

1 Response to comments of Referee #1

- The article is very well written and discusses a problem with a well-established methodology by subdividing the perturbation equations into several layers where the analytical solution is found, reducing the differential problem to an algebraic linear system.

We would like to thank the referee for the kind words, and for the helpful feedback.

- 1. Section 2.1 and 2.2 do not have any graphical schematic to help the reader to understand the layers subdivision. I think that adding those (at least for one section) will facilitate the understanding

We have added a figure from Allaerts and Meyers (2019) to clarify the structure of the TLM, and referenced it at line 101:

“... For this reason, the lower layer is also called the wind-farm layer. The resulting approximation of the ABL is visualized in figure 1. The flow in the two layers is governed by ...”

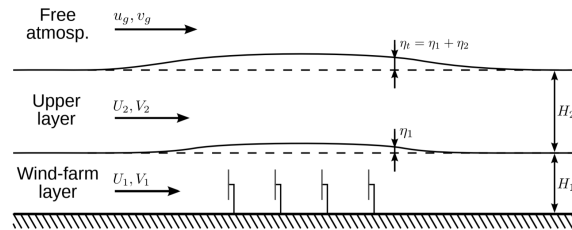


Figure 1: Schematic representation of the three-layer model. Figure from D. Allaerts and J. Meyers, Journal of Fluid Mechanics, 862, 990-1028 (2019).

- 2. Equation 14. The total derivative operator is undefined. The authors are also focusing on stationary waves, right?

We indeed only analyse steady-state flows. To clarify this, we have added the definition of the total derivative where it is first used, at line 152:

“where $\frac{D}{Dt}$ is the inertial derivative, which for steady systems simplifies to $\frac{D}{Dt} = \bar{u}_i \frac{\partial}{\partial x_i}$, and where w' is the vertical velocity perturbation of the wave.”

- 3. Equation 20. Are the derivatives evaluated at $Z=H$?

Since Φ describes the relation between pressure and displacement at the top of the ABL, they are indeed defined there. To make this clear, we have changed equation 20 to:

$$\hat{\Phi} = \left[\frac{\Omega}{k^2 + l^2} \left(\Omega \frac{dW}{dz} - \frac{d\Omega}{dz} W \right) \right] \Big|_{z=H}$$

- 4. The method described by equation 20 is very common in acoustics (see the book of Salomons, Computational Atmospheric Acoustics about the FFP method)

We thank the referee for pointing this out to us. We added the following sentence to the section where the piecewise method is introduced, on line 175:

“This approach is also commonly used in acoustics, where it is called the FFP method (Salomons, 2001).”

- 5. around line 245: since the authors give importance to the computational time, it is worth to state what solver was used to solve the banded matrix? Was that the native numpy routines or did they use a home-made algorithm?

We agree with the referee that this is important to add. We changed line 243 to:

“The total computation for a single profile of $m^2(z)$ with $n = 100$, including the building of the matrix, takes roughly 0.4s on a personal laptop with 16GB of RAM and an Intel core i7 2.60GHz, using the standard numpy solve routine sped-up with the numba package (Lam et al., 2015; Virtanen et al., 2020).”

We also changed the number of sublayers used in this example calculation to be more representative of the cases analysed in section 3.3 and 4.

- 6. Line 254. I would replace frequency with wavenumber since frequency is more related to temporal variations, while your method is for stationary waves. This applies to the entire manuscript

We agree that our phrasing is unclear, but disagree with the referee’s proposed change. Since Ω is the convective derivative operator in Fourier space, it denotes the temporal frequency the flow experiences for a given wavenumber. Therefore, we changed the wording throughout the paper to describe Ω as the *intrinsic* frequency.

- 7. Line 289. The agreement is qualitatively well but not perfect. How can one improve the agreement? By adding more layers? or there is a limitation in the original data from Wells and Vosper?

We find that using a vertical spacing of 100m slightly improved results, as the altitude of the change in Brunt-Vaisala frequency then exactly corresponds to a sublayer interface. However, aside from this, using up to ten times finer grids did not change anything. Furthermore, earlier work by Gill (1982) and Leutbecher (2001) for the case of constant wind velocity agrees with our results.

To address this in the paper, we have updated figure 2 to now include the results of Leutbecher (2001) for the case with uniform velocity (left panel). This is referenced in the text on line 286 as follows:

“Our results, obtained on the same grid as above with our method adapted to the hydrostatic regime, and those obtained by Wells and Vosper (2010), and Leutbecher (2001) for the constant wind case, are shown in figure 2.”

- There are some typos here and there. I have found two at page 5 at rows 2 and 6 where coefficients and inversions should be singular.

We thank the referee for finding these. The typo *inversions* has been corrected to *inversion*. The stratification coefficients should be plural however, as there are different coefficients for all the wavenumbers.

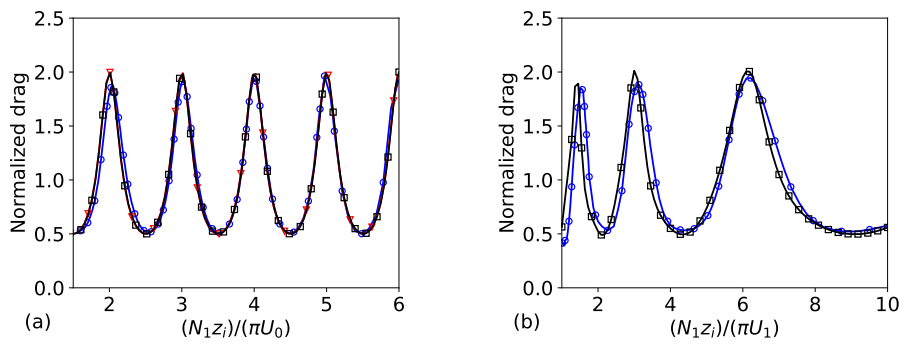


Figure 2: Mountain wave drag on a small ridge with a two-layer Brunt-Väisälä frequency profile and constant background wind (a) and vertical wind shear (b), normalized by the drag for constant background profiles. The black lines with squares show our results, in which the wave drag is normalized with the drag for constant background wind u_0 and stratification strength N_1 . The blue lines with circles show the results of Wells and Vosper (2010), and the red line with triangles in the left figure shows the results of Leutbecher (2001).