Response to Referee Comment 'RC2'

Mark Kelly M. Paul van der Laan

April 14, 2023

This is the authors' response to Ganesh Vijayakumar's review (RC2) of our manuscript **"From shear to veer: theory, statistics, and practical application"** (WES 2022-119). We include the referee's comments *in italic script*, followed point-wise by our responses in blue.

The authors are thankful for the time and effort spent—and constructive criticism made—by the reviewer.

Overall review ('*overview*')

This paper is a very large body of work trying to connect the veer to shear for wind energy applications. This is very relevant for modern and next generation of turbines that will be very large and operating outside the surface layer of the atmospheric boundary layer (ABL). The first part of this paper is highly theoretical followed by comparisons to RANS simulations of the ABL. The latter part of this paper analyzes the experimental data from several wind farms and tries to connect the shear and veer based on the derivations in the theoretical sections. This is very good work and deserves to be published. However, I feel that there's likely a major flaw in Equation 14 that is the basis for Equations 38 and 39 that connect to the theme and the title of the paper. This is outlined in further detail in the next section under "Page 7, Line 165". If the authors could address this and the other specific comments in the next section, this paper should be published and would prove a great addition to the literature.

The authors again thank the reviewer for these positive comments. We disagree that there is a 'major flaw' in equation 14, but rather a small factor was not included. We note it was also confirmed by the RANS simulations, and respond in detail to this specific point below, including a minor update/correction to (14) and explanation in the text as well as corresponding update to eq.39 (which is now 40; eq.38 was not affected). We address the other specific comments of the reviewer below as well.

Specific comments

- ***Page 4, Line 88:*** *What is the purpose of Section 2.1.1? I'm not sure where this discussion is used in the rest of the paper.*

  As expressed in the reply to reviewer 1 (RC1), we have added to and modified the introduction section, to clarify the intent and structure of this work; this includes motivating section 2.1.1. Specifically, we now state "*In Sec. 2, after reviewing expression of the shear exponent and its relation to stability and turbulence, we derive new relations for veer;*" the relation of shear to stability connects to/informs the analysis of observed α with stability in Section 3.2.

  Also, since this paper considers non-zero veer, the balance of terms in the turbulent kinetic energy equation will be

  $$\frac{\mathrm{d}e}{\mathrm{d}t} = 0 = -\langle uw\rangle\frac{\mathrm{d}U}{\mathrm{d}z} - \langle vw\rangle\frac{\mathrm{d}V}{\mathrm{d}z} + B + T - \epsilon.$$

  Since $\mathrm{d}U/\mathrm{d}z \neq \mathrm{d}|S|/\mathrm{d}z$*, I think the expression in Equation 4 of the manuscript will no longer hold.*

  Regarding the TKE budget, the text continuing just below Equation 4 states "for a given height z, where the streamwise direction is defined by the mean wind $U(z)$ and we have suppressed z-dependences for brevity", so that $\langle vw\rangle dV/dz \to 0$ in the surface layer for such coordinates. However, you are correct in that a cross-wind term will linger; the expression (4) for shear exponent becomes

$$\alpha = \frac{z}{U}\frac{\left(\varepsilon - B - T + \langle vw\rangle \frac{dV}{dz}\right)}{-\langle uw\rangle}.$$

This will not qualitatively affect the effect of buoyancy on shear exponent, but is now addressed in the text. In section 2.1.1 we add *"We point out that Kelly et al. (2014a) ignored cross-wind stress $\langle vw\rangle$ when deriving (4), however it still shows that shear will increase in stable conditions ($B < 0$) and decrease in unstable conditions ($B > 0$), as will be demonstrated using observations in Sec. 3.2; further, as we will see in section 2.3, this is also related to the veer."* At the end of section 2.3.0 (just before §2.3.1) we also add a paragraph which includes a new equation (eq.17) updating the form of eq.4 to include the lateral (veer-associated) contribution: *"One last relation between shear and veer can also be elucidated, by considering a corrected version of (4). By keeping the lateral shear term $\langle vw\rangle\partial V/\partial z$ in the TKE rate equation, then again using coordinates defined wth x in the mean direction at height z and subsequently $\partial V/\partial z \to U\partial\varphi/\partial z$, then (4) contains an additional contribution, becoming*

$$\alpha\big|_{\mathbf{e}_x \| \boldsymbol{U}(z)} = \frac{\varepsilon - B - T}{-U\langle uw\rangle_{\|}/z} - z\frac{\partial\varphi}{\partial z}\frac{\langle vw\rangle_{\perp}}{\langle uw\rangle_{\|}}. \tag{17}$$

Recalling in the ABL that $\langle uw\rangle_{\|} < 0$ (momentum gets transferred towards the surface), because $\langle vw\rangle_{\perp} > 0$ in the ABL (Wyngaard, 2010) we see as in (14)–(16) that negative $\partial\varphi/\partial z$ (clockwise veer) is associated with positive shear; we remind that the sign of $\partial\varphi/\partial z$ is flipped in typical wind energy coordinates (left-handed, with 0 degrees corresponding to wind from the north and increasing clockwise). Although we have provided (17) to both improve (4) from Kelly et al. (2014a) and offer insight into how shear and veer are linked within the context of TKE, we advise that it is not easily utilized compared to forms like (14); the latter will be applied and investigated further in later sections."

- ***Page 5, Line 132:*** *I'm not sure of the purpose of the expressions with the complex math as they're not used further in the manuscript. However,*

$$\mathrm{d}\varphi = \frac{U\ \mathrm{d}V - V\ \mathrm{d}U}{|S|^2} = -i\frac{S^*\ \mathrm{d}S - S\ \mathrm{d}S^*}{2|S|^2},$$

where the manuscript uses i instead of $-i$ in the second term.

As also mentioned in the authors' reply (AC1) to reviewer 1, we have corrected the minus sign error and also removed the last (complex) expression on the right-hand side due to it no longer being used.

- ***Page 7, Line 165:*** *The derivative of Equation 13 is taken here after the evaluation at the height of interest to arrive at Equation 14. I don't think this is correct. If you followed the math described at the top of Page 7 , it will instead be*

$$\gamma = \cos^{-1}\left[\frac{|S|}{|G|} + \frac{1}{f|S||G|}\left(U\frac{\partial\langle vw\rangle}{\partial z} - V\frac{\partial\langle uw\rangle}{\partial z}\right)\right]$$

$$\frac{\partial\gamma}{\partial z} = \left(-\frac{|S|\alpha}{|G|z} - \left(U\frac{\partial\langle vw\rangle}{\partial z} - V\frac{\partial\langle uw\rangle}{\partial z}\right)\frac{1}{f|S||G|}\frac{\alpha}{z}\right.$$
$$\left. - \frac{1}{f|S||G|}\left(\frac{\partial U}{\partial z}\frac{\partial\langle vw\rangle}{\partial z} - \frac{\partial V}{\partial z}\frac{\partial\langle uw\rangle}{\partial z} + U\frac{\partial^2\langle vw\rangle}{\partial z^2} - V\frac{\partial^2\langle uw\rangle}{\partial z^2}\right)\right)$$
$$\left[1 - \left(\frac{1}{f|G|}\frac{\partial\langle vw\rangle_{\perp}}{\partial z} + \frac{|S|}{|G|}\right)^2\right]^{-1/2}$$
$$= \left(-\frac{|S|\alpha}{|G|z} - \frac{\alpha}{f|G|z}\frac{\partial\langle vw\rangle_{\perp}}{\partial z} - \frac{1}{f|S||G|}\left(\frac{\partial U}{\partial z}\frac{\partial\langle vw\rangle}{\partial z} - \frac{\partial V}{\partial z}\frac{\partial\langle uw\rangle}{\partial z} + U\frac{\partial^2\langle vw\rangle}{\partial z^2}\right)\right)$$
$$\left[1 - \left(\frac{1}{f|G|}\frac{\partial\langle vw\rangle_{\perp}}{\partial z} + \frac{|S|}{|G|}\right)^2\right]^{-1/2}.$$

Out of the 5 terms in the numerator of the last expression, you only have terms 1 and 5 in Equation 15. I'm not sure how the terms 2, 3 and 4 are eliminated. Could you please explain/describe the math here.

As mentioned on line 159 in the original submission, we derived Equation 14 directly from (11), not following Eq. 13, though you appear above to also start a derivation from (11). Also in your final sentence, we presume you mean eq.14 not eq.15—the terms 1 and 5 that we had pertain to eq.14 (thus eq.15 remains as previously written, since the product terms to be differentiated are all still within parenthesis; upon using the chain-rule more terms would arise).

We point out that there are several terms lingering in your version that disappear from (14) when picking $S(z) \parallel U(z)$, namely the $V\partial\langle uw\rangle/\partial z$ terms; this can be seen because $|S| \to U$ as well as $V \to 0$. The disappearance of V terms, and also that $|S|^{-1} \to U^{-1}$ cancels out a U (which can be seen more easily before taking the vertical derivative, or alternately after differentiating via recombining), eliminates these terms.

However, there is one term that does not disappear and which we neglected in eq. 14, that we assumed to be negligible: term 4 in the numerator on your last line, $\frac{1}{f|S||G|}\frac{\partial V}{\partial z}\frac{\partial\langle uw\rangle}{\partial z}$. Noting in this coordinate system (i.e., $\mathbf{x} \parallel U(z)$) that (8) means $\partial V/\partial z \to U\partial\varphi/\partial z = |S|\partial\varphi/\partial z$ then this term can be re-written; collecting this on the left-hand side we then have a factor $\left(1 + \frac{1}{f|G|}\frac{\partial\langle uw\rangle_\parallel}{\partial z}\right) \Big/ \sqrt{1 - \left(\frac{|S|}{|G|} + \frac{1}{f|G|}\frac{\partial\langle vw\rangle_\perp}{\partial z}\right)^2}$ multiplying the veer (where we use the subscript "$_\parallel$" for consistency and to remind of the coordinate system), which dividing out becomes part of the denominator on the right-hand side. Then (14) becomes

$$\left.\frac{\partial\varphi}{\partial z}\right|_{\mathbf{e}_x\parallel\mathbf{U}(z)} = \frac{-\frac{|S|}{|G|}\frac{\alpha}{z} - \frac{1}{f|G|}\frac{\partial^2\langle vw\rangle_\perp}{\partial z^2}}{\sqrt{1 - \left(\frac{|S|}{|G|} + \frac{1}{f|G|}\frac{\partial\langle vw\rangle_\perp}{\partial z}\right)^2} - \frac{1}{f|G|}\frac{\partial\langle uw\rangle_\parallel}{\partial z}},$$

which is the same as the original submission's (14) but with an additional (small) stress divergence term in the denominator. This has been updated in the text. We point out/remind that this additional term is of similar magnitude as $\frac{1}{f|G|}\frac{\partial\langle vw\rangle_\perp}{\partial z}$; thus (14) is still just a 'competition' between the two terms in the numerator, as written and discussed in the first article draft. This is one reason why eq.38 (now 39) worked as well as it did, as one can see via simple binomial expansion that the denominator can be approximated by $1 - |S|/|G| - \frac{1}{f|G|}\frac{\partial}{\partial z}\left(\langle uw\rangle_\parallel + \langle vw\rangle_\perp\right)$, with $|S|/|G|$ dominant over the stress divergences; this is also seen via the scale analyses in Sect. 2.4 and 3.3.1–3.3.2, due to c_G being between 0.02 and 0.06.

The slight modification/correction to (14) also affects eq. 39 (now eq.40) in section 3.3.1, which has been modified accordingly. We note that due to the limited sensitivity to h, the constants in eq.39 (now 40) did not need to be updated when this was modified, and the dashed lines in Fig.15 show negligible difference.

Also, what is the connection to Equation 10 of the manuscript here. That appears to be far simpler, albeit lacking any shear exponent!

As already mentioned just below eq. 10 in the text, this relation illustrates that the curvature (second derivative) of stress profiles along with the Coriolis effect are "the basis for mean veer", starting with simple conditions (barotropic, over a homogeneous surface). I.e., it motivates physical understanding, and as noted (eq.10 lacks shear so it isn't "directly useful") it also motivates the more complicated/indirect derivation which immediately follows in section 2.3.

- ***Page 10, Line 255:*** *Typo in "moreso"? Should be "more so"?*

  Corrected.

- *Page 12, Line 294 The velocity profiles based on the shear exponent and the log-law are*

$$|S| = |S|_{\text{ref}}\left(\frac{z}{z_{\text{ref}}}\right)^{\alpha}, \text{ and}$$
$$|S| = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right)$$

respectively. There is quite a bit of discussion on Page 3 Lines $80-87$ *describing the invalidity of the loglaw above the surface layer and how the shear exponent is better suited for wind energy applications. How do you explain using the log law expression again, especially without the corrections for stability as in M-O law?*

As discussed in the end of Section 3.3.1 and particularly in the 'Ongoing and future work' part of Sec. 4, various stability-corrected forms were adapted, derived, and used; in the latter section we wrote *"Some relations including stability within $|S|/|G|$ in the shear contribution to veer were developed and tested; however these were not included here, as they did not offer improvement, are seen to be beyond the scope of the current work, and might also require stability effects to be explicitly incorporated within the cross-stress terms. Ongoing work involves addressing the latter: i.e., self-consistent α-based description of stability within the veer formulations, within both the shear and cross-stress contribtions in concert with the stability-perturbed geostrophic drag law..."*

- ***Page 12, Lines 296-302:*** *The estimate for the first vertical derivative of the Reynolds stresses seems reasonable as $\partial\langle uw\rangle/\partial z \approx u_*^2/h$, although the sign is likely negative. However, the estimate for the second derivative is not very clear. In addition, considering the mean momentum equation from Equation 7,*

$$\frac{\partial\langle vw\rangle}{\partial z} = -f\,(U-U_G)$$
$$\frac{\partial^2\langle vw\rangle}{\partial z^2} = -f\frac{\partial U}{\partial z} = -f\alpha\frac{U}{z}.$$

This tells me the sign of this constant c_{vw} must be negative. This might make the numerator of Equation 14 almost zero, suggesting other terms are dominating as expressed in my concerns for Page 7, Line 165. Could you please expand the explanation for your estimate of the second derivative of the stresses?

The sign of $\partial\langle uw\rangle/\partial z$ is not negative, as seen in e.g. the textbook by Wyngaard (2010), which we reference; we remind that $\langle uw\rangle$ begins as u_*^2 at the surface, increasing towards zero (magnitude decreasing with z) as one moves away from the surface; thus $\partial\langle uw\rangle/\partial z$ is positive ($\approx u_*^2/h$, as written). Indeed the sign of c_{vw} is negative, as we had stated on lines 612–613 (*"Given the negative curvature of lateral stress, $\partial^2\langle vw\rangle/\partial z^2 < 0$ (Wyngaard, 2010), c_{vw} is negative and of order 1..."*) As written on line 617, *"The value of c_{vw} giving the estimates shown in Fig. 15 was -0.7 for all sites"*.
The estimate of the stress curvature (second derivative) was explained and given on lines 299–304 as well as eq.34.

- ***Page 12, Line 306:*** *Based on the expressions in Equation 31, I got this to be*

$$\sin\varphi_G = \frac{-c_G}{\kappa}\left\{B\cos\varphi_0 - \left[\ln\frac{u_*}{fz_0} - A\right]\sin\varphi_0\right\}$$

The multiplication of B by $\cos\varphi_0$ is missing. Could you please check again.

We agree: checking again we see the factor of $\cos\varphi_0$ on B was omitted when transferring from handwritten to LaTeX; this has been corrected.

- ***Page 12, Line 315:*** *A summary of the expressions to be evaluated after substitutions listed in Section 2.4 would be extremely helpful before evaluating them in Section 3.1.*

To limit the length of the paper we have not added more expressions, preferring to refer to them directly when using them to make the practical/applied equations 38–39 (now eq.39–40).

- ***Page 14, Line 355****: A more thorough overview of how the RANS simulations covers the space of interest would be helpful here.*

  Lines 349–351 (now l.392–394 in updated manuscript) also described how the RANS simulations cover the space of interest. Further, Fig. 6c visually displays this also in terms of shear and veer.

- ***Page 14, Line 358****: How does the expression in Equation 14 compare to Equation 9 in the right plot in Figure 2? Based on my concerns expressed for Page 7, Line 165, I expect the correlation to be not as good as for Equation 9.*

  As mentioned in our response above, inclusion of the lateral shear piece introduces a (small) modification into the denominator of (14); since it is in the denominator it does not affect the (numerator) components shown in Fig. 2 nor the balance shown in Fig. 5. Thus the correlation is just as good.

- ***Page 15, Line 374****: Behavior that is "not inconsistent"? Do you mean consistent?*

  As addressed in the authors' reply to reviewer 1 ("AC1" on 21 Mar.2023), this has been changed to 'consistent.'

  There is no expression derived in Section 2 that gives $Ro_h^{1.4}$. This is also likely affected by concerns for Page 7 , Line 165.

  We have now specifically noted *"the veer is empirically found to be proportional to $Ro_h^{1.4}$..."* This is not connected to eq.14 nor its derivation (your concerns around line 165), but rather comes directly from the RANS output which are independent of such derivation (they just solve the mean Navier-Stokes equations).

- ***Page 26, Line 583 and Page 28, Line 609****: This simplification in Equations 38 and 39 are likely in error. Could you please check my concerns for Page 7, Line 165 before confirming these results.*

  As mentioned above addressing your previous concerns: the simplification in eq.38 (now 39) was not affected; the small change to eq.39 (now 40), which we updated in the manuscript, did not cause a noticeable difference.

- ***Page 29, Figure 16****: The standard deviation of the veer is almost the same or much larger than the mean from the experiments. It would be hard to consider this as proof of the expressions derived in Equations 38 and 39.*

  These are two different things, and remind that the simplified expressions are providing an α-based estimate for the mean. The standard deviation conditioned on shear exponent $\sigma_{\Delta\varphi/\Delta z|\alpha}$ was discussed in lines 635–645 (now 683–693) and shown in Fig. 16. We considered developing an empirical model for $\sigma_{\Delta\varphi/\Delta z|\alpha}$, but deemed this to be beyond the scope of the current paper and is ongoing work. We also note that we gave a form to estimate the standard deviation of veer conditioned on wind speed, which was eq.37 (now 38).

Again we express appreciation to the Ganesh Vijayakumar for the review and his checking of the derivations, which helped to both improve the article as well as support the scientific method.

with kind regards,

Mark Kelly and M. Paul van der Laan
Department of Wind and Energy Systems
Risø Lab/Campus, Danish Technical University