Supplementary comments

There is a lot of food for thought in;

https://www.researchgate.net/publication/344364853_Rethinking_Ducted_Turbines_The_Fundamentals_of_Aerodynamic_Performance_and_Theory

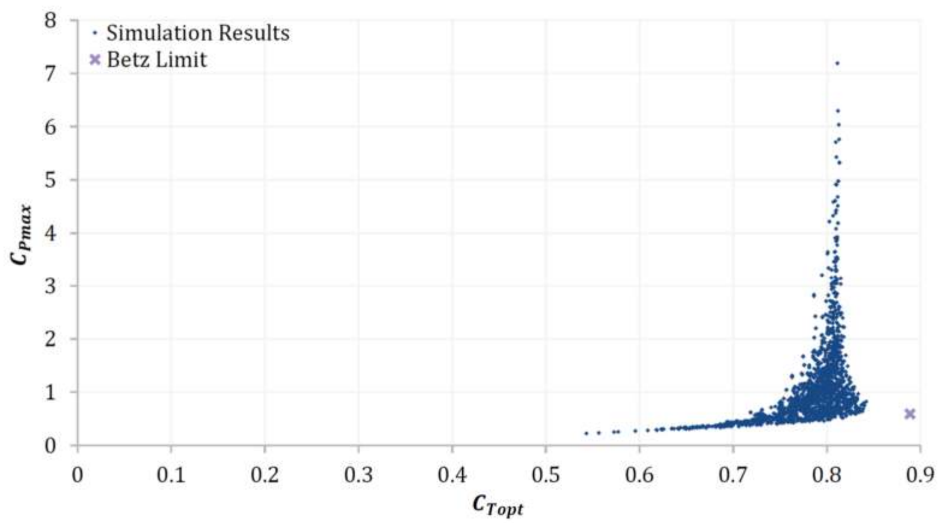

Figure 7.4: Optimum rotor loading for 1776 aerofoil and diffuser parameter duct shapes.

The value of 8/9 is only approached as the area ratio tends to 1 and length to zero equivalent to having no duct – open flow - and ducts that may be considered useful did not maximise Cp at Ct above ~ 0.84.

For a while I thought that a fully optimised finite duct in inviscid flow would achieve Cp max at Ct = 8/9. I was able to construct an analytic potential flow model of an ideal planar duct of infinite length which always maximised Cp at Ct =8/9 regardless of area ratio. However I now suspect, as suggested by the figure above, that no finite duct, in inviscid flow, will maximise rotor Cp with a loading as great as Ct=8/9.

What is very clear and has been for a long time is that the optimal loading of a rotor in a duct is not far different from in open flow. I believe that experimental results (Foreman etc) in support of 8/9 and experimental values above 8/9 around unity (Phillips) arise from viscous flow effects that energy in some of the flow that does not pass through the duct can assist flow through the duct. I refer here to "simple" ducts without slotted aerofoils of boundary layer injection which obviously exploits external flows.

The most efficient ducts, as measured by rotor Ct which is identically equal to the fraction of source kinetic power extractable at the rotor plane, are aerofoil ducts but, in inviscid flow, ducts for a given rotor size and specific area ratio that maximise rotor Cp are right angle ducts. Ct at Cp max may be below 0.8 with these ducts, implying that they extract source energy less efficiently, but they maximise Cp overall by inducing flow from a relatively larger source area than the more efficient aerofoil ducts. There is some experimental evidence of this from data on the WindLens duct (Jamieson, "Innovation of Wind Turbine Design" 2nd edition 2018) and results from this lead me to think that the most effective real ducts will combine some aerofoil shape expansion with a more abrupt exit flange.