 \int_0^T \iint_\Omega \Big[\mathcal{F}^{-1}(\hat{\Phi})(-\boldsymbol{x}, -t) * \nabla \Pi_1\Big] \cdot \delta \boldsymbol{u}_1 \, d\boldsymbol{x}dt.$$

"

- 4. Line 235: Can you comment on the numerical resources (time and number of processors if parallelized) it takes to compute an optimal set-point distribution? I am asking because a possible application could be using weather forecasts to update the set-point distribution when gravity waves are to be expected (e.g., forecast predicts shallow boundary layers in the next few hours).
  To include these information in the text, we have added the following paragraph at the end of section 3.1 (P10-L257):

  "*The solver (which is not parallelized) takes a couple of hours to solve the equations for* *a grid with resolution of 250 m (6.4×10^6 DOF per layer). Since convergence is reached after approximately 20 function evaluations (which means that we solve state and adjoint equations 20 times), the optimizer takes a couple of days to compute an optimal thrust set-point distribution. However, after this work was performed, we have upgraded the forward solver which is now 700 times faster than our previous version. Optimization of the backward solver is planned for the future, and we expect that this will lead to an optimization algorithm that will only take several minutes for the same case.*"

- 5. Line 241: How did you select the relative turbine spacing?
  In the study of Allaerts and Meyers (2019), the authors considered a rectangular shaped farm with length $L_x = 20$ km and width $L_y = 30$ km containing $N_t = 486$ turbines. These numbers where chosen to represent roughly the number of turbines and the area covered by the Belgian-Dutch wind-farm cluster. In our study, we select a wind-farm with the same dimensions. Moreover, to have a similar density of turbines in the farm, we fix the dimensionless turbine spacings to $s_x = s_y = 5.61$ so that $L_x L_y / s_x s_y D^2 \simeq 486$. To include this information, we have added the following sentence at P10-L267:

  "*The wind turbine relative spacings along the x- and y-direction are $s_x = s_y = 5.61$ (both non-dimensionalized with respect to the turbine rotor diameter D), so that the density of turbines in the farm is similar to the one of Allaerts and Meyers (2019) (i.e. leading to $\beta = 0.01$ in Eq. 8, setting both the wake efficiency η_w and γ to 0.9 as in Allaerts and Meyers (2018)). Note that we do not define a specific layout or a number of turbines but we only fix the density of turbines in the farm.*"

- 6. Line 301: Favourable pressure gradients are also present in the bulk of the wind farm, whereas the velocity deficits continue to increase throughout the farm and only recover behind the farm. I believe the favourable pressure gradients do not necessarily accelerate the flow, but are instead balanced by a higher thrust force. Physically, this would correspond to the favourable pressure gradient re-energizing the wake flows and thereby reducing the turbine losses in the bulk of the farm. Can you comment on this?
  We agree with the referee and we confirm that the sentence "*In both sub- and supercritical case, favourable pressure gradients develop within the wind-farm area which tend to accelerate the flow in the wind-farm exit region*" is misleading. In fact, favourable pressure gradients re-energize the waked flow reducing the velocity deficits in the bulk of the farm. This also partially explains the higher velocity deficits observed for the supercritical case over the wind-farm area if compared with the ones obtained in the subcritical case (the favourable pressure gradient is stronger in subcritical conditions than supercritical ones). Therefore, we have modified the sentence mentioned above to (P14-L341):

  "*In both sub- and supercritical case, favourable pressure gradients reduce the velocity deficits in the bulk of the farm.*"

- 7. Section 4.1: Did you consider optimizing for a uniform set-point distribution? What are the maximum gains to be expected there, and hence how much more is there to be gained by using a non-uniform set-point distribution? How would that uniform value compare to the average of the non-uniform distribution, and would the uniform value depend on the atmospheric condition as well?
  We thank the referee for the very insightful questions. To answer these questions, we have added the following section in the appendix (P28-L652):

  "*In the current section, we use the optimization framework derived in Section 2.2 to find an*

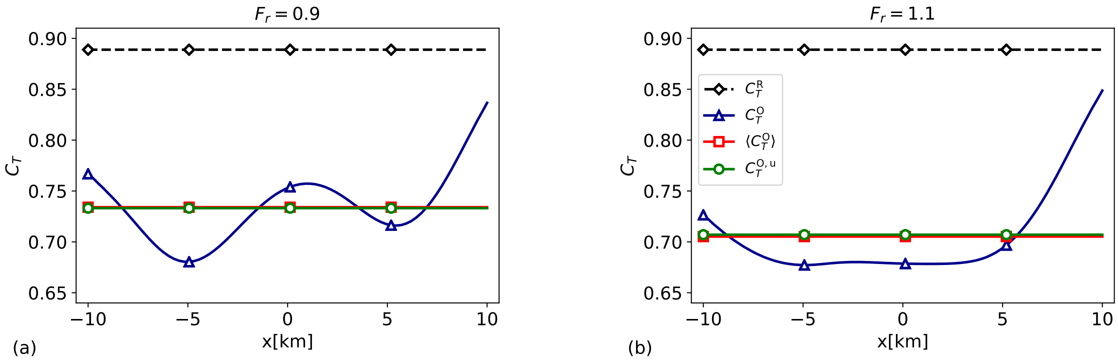

Figure 1: Reference thrust set-point (C_T^{R}), optimal non-uniform thrust set-point (C_T^{O}) and its averaged value over the wind-farm area $(\langle C_T^{\mathrm{O}} \rangle)$ and optimal uniform thrust coefficient distribution $(C_T^{\mathrm{O,u}})$ in (a) subcritical and (b) supercritical flow conditions. The C_T^{O} profiles are taken through the center of the farm $(y = 0)$.

optimal uniform and steady thrust-coefficient distribution that minimizes the gravity-wave induced blockage effects. To avoid confusion, we will denote with C_T^{O} and $C_T^{\mathrm{O,u}}$ the optimal non-uniform and uniform distribution, respectively. The wind-farm layout and the atmospheric state are the ones detailed in Section 3.
Figure 1(a,b) displays the optimal spatially invariant $C_T^{\mathrm{O,u}}$ together with the streamwise profile of C_T^{O} through the center of the farm, and its averaged value over the wind-farm area $\langle C_T^{\mathrm{O}} \rangle$ for the sub- and supercritical case, respectively. Moreover, C_T^{R} denotes the thrust distribution used in the reference model. Interestingly, $C_T^{\mathrm{O,u}}$ corresponds to the average of the non-uniform distribution in both cases. Since C_T^{O} is sensitive to the atmospheric conditions, we expect $C_T^{\mathrm{O,u}}$ to depend as well on the atmospheric state (in fact, we observe a different value of $C_T^{\mathrm{O,u}}$ in sub- and supercritical conditions).
In the current example, the energy gain $\mathcal{G}$ (see Eq. 21 on article) over the reference model configuration obtained with the non-uniform distributions C_T^{O} are 5.3% and 7% for the sub- and supercritical case, respectively. For the optimal uniform distributions, we obtain an energy gain of 5% and 6.6%."

- 8. Section 4.1: How does the power performance of the optimal set-point distribution compare to the idealized power output when all turbines would be operating in isolated conditions? I.e. how much of the power loss due to flow blockage is irreversible?
  The usage of a box-function model makes it difficult to answer this question. In fact, this model uniformly spreads the force over the simulation cells in the wind-farm area and does not represent the disturbances caused by each turbine in detail. Therefore, the concept of turbines operating in isolated conditions is not reproducible. What we can do is to consider "turbines" which operate in idealized conditions, by using a uniform thrust set-point distribution with $C_T = 0.88$. This is what we have defined as reference case and the energy gains are referred to this state. The comparison suggested by the referee would be realizable if analytical wake models would be used (such as, the Gaussian wake model proposed by Niayifar and Porté-Agel (2014)). In order to not overload the discussion in the manuscript, we decided to not further explicitly comment on this. However, the wind-farm force model used in the optimization solver will be improved in the future, as suggested in the conclusions, and this will allow us to answer this question in more detail.

- * Line 389: Typo in “dispersive”
  * Label A12 and A13 reference the same equation split over two lines.
  * Line 573: Typo in “through”

Thank you, we have corrected these errata.