Response to Referee 2

We greatly appreciate the time taken by the referee to read our manuscript. We have taken into consideration and addressed all comments, questions, and suggestions from the reviewer, and we feel that the revised manuscript is now substantially stronger as a result. Changes made to the text at the request of the reviewer have been highlighted in blue in the revised manuscript. In the following, reviewer comments are repeated in italics and our responses are provided in the bulleted sections of text.

Comments

1) *The paper is well written and explores an idea worth considering. My main concern is the omission of discussions surrounding accuracy.*

- In an effort to include greater discussions surrounding accuracy, we included material to assess the accuracy of the AEP approximation, as well as the accuracy of the final solution with respect to the imposed constraints. This material is described in the next two bullet points.
- We have included convergence analysis of the Monte Carlo and quadrature approximation in the Application in Figure 2 P10L221-230 in an effort to directly compare the accuracy of the deterministic and stochastic approaches in computing AEP.
- We also included analysis of how well the constraints are satisfied in the Results in Figures 11 and P17L323-P18L333. The left panel of Figure 11 shows the relationship between final scheduled learning rate and the constraint violations of the final solution. The right panel of Figure 11 shows the trade-offs between the AEP and constraint violation of the solutions associated with different initial and final learning rates.

2) - *operating on continuous PDFs isn't so much of an advantage as it is made out to be. And really pretty much any of the UQ methods could be thought of acting on a continuous input (the discretization can be considered part of the method, in the same way that random sampling is part of the method of MC).*

- We have removed references to the benefits of the continuous formulation in the abstract and future work section of the conclusions.
- We still believe that this method will scale favorably as more atmospheric conditions (shear, stability, etc.) are considered, as noted on in the conclusions on P19L330-341.

3) - *the literature review on SGD has good detail, but the lit review is light on other topics that this paper is based on. Namely, the proposed approach is a gradient-based one and there is no discussion of any prior work in this regard, other than the reference to the review paper by Herbert-Accero (which is a somewhat ironic choice as that review paper shows very few gradient-based approaches and is pretty dismissive of the approach in general).*

- We added additional review of work that uses gradients in the introduction on P2L48-P3L62 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]
- We added references to give a greater perspectives on this rich topic on P1L14 [22, 23].

4) - *Algoirthm 1 could use more introduction. Although I am already familiar with SGD, as noted by the authors it is not in regular use by the wind farm optimization community and so needs more explanation than just an algorithm dump.*

- We agree that this is a good idea. We added an introductory paragraph before Algorithm 1 in the Methodology on P4L97-107 to provide context to the algorithm and explain why our formulation is novel.

5) - *The SGD method notes a mix of AD and finite difference, but in the determinisitic approach it is not stated how you compute the derivatives and of course this will affect the time performance.*

- This is an important point to consider. Since submitting the manuscript, we have implemented the SGD algorithm in TOPFARM (which is now noted on P2L49), which also has an SLSQP implementation through the OpenMDAO package.
- We have updated the manuscript to reflect the latest benchmarking results using TOPFARM, where the gradients of the constraints are computed analytically (which is noted on P9L217-218).
- This is in contrast to the original manuscript, where constraints were computed via finite difference. The TOPFARM implementation substantially speeds up the SLSQP algorithm. However, as larger wind farms are examined, the same overall trend emerges – the computational cost of SGD scales more favorably than SLSQP.

6) - *Nowhere is the accuracy of the method shown. Undoubtedly MC can be much faster than a dense rectangule rule, but of course the real question is at what level of accuracy? One would like to see how many random samples are required to achieve the same accuracy.*

- We added analysis and discussion of the accuracy and expense of computing AEP and the associated gradients in the Application in Figure 2 and P10L221-230.

7) - *I realize this is somewhat difficult to directly compare as you are using a stochastic gradient and so allowing for inaccuracy is built into the method, but this is still a topic that one should be more transparent on. Especially since the main comparison is a very dense rectangle rule combined with SLSQP. I don't have a problem with that choice, but lots of things are faster than that combination. It shows promise, but must be careful on concluding anything more than that.*

- We agree that it is best to be as transparent as possible regarding the accuracy of these methods. We also agree that it is difficult to compare accuracy of the methods, as the SGD approach is designed to expect significant errors in the gradient evaluation, using a moving average to find the optimum.
- We included some discussion of the convergence of the AEP using quadrature and Monte Carlo approaches on P10L221-230/Figure 2. These additional data show that the quadrature rule is more computationally efficient than Monte Carlo simulation at accurately determining the AEP.
- We included a note in the previously mentioned paragraph that the advantage of the SGD formulation is that it allows for random samples in each iteration, and the average error is zero.

8) - *The plots on different learning rates, and the discussion on how hyperparameters were selected are appreciated.*

- We have added a dedicated sensitivity analysis to the results.

References

[1] Ryan N King, Katherine Dykes, Peter Graf, and Peter E Hamlington. Optimization of wind plant layouts using an adjoint approach. *Wind Energy Science*, 2(1):115–131, 2017.

[2] Jeffery Allen, Ryan King, and Garrett Barter. Wind farm simulation and layout optimization in complex terrain. In *Journal of Physics: Conference Series*, volume 1452, page 012066. IOP Publishing, 2020.

[3] Neil Wu, Gaetan Kenway, Charles A Mader, John Jasa, and Joaquim RRA Martins. pyoptsparse: A python framework for large-scale constrained nonlinear optimization of sparse systems. *Journal of Open Source Software*, 5(54):2564, 2020.

[4] Can Zhang, Stephan C Kramer, Athanasios Angeloudis, Jisheng Zhang, Xiangfeng Lin, and Matthew D Piggott. Improving tidal turbine array performance through the optimisation of layout and yaw angles. *International Marine Energy Journal*, 5(3):273–280, 2022.

[5] Ti Zilong and Deng Xiao Wei. Layout optimization of offshore wind farm considering spatially inhomogeneous wave loads. *Applied Energy*, 306:117947, 2022.

[6] Konstanze Kölle, Tuhfe Göçmen, Irene Eguinoa, Leonardo Andrés Alcayaga Román, Maria Aparicio-Sanchez, Ju Feng, Johan Meyers, Vasilis Pettas, and Ishaan Sood. Farmconners market showcase results: wind farm flow control considering electricity prices. *Wind Energy Science*, 7(6):2181–2200, 2022.

[7] Caitlyn E Clark, Garrett Barter, Kelsey Shaler, and Bryony DuPont. Reliability-based layout optimization in offshore wind energy systems. *Wind Energy*, 25(1):125–148, 2022.

[8] Eric Simley, Dev Millstein, Seongeun Jeong, and Paul Fleming. The value of wake steering wind farm control in us energy markets. *Wind Energy Science Discussions*, pages 1–26, 2023.

[9] Rémi Lam, Matthias Poloczek, Peter Frazier, and Karen E Willcox. Advances in bayesian optimization with applications in aerospace engineering. In *2018 AIAA Non-Deterministic Approaches Conference*, page 1656, 2018.

[10] Jichao Li and Mengqi Zhang. Data-based approach for wing shape design optimization. *Aerospace Science and Technology*, 112:106639, 2021.

[11] Paul A Fleming, Andrew PJ Stanley, Christopher J Bay, Jennifer King, Eric Simley, Bart M Doekemeijer, and Rafael Mudafort. Serial-refine method for fast wake-steering yaw optimization. In *Journal of Physics: Conference Series*, volume 2265, page 032109. IOP Publishing, 2022.

[12] Riccardo Riva, Jaime Liew, Mikkel Friis-Møller, Nikolay Dimitrov, Emre Barlas, Pierre-Elouan Réthoré, and Arvydas Beržonskis. Wind farm layout optimization with load constraints using surrogate modelling. In *Journal of Physics: Conference Series*, volume 1618, page 042035. IOP Publishing, 2020.

[13] Andreas Wolf Ciavarra, Rafael Valotta Rodrigues, Katherine Dykes, and Pierre-Elouan Réthoré. Wind farm optimization with multiple hub heights using gradient-based methods. In *Journal of Physics: Conference Series*, volume 2265, page 022012. IOP Publishing, 2022.

[14] J. Criado Risco, R. Valotta Rodrigues, M. Friis-Møller, J. Quick, M. Mølgaard Pedersen, and P.-E. Réthoré. Gradient-based wind farm layout optimization with inclusion and exclusion zones. *Wind Energy Science Discussions*, 2023:1–24, 2023.

[15] José F Herbert-Acero, Oliver Probst, Pierre-Elouan Réthoré, Gunner Chr Larsen, and Krystel K Castillo-Villar. A review of methodological approaches for the design and optimization of wind farms. *Energies*, 7(11):6930–7016, 2014.

[16] David Guirguis, David A Romero, and Cristina H Amon. Toward efficient optimization of wind farm layouts: Utilizing exact gradient information. *Applied energy*, 179:110–123, 2016.

[17] Peter Graf, Katherine Dykes, George Scott, Jason Fields, Monte Lunacek, Julian Quick, and Pierre-Elouan Rethore. Wind farm turbine type and placement optimization. In *Journal of Physics: Conference Series*, volume 753, page 062004. IOP Publishing, 2016.

[18] Pieter Gebraad, Jared J Thomas, Andrew Ning, Paul Fleming, and Katherine Dykes. Maximization of the annual energy production of wind power plants by optimization of layout and yaw-based wake control. *Wind Energy*, 20(1):97–107, 2017.

[19] Nicholas F Baker, Andrew P Stanley, Jared J Thomas, Andrew Ning, and Katherine Dykes. Best practices for wake model and optimization algorithm selection in wind farm layout optimization. In *AIAA Scitech 2019 forum*, page 0540, 2019.

[20] Andrew PJ Stanley, Owen Roberts, Jennifer King, and Christopher J Bay. Objective and algorithm considerations when optimizing the number and placement of turbines in a wind power plant. *Wind Energy Science*, 6(5):1143–1167, 2021.

[21] Carsten Croonenbroeck and David Hennecke. A comparison of optimizers in a unified standard for optimization on wind farm layout optimization. *Energy*, 216:119244, 2021.

[22] Michele Samorani. The wind farm layout optimization problem. *Handbook of wind power systems*, pages 21–38, 2013.

[23] Andrew Ning., Katherine Dykes., and Julian Quick. *Systems engineering and optimization of wind turbines and power plants*, pages 235–292. Wind Energy Modeling and Simulation - Volume 2: Turbine and System. Institution of Engineering and Technology, 2020.