

The authors present expression of the "general" momentum theory of an actuator disk (including the pressure terms acting on the control volume surfaces, and without assuming that the pressure is recovered in the far wake). In their analyses, they relate the contribution of the pressure term to an integral over the axial and radial momentum in the radial direction. The formulae are presented in integral and differential form. The authors then proceed to studying a finitely bladed rotor with expanding wake but constant pitch.

The work is highly relevant and thorough. I have several general comments that I hope can improve the paper:

- I would advise to split this paper into two. The actuator disc and finite number of blades part are somewhat related, but each part could very well be put into separate, shorter papers. Mixing disc and finitely-bladed rotors adds in complexity and can confuse the reader.

We have clarified the split between infinite and finite number of blades in response to the first reviewer. We appreciate that the paper is long but hope it is sufficiently unified to justify keeping together. There is a practical issue in that the grant that the remaining funds for this work are insufficient to pay for a second paper.

- The paper contains a lot of maths and is not easy to follow without a definite engagement from the reader. I would recommend to guide the reader more between equations: stating what equations are used, adding intermediate steps and definitions, translating definitions into maths, etc. I think it would help the reader, if going from one equation to the next is straightforward. I've added specific comments in the pdf for equations that in my opinion need more guidance.

I enclose some specific comments in the pdf attached to this review. I'd like to congratulate the authors for their interesting work. I'll be looking forward to review a revised version of this paper.

(combined answer to last two queries) We have looked at all the annotations on the pdf in making our revisions and list our detailed responses below.

Page 2: Is there an assumption related to the pressure on the control volume boundary? Could you mention why no pressure term is present in this equation? (I apologize, I haven't reviewed your previous work yet).

Could you briefly mention in the text what was used to derive this formula? (conservation of axial momentum and some angular momentum considerations?)

(combined answer to last two queries) As noted in the text the impulse equations for force have had the pressure removed in their basic formulation. In the paragraph preceding equation (1), we have added a sentence and a reference to Noca's thesis (1997) to highlight that the pressure removal is done by substitution of various identities into a standard momentum CV analysis. For more details, see LW or Noca (1997). In response to the following comments about pressure, we likewise point the reviewer to LW. LW

mention that the impulse formulation appears to give no benefit in analyzing angular momentum but this equation is not used in the current manuscript.

Could you describe "circumferential" using a coordinate system to avoid any confusion on what is meant here?

We have added text to clarify the definition of the circumferential co-ordinate. We use  $x$  instead of  $r$  for consistency with LW.

I would suggest using  $r$  instead of  $x$ .

We have kept the notation used in LW to make it easier for readers to switch from that paper to this one.

Page 3: Are you using an actuator disk assumption? You mention blades here, are they lifting lines? It seems like assumptions on the rotor loading need to be stated.

We have clarified the assumptions relating to the actuator disc. Equations (1) – (3) are applicable to an actuator disc.

the choice of  $u$  and  $v$  does not really match the convention where " $u$ " is along " $x$ " (since you chose  $x$  along  $r$ ).

As explained above, we have kept the notation used in LW to make it easier for readers to switch from that paper to this one.

Could you justify assumption 8?

LW show that this assumption is required to recover the Kutta-Joukowski expression for local thrust that is conventionally employed in blade-element momentum analyses. A note to this effect has been added in the paragraph after the list of assumptions. To remove the assumption would require a model for the vorticity crossing the other faces, which we do not have.

Is there an assumption relating the pressure in the far wake (recovered and equal to free stream)? Coming back up, it seems you don't, since you keep  $P_{infty}$ .

We do not assume equality of free-stream and far-wake pressure – they differ because of the new Equation (19). The situation has been clarified by adding that we treat all pressures as gauge pressures relative to the free-stream.

It seems you use non dimensionalized velocities, I don't think it was mentioned above (I might have missed it).

Thanks for pointing this out. The original text moved from dimensional to non-dimensional quantities without comment. The text has been revised.

Page 5: It seems to me that an assumption on the pressure on the side of the control volume (at radius=infinity and infinity upstream) needs to be mentioned here.

Please see the comments above about pressure.

Also,  $P_D$  seem to be the pressure "minus" the infinity upstream pressure  $P_0$ . This could be precised in the text for clarity.

You are correct and the text has been modified to indicate that all pressures are gauge pressures.

It might be worth stating in the text what this means (there is no pressure jump outside of the actuator disk)

A sentence to that effect has been added immediately below Equation (16).

I believe BS ends at  $z=0+$ , could you mention this in the text?

No, the BS extends to the far-wake but its involvement with the thrust is determined by consideration of the CVs in Figure 1, which end in the immediate vicinity of the rotor.

Could you detail how the normal to the streamtube projected on the axial direction is obtained here?

This has been done.

Page 6: Can you justify why this generalization is possible. I'm guessing it comes from the fact that all the equations used are valid upstream (with  $T=0$ ). But this doesn't appear obvious straightaway to the reader since you made use of  $S_D$  multiple times in the developments above.

You are correct and a phrase has been added about  $T = 0$ .

Page 7: Can you introduce a couple of temporary steps here, showing first an equation similar to (12), and then showing how the terms are expressed? It is not straightforward to me to see how the term  $\int_{-\infty}^{\infty} P dx/dz x dz$  term got manipulated here.

The first part of the new Equation (22) is just a standard application of the axial momentum balance to a CV enclosing the far-wake. The second part uses (19) to remove the pressure. We do not think that additional justification is needed for these well-known results.

It this derived from equation (22), can you make the assumptions more explicit so that the reader can easily go from 22 to 23?

The new (24) comes from the full form of the impulse version of the T equation. The appropriate reference to the equation in LW has been added.

Intermediate steps/ helps would be at here again to guide the reader and see how  $u$  and  $\Delta P$  where introduced based on the variables in the bracket.

The description of the steps in obtaining (27) from (26) have been expanded.

This statement holds out outside and inside the wake? or only outside of the wake region?  
Could you please precise?

As noted above, all the discussion that uses the term “redistribution” has been revised.

Page 18: I'm guessing you are doing the theta integration here to find the average, is this correct? can you precise in the text?

We hold that the description of the process is sufficient. Immediately below the new (41) we explain that the sin terms vanish on circumferential integration and we have carefully noted that the arguments for  $i_a$  etc show whether they are for velocities at a point or are circumferential averages. Further, section 4.1 starts by stating the goal of obtaining the circumferential averages.