Response to Referee 1

We greatly appreciate the time taken by the referee to read our manuscript. We have taken into consideration and addressed all comments, questions, and suggestions from the reviewer, and we feel that the revised manuscript is now substantially stronger as a result. Changes made to the text at the request of the reviewer have been highlighted in blue in the revised manuscript. In the following, reviewer comments are repeated in italics and our responses are provided in the bulleted sections of text.

Major Comments

The article describes using Stochastic gradient descent (SGD) (it is misspelled as "decent" in line 27) to optimize the layout of a wind farm with the objective of maximizing annual energy production (AEP).

• We corrected the typo to read "descent" instead of "decant" and ensured this mistake did not occur elsewhere.

SGD is a pretty old algorithm; however, the authors used a modified SGC called Adam SGD, developed in 2014.

- We added to the introduction on P2L43-47 and Methods on P4L96-106 to emphasize that the Adam SGD algorithm is primarily used in the context of unconstrained optimization.
- We are not aware of a similar implementation of the Adam algorithm that allows for deterministic constraints. Therefore, although we feel the primary novelty of this paper is applying SGD to wind farm optimization, we believe that the paper is also novel in that the particular implementation of the Adam algorithm is unique with respect to how the deterministic constraints are considered, which we have now noted on P245-L47 and P4L103-108.

My primary concern with this article is the novelty. This paper is a single-objective layout optimization that compares the AEP production and computing time for the said algorithm and a second one serving as the baseline.

- We added to the introduction to highlight that the novelty of the manuscript is the application of SGD to wind farm optimization (P2L43-47).
- We added an introduction to the presented SGD algorithm in the Application on P4L103-108 to explain that our algorithm is an extension of the classic adam algorithm.

The second algorithm that the article uses as its baseline is Scipy Sequential Least Squares Programming. The two algorithms lead to layouts that generate almost equal AEP (Adam SGD produces 0.2% more, which is not significant). The computational time for the Adam SGD algorithm is nearly 0.5 hours. In comparison, Scipy Sequential Least Squares Programming takes almost 10 hours to identify an optimal layout. This data is for the 100-turbine case, which yields the maximum difference among all presented instances.

• Since submitting the original manuscript, we have implemented the SGD approach in TOP-FARM. We have updated the results to show the latest results of the SGD/SLSQP analysis, fully implemented within TOPFARM.

- The previous analysis included an SLSQP implementation that computed constraint gradients using finite differences. This amounted to long deterministic optimization times.
- TOPFARM computes the constraint gradients analytically (as is now noted on P9L217-218), significantly reducing the deterministic computational cost, and slightly increasing the SGD computational cost.

As the authors also acknowledge, the Scipy Sequential Least Squares Programming is a 30-year-old Fortran code, which one might be able to modify to make it more efficient. Also, other modern algorithms can generate equally efficient layouts as quickly as Adam SGD. Hence, showing that Adam SGD finds layouts that are approximately as efficient as those found by another specific algorithm but does so more quickly does not offer enough novelty, in my opinion.

- Since writing the article, we have incorporated SGD into the TOPFARM framework, allowing for the examination of larger wind farms. Our results indicate that SGD consistently produces about 0.3-0.5% more AEP than the deterministic counterpart. While we believe this is a useful data point, we mainly wish to highlight the difference in computational cost between the two approaches
- We agree that the comparison to SLSQP may have seemed arbitrary in the original text. We modified the introduction (P2L48-57) and Methodology (P7L171-174) to explain that SGD was developed for the TOPFARM framework, and that TOPFARM and other engineering design codes are often run using SLSQP.
- We include references to some studies that have used TOPFARM with SLSQP in the Introduction on P2L48-54 to help give context for this work.

In addition, the industry and the public, including investors, are not concerned with spending several more hours or days finding more efficient algorithms since they invest hundreds of millions of dollars in a wind farm. Speeding up the optimizations is crucial for real- time control of wind farms, not layout optimization, since that is only a one-time job.

- Based on our conversations with companies that design wind farms, computational time is a factor when determining the appropriate optimization approach. While developers may not have significant time constraints in developing a layout, turbine manufacturers are often asked to provide suggested layouts, which can amount to a large number of wind farms to optimize.
- In fact, this work was partially funded by a wind energy company interested in speeding up their optimization capabilities. At the extreme, it is obviously impossible to work with an optimization that takes years to complete.
- We note on P6L134-136 and P18L337-338 that our SGD implementation allows for a computational budget to be decided upon before starting the optimization, which can allow for greater organization when planning project design timelines.

With that said, again, I must mention that this article does not demonstrate that Adam SGD is the fastest optimizer; it shows that Adam SGD is faster than one other algorithm, leaving the reader to believe there are other algorithms that are faster than Adam SGD.

- We added text to P2L45-46 to attempt to clarify that this manuscript primarily seeks to demonstrate that the SGD algorithm seems particularly well-suited for wind farm optimization.
- We elaborate on this point on P2L43-47, explaining that SGD is distinct in that it naturally allows the problem to be extended to allow for more uncertain variables.

Minor Comments

1) My other concern is about the spacing constraint: "We require each turbine to be spaced at minimum two rotor diameters apart (ND = 2)" [Lines 163-164] But then, the article reads: "All power plants considered in this study have square boundaries so that, in a grid, the turbines would be spaced five rotor diameters apart ($\Delta = 5$)." These statements confuse me. Is the spacing 2 or 5 diameters? This matters since ND = 2 is exceptionally tight, and I am not aware of any real wind farm with turbines that are so close. Lillgrund, of which the authors used its wind data for this work, is considered a packed wind farm, in which I believe the two closest turbines in one specific direction are yet apart by more pg. 2 than 4D. Also, what do authors mean by "in a grid?" Did they define a background grid to discretize the search area or the search area was continuous?

- We added text in the application P8L193 and P8L98 to better clarify that the minimum rotor diameter (N_D) is a constraint imposed on the optimization problem and that the averaging spacing (Δ) was used to initialize the turbine layout and define the boundary locations.
- Thanks to the reviewer's comment, we realized that the N_t was prematurely assigned as 2 in Equation 19 in the original manuscript. We have corrected this in the manuscript, which hope will also enhance clarity regarding the difference between Δ and N_D .

2) Line 20: "It can be costly to compute the power associated with each speed and direction combination, as the number of these combinations can be large." - I don't believe these computations are costly, and a modern processor can quickly handle such calculations. Also, the wind speed and direction are measured using sensors with limited resolution and other uncertainties, and that's why they are reported in a discretized way using bins

- We agree that it is somewhat subjective what is "costly" and what is not. To avoid this ambiguity, we modified the text (P1L20-23) to instead state the following.
- The cost of wind farm optimization scales with the number of speeds and directions.
- We have encountered memory limitations when computing gradients in large wind farms.
- As some wind farm models (e.g., RANS) can be time-consuming to run, it can be useful to investigate approaches designed to minimize the number of required flow cases to consider when computing AEP, which examined by the cited studies [1, 2, 3].

3) Lines 23: "to avoid discretizing the input distributions into evenly spaced intervals. In this study, we present an approach for wind farm optimization that estimates the gradient of the AEP using Monte Carlo simulation. This does not require that the input be discretized at all, and allows for the consideration of an arbitrarily large number of atmospheric conditions". Input data are measured in a discretized way. The guide to meteorological instruments and methods of observation recommends measuring wind direction with an accuracy of 5 degrees. Hence, why would feeding the data that is measured discretized in the first place in a "continuous" format into a model help the accuracy? [4]

- We agree that this is a potentially confusing point. To add clarity, we stress that it is important to discretize the atmospheric conditions when measuring the frequency of occurrence, similar to the steps taken to construct a power curve, in the Methodology on P9L206-210.
- We further clarified on P9L204-216 and in the figure caption of Figure 1 that, after the measurements are taken, TOPFARM has the functionality to interpolate these probability

densities, so that the range of potential atmospheric conditions can be fully represented, and that we use a linear interpolation scheme with 1 m/s and 1° bins.

4) Figure 1: I do not think this figure reflects a realistic representation of Lillgrund's wind conditions. I know that the westerly and southwesterly winds are stronger at this site; however, the data presented in this figure is too extreme. I think this is (at least partly) caused by what I explained in the two previous points. This requires the authors to explain precisely how they converted the data measured discretized to a continuous format. It has to be done by some sort of interpolation. How did they conduct this interpolation? Does the data presented in this figure even sum up to 1 (100%), since the largest frequency shown is about 0.005?

- The revised version of Figure 1 has 360 direction bins and 5 wind speed bins, so we can expect the probability mass to have an average value of $\frac{1}{360\times 5} \approx 0.0005$. We affirm that our plotting routine shows a correct probability mass function.
- The revised figure caption explicitly states that this probability mass is obtained via linear interpolation of a probability mass developed using a coarser discretization.
- The original plot considered wind speeds strictly greater than 5 m/s. We revised the plot to include low wind speeds, which we believe now better resembles other published works (e.g., Figure 2(A) in [5]).
- We added a note about the context of this data. We associate this data with Lilligrund because that is how the data is labeled in the PyWake software's selection of example data sets. The data is based on seven months of measurements from the Lilligrund site, as described in Section 2.1 of [6].

5) Line 40: "This term introduces sudden steep gradients in the optimization space, necessitating the use of smaller momentum parameters than are typically employed in the Adam SGD algorithm." The article presumes the reader knows the used algorithm; hence, it employs undefined terms. For instance, in the statement above, what are the momentum parameters, and why/how would smaller ones help? This is not defined by this line of the article.

- We agree that this was a potentially confusing point. To avoid this confusion, the sentence in question has been removed from the introduction of the manuscript.
- The concept of momentum is now first introduced in the Methodology in P4L97-98.

6) The introduction does not cover the literature on "wind farm layout optimization (hereafter WFLO)." I also find the introduction scattered and distracted with information that does not belong to the introduction, in particular, or this paper, in general. For example, I think the quick review given on lines 59-61 on applications of SGD in areas other than WFLO is distracting. In these applications, SGD tunes a neural network's hyperparameters. Instead, the introduction needs to remain focused on the literature of WFLO, identify gaps in WFLO, and clarify how this new optimization using SGD addresses the identified gaps.

• We added to the introduction to give a better overview of WFLO, including the following references: [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 15, 17, 18].

- we added material to the introduction on P2L43-47 to more clearly ground our problem and why it addresses an open question: The wind farm layout optimization problem presents a setting where the objective (annual energy production) can be formulated as being stochastic (e.g., the AEP is derived from a probability density function), while the constraints (e.g., boundaries and minimum turbine spacing) are firmly deterministic. This manuscript explores the potential benefits of formulating the wind farm layout optimization problem in this way.
- We believe that the revised introduction gives better context to the layout optimization problem, motivates the use of SGD, and makes clear the challenges of SGD with constraints.

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