From shear to veer: theory, statistics, and practical application

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Abstract.

In the past several years, wind veer — sometimes called 'directional shear' — has begun to attract attention due to its effects on wind turbines and their production, particularly as the length of manufactured turbine blades has increased. Meanwhile, applicable meteorological theory has not progressed significantly beyond idealized cases for decades, though veer's effect on the wind speed profile has been recently revisited. On the other hand the shear exponent (α) is commonly used in wind energy for vertical extrapolation of mean wind speeds, as well as being a key parameter for wind turbine loads calculations and design standards.

In this work we connect the oft-used shear exponent with veer, both theoretically and for practical use. We derive relations for wind veer from the equations of motion, finding the veer to be composed of separate contributions from shear and vertical gradients of cross-wind stress. Following from the theoretical derivations, which are neither limited to the surface-layer nor constrained by assumptions about mixing length or turbulent diffusivities, we establish simplified relations between the wind veer and shear exponent for practical use in wind energy. We also elucidate the source of commonly-observed stress-shear misalignment and its contribution to veer, noting that our new forms allow for such misalignment. The connection between shear and veer is further explored through analysis of one-dimensional (single-column) Reynolds-averaged Navier-Stokes solutions, where we confirm our theoretical derivations as well as the dependence of mean shear and veer on surface roughness and atmospheric boundary layer depth in terms of respective Rossby numbers.

Finally we investigate the observed behavior of shear and veer across different sites and flow regimes (including forested, offshore, and hilly terrain cases) over heights corresponding to multi-megawatt wind turbine rotors, also considering the effects of atmospheric stability. From this we find empirical forms for the probability distribution of veer during high-veer (stable) conditions, and for the variability of veer conditioned on wind speed. Analyzing observed joint probability distributions of α and veer, we compare the two simplified forms we derived earlier and adapt them to ultimately arrive at more universally applicable equations to predict the mean veer in terms of observed (i.e., conditioned on) shear exponent; lastly, the limitations, applicability, and behavior of these forms is discussed along with their use and further developments for both meteorology and wind energy.

1 Introduction

The shear exponent has generally not been used or accepted by meteorologists, as it does not (directly) relate to the physics of atmospheric flow, nor to the most important boundary condition—the surface. Regarding the latter, in contrast with similarity theory (**?**), the shear exponent does not contain explicit information about the surface roughness. However, the shear exponent can be related to surface properties in a generalized way, as well as to turbulent kinetic energy and atmospheric stability (buoyancy) as shown by e.g. **?**. This is particularly useful above the atmospheric surface layer (ASL), where micrometeorological theory based on ASL assumptions fails—and where the effects of the surface are neither dominant nor simple enough to be characterized through accepted ASL parameterizations. As practiced in the wind energy resource assessment community for decades, the shear exponent can thus be preferable over similarity theory for use in vertical extrapolation (**???**) with quantification of uncertainty in its use more recently reinforcing such (**??**). Shear is also a key parameter for flow characterization towards loads simulations, being seen to systematically affect various turbine loads (e.g. **??**).

Veer has received much less attention than shear, though its potential importance to wind energy has been noted more recently. In the meteorological literature, where veer is often labelled as ‘directional shear’ or ‘turning,’ **?** reviewed the distinction between veer and vertical gradients of wind speed, listing studies of meteorological phenomena that considered veer (though they focused on convective storms). While some works in meteorology have investigated veer, these have tended to focus on the angular difference between winds at the top of the atmospheric boundary layer (ABL) and the surface (e.g. **????**),and are not generally suited for engineering applications. For wind energy, **?** looked at the veer (and shear) along with power production measured over a six-month period, finding a minor but non-negligible effect of veer on power production for a utility scale turbine. **?** found positive veer over the upper half of a single (2.5MW) clockwise-turning turbine rotor to reduce power production, opposite and slightly larger than the corresponding effects of negative veer there; they also showed the rotor’s lower-half veer was less significant than the upper half. **?** examined measurements from a lidar offshore between islands southwest of Hong Kong, observing larger veer when hilly terrain was upstream compared to more open sea conditions; they also noted seasonal variations. For power production, the veer was incorporated into rotor-equivalent wind speed (REWS) by **?**, whom found it to generally decrease production at two sites; **?** found similar results from weather assimilation model output over the USA, along with higher production at night and lower power during daytime at most locations. Wind veer has also been examined with regard to its connection with the distortion and lateral movement turbine wakes via measurements and simulations (e.g. **??**), also including yaw-misalignment affects (**??**).

In this paper we ~~elucidate analytical and statistical connections between a number of~~ investigate wind veer, showing its joint behaviors with and connection to shear and key parameters used to describe atmospheric boundary layer flow~~, with focus on the vertical variation of wind velocity. This follows the earlier work of **?** that gave forms for low-order statistics of shear exponent α, relating α to turbulence intensity and stability; here we derive new~~ . In Sec. 2, after reviewing expression of the shear exponent and its relation to stability and turbulence, we derive new relations for veer; we show veer to be composed of shear-driven and Coriolis-associated stress gradient contributions. The theoretical behavior of veer is also derived for canonical cases such as Ekman and surface-layer flow, as well as the effect of shear-stress misalignment on veer. Further, in Sec. ?? practical relations from micrometeorology are elucidated, towards evaluation of the expressions developed for veer. Section ?? includes analysis of veer, exploring and connecting the developed relations to both computational modelling and observations. Section ?? gives RANS (mean) simulation results over flat terrain in neutral conditions for hundreds of combinations of surface-Rossby number and ABL-depth Rossby number, showing the dependence of veer on the latter as well as the counteracting behavior of veer's two primary components. Section ?? begins with analysis of multi-year observations from six different flow regimes across four sites showing the statistical behavior of shear with stability, and subsequently that of veer, also providing new empirical relations for the turning of the wind in terms of shear, going beyond classical Ekman-type analysisprobability of occurrence of larger veer (due to the effect of stable conditions) and for the variability of veer with wind speed. The observational analysis concludes in Sec. ?? with simplified practical relations for veer based on observed shear, including comparison with joint distributions of veer and shear across the six flows analyzed. Finally the results summarily discussed and conclusions given, with ongoing and future work also described for the reader.

2 Theory and development

In this section we define the shear exponent and veer, then derive relations for veer in terms of shear and vertical gradients of stress, as mentioned in the previous paragraph. Section 2.3 provides a number of expressions for veer; this is done to facilitate its calculation and interpretation in the different coordinate systems typically considered in wind energy flow analyses, and we also include forms that are independent of coordinate system. Because coordinates aligned with the mean wind for a given height of interest (e.g. hub height) are commonly used in wind energy, and because expressions for veer in such a coordinate system are simpler to express and calculate, we ultimately arrive at two forms in such a system (eqs. 14 and 16); due to its robustness, one of these (eq. 14) will later be shown in section ?? to be further simplifiable and usable (as eq. ?? or ??) in comparison with measurements.

2.1 Shear exponent

Just as potential temperature—the buoyancy variable commonly-used in meteorology—was labeled the "meteorologist's entropy" by ?, one could call the shear exponent (α) the "wind engineer's phi-function." Specifically this follows from the definition of shear exponent

$$\alpha \equiv \frac{\partial U/\partial z}{U/z} = \frac{\partial \ln U}{\partial \ln z} \tag{1}$$

and the dimensionless wind speed gradient

$$\Phi_m \equiv \frac{dU/dz}{u_{*0}/\kappa z} = \frac{\kappa U}{u_{*0}}\alpha; \tag{2}$$

used in meteorology, where u_{*0} is the surface layer friction velocity (square root of kinematic shear stress), $\kappa = 0.4$ is the von Kármán constant, and z is the height coordinate[1]. We remind that (1) is derived from the power-law expression for wind speed

$$\frac{U}{U_{\mathrm{ref}}} = \left(\frac{z}{z_{\mathrm{ref}}}\right)^{\alpha}, \tag{3}$$

which is assumed to be valid over some extent around height z_{ref}, with $U_{\mathrm{ref}} \equiv U(z_{\mathrm{ref}})$. The power-law (3) with shear exponent (1) has been used in wind engineering for decades (e.g. **??**) due to its simplicity, and because it doesn't require any information other than the wind speed at two heights. Although (1) and (2) might appear to be quite alike, one can see a phenomenological difference when comparing the wind speed profiles resulting from these relations. In Monin-Obukhov ("M-O") theory Φ_m is a function of the stability z/L which is proportional to surface heat flux H_0 divided by u_{*0}^3, i.e. the reciprocal Obukhov length is $1/L = \kappa(g/T_0)H_0/u_{*0}^3$ where T_0 is the background temperature and g is the gravitational acceleration (**?**); the Φ_m function and corresponding M-O wind profile (which arises via integrating dU/dz in (2) from a height equal to the roughness length z_0 up to height z) thus require a number of assumptions and more information than calculation of α via (1) or use of the power-law (3). Monin-Obukhov wind profiles also require the surface roughness length (z_0), while the friction velocity u_{*0} (and thus shear stress) is assumed to be constant in the surface layer where M-O theory is most valid[2]; further, the assumptions of stationarity and a uniform flat surface are implicit in use of M-O theory. Following surface layer theory one could write an equivalent shear exponent $\alpha_{\mathrm{ASL}} = \Phi_m(z/L)\big/[\ln(z/z_0) - \Psi_m(z/L)]$ where $\Psi_m = \int_{z_0}^{z}[1-\Phi_m(z'/L)]d\ln z'$ is the M-O wind speed correction function; the analytic forms for Φ_m and Ψ_m differ in stable and unstable conditions, and have been determined empirically in decades past (**???**). But Monin-Obukhov similarity theory and its assumptions (such as constant u_*), as well as established forms for Φ_m, fail above the surface layer;[3] this motivates use of α in applications such as wind energy, as (1) does not directly rely on surface-layer assumptions.

2.1.1 Relation to stability and turbulence

As shown by **?**, in horizontally homogeneous conditions the steady or mean balance of turbulent kinetic energy (TKE) can be written in terms of shear exponent as

$$\alpha = \frac{z}{U}\frac{(\varepsilon - B - T)}{-\langle uw\rangle} \tag{4}$$

for a given height z, where the streamwise direction is defined by the mean wind $U(z)$ and we have suppressed z-dependences for brevity; here $\langle uw\rangle$ is the turbulent horizontal momentum flux (kinematic stress), T is the total (turbulent plus pressure)

[1]The full derivative (d/dz) is used in (2) ~~because of~~ due to the ~~assumption of~~ horizontal homogeneity assumed by Monin-Obuhkov similarity theory, from which Φ_m arises.

[2]The 'constant-flux layer' in surface-layer theory does not require exactly constant fluxes with height, as is often presumed. The label and assumption are that the *non-dimensional* fluxes, normalized by ABL scales, are constant with z (**??**); i.e., the ASL is the layer over which the decrease in u_*^2 is small compared to u_{*0}^2, roughly the bottom 10% of the ABL.

[3]We note that **?** adapted M-O theory to long-term means and **?** extended this beyond the surface layer within the European Wind Atlas (WAsP) framework, thus addressing the stationarity and surface homogeneity aspects. However, the purpose and scope of the current article is to examine the commonly-used shear exponent and its connection with veer, not on vertical extrapolation methods per se.

transport, B is buoyant production, and ε is the viscous dissipation rate of TKE. We point out that **?** ignored cross-wind stress $\langle vw \rangle$ when deriving (4), however it still shows that e.g. shear will increase in stable conditions ($B < 0$) and decrease in unstable conditions ($B > 0$), as will be demonstrated using observations in Sec. **??**; further, as we will see in section 2.3, this is also related to the veer. Within the ASL under these conditions where M-O theory is valid and $\langle vw \rangle \to 0$, using the neutral value of dissipation rate as $\varepsilon_0 \equiv u_{*0}^3/(\kappa z)$ along with the dimensionless functions $\Phi_\varepsilon \equiv \varepsilon/\varepsilon_0$ and $\Phi_T \equiv T/\varepsilon_0$ (**?**), we can express an ASL version of (4) as

$$\alpha_{\mathrm{ASL}} = \frac{u_{*0}}{\kappa U}\left(\Phi_\varepsilon + \frac{z}{L} - \Phi_T\right) \approx I_u \left(\Phi_\varepsilon + \frac{z}{L} - \Phi_T\right) \tag{5}$$

since by definition $B/\varepsilon_0 = -z/L$ and $u_{*0}^2 = -\langle uw \rangle$; here $I_u \equiv \sigma_u/U$ is the streamwise turbulence intensity. The dimensionless dissipation rate (M-O function) $\Phi_\varepsilon \geq 1$ is roughly $1 + 5z/L$ in stable conditions and increases more weakly with $-z/L$ in unstable conditions (**??**); meanwhile the transport is negligible in stable conditions but $\Phi_T > 0$ in unstable conditions (e.g. **?**). Thus in stable conditions ($L^{-1} > 0$) one can see α is larger than in neutral conditions, while in unstable conditions α becomes smaller. Above the ASL this will also generally be the case, though analytic nondimensional forms become difficult to derive, while the flow becomes affected by more terrain upwind and associated inhomogeneities; furthermore in stable conditions the local stability (at a given z) becomes increasingly more important than surface-based z/L (**?**). As will be shown below, the most common and mean conditions at contemporary rotor heights qualitatively follow (5), but due to these and other non-ideal effects (e.g. nonstationary transients) large deviations can occur. We note that in this work we are not searching for analytical forms for α or surface-layer behavior; rather, we are concerned with how α relates to the ***veer***, especially over heights corresponding to wind turbine rotors, a portion of which commonly extends beyond the ASL.

2.2 Veer

For the simplified general case of Coriolis-affected mean flow, we write the horizontal mean velocity vector $\{U, V\}$ as a complex number, $S \equiv U + iV = |S|e^{i\varphi}$. For a mean wind direction defined at some height z, the veer can be defined as a directional shear $\partial\varphi/\partial z$ through the wind direction

$$\varphi(z) = \arg[S(z)] = \arctan\left[\frac{V(z)}{U(z)}\right]. \tag{6}$$

In most of the micrometeorological literature, the mean wind direction is defined based on the surface stress (i.e. via the winds closest to the surface, so $\varphi_0 \equiv \varphi(0) = 0$). We follow this convention unless stated otherwise, as done for some expressions later in section 2.3; one could also choose to define the coordinate system based on the geostrophic wind direction (e.g. **?**).

As is classically known in micrometeorology (e.g. **?**), the veer across the entire ABL depends primarily on the Coriolis parameter f (thus latitude), geostrophic wind speed $|G|$, and surface roughness length z_0, but is also affected by the ABL depth h and stability (as confirmed via Reynolds-averaged Navier-Stokes simulations by **?**). The veer across a fraction $\Delta z/h$ of the ABL will also depend on these parameters; thus for a given site and height, $\Delta\varphi/\Delta z$ will have a distribution due to variations in these parameters.This will become clearer below as we examine the relationship between veer and shear.

The Coriolis-affected mean momentum balance can be written in the form

$$\frac{\partial S}{\partial t} = 0 = -if(S-G) - \frac{\partial \langle sw \rangle}{\partial z} \tag{7}$$

for stationary and horizontally homogeneous conditions (thus neglecting advection). Here the kinematic horizontal pressure gradient $\nabla p/\rho = f\{V_G, -U_G\}$ is also written like a velocity in complex form as $G \equiv U_G + iV_G = (-\partial p/\partial y + i\partial p/\partial x)/(\rho f)$. The mean stresses are dominated by vertical momentum transport $\langle sw \rangle$, where w denotes (turbulent) vertical velocity fluctuations and $s \equiv u + iv$ the horizontal velocity fluctuations.

At a given height z, taking the differential of (6) (recalling $d\arctan x = dx/[1+x^2]$ and using the chain rule) gives

$$d\varphi = \frac{UdV - VdU}{|S|^2}; \tag{8}$$

here the superscript asterisk denotes complex conjugate. Applying $\partial/\partial z$ to (8) and (7) and combining provides a basic expression for veer:

$$\frac{\partial \varphi}{\partial z} = \frac{U}{|S|^2}\left[\frac{1}{f}\frac{\partial^2 \langle uw \rangle}{\partial z^2} + \frac{\partial V_G}{\partial z}\right] + \frac{V}{|S|^2}\left[\frac{1}{f}\frac{\partial^2 \langle vw \rangle}{\partial z^2} - \frac{\partial U_G}{\partial z}\right]. \tag{9}$$

In the case of zero geostrophic shear ($d\mathbf{G}/dz = 0$), if the coordinate system's x-axis is defined by the mean wind direction at the height z where the veer is sought, then (9) can be written more simply as

$$\left.\frac{\partial \varphi(z)}{\partial z}\right|_{d\mathbf{G}/dz \to 0} = \frac{1}{f|S|}\frac{\partial^2}{\partial z^2}\langle uw \rangle|_{\mathbf{e}_x \| \mathbf{U}(z)}. \tag{10}$$

Though (9) and (10) are not directly very useful for relating veer to shear, they illustrate that the *curvature* of stress profiles and Coriolis effect are the basis for mean veer following (7), and also that geostrophic shear can further contribute to veer (e.g. due to baroclinity, **???**).

2.3 Relating veer to shear

Towards relating the veer to shear, one can alternately derive the veer by first taking the time derivative of (8); using the real and imaginary parts of (7), in the horizontally homogeneous limit (ignoring advection) one obtains a rate equation for mean wind direction:

$$\frac{\partial \varphi}{\partial t} = \left[\frac{V}{|S|^2}\frac{\partial \langle uw \rangle}{\partial z} - \frac{U}{|S|^2}\frac{\partial \langle vw \rangle}{\partial z}\right] + f\left(\frac{|G|}{|S|}\cos\gamma - 1\right). \tag{11}$$

The 'turning' angle $\gamma \equiv \varphi - \varphi_G$ between geostrophic and mean wind directions (e.g. **?**) arises through[4]

$$U_G U + V_G V = \mathbf{U}\cdot\mathbf{G} = |S||G|\cos\gamma$$

by taking $\partial/\partial t$ of (6) or equivalently $U\partial V/\partial t - V\partial U/\partial t$ via (8). The geostrophic wind direction is defined as $\varphi_G \equiv \arctan(V_G/U_G)$, and the 'cross-isobar' angle, i.e. the turning over the whole ABL ($\gamma_0 = \varphi_0 - \varphi_G$), is generally less than $45°$ (**?**)[5]; in a right-

[4]The turning angle can also be expressed in complex notation, recalling that the angle between vectors written in complex notation (here $\mathbf{U} \to S$ and $\mathbf{G} \to G$) can be recovered by taking $\Re\{G^* S\}$, i.e. $|G||S|\Re\{e^{-i(\varphi+\gamma)}e^{i\varphi}\} = |G||S|\cos\gamma$.

[5]The ABL turning angle γ_0 cannot exceed $45°$, according to the Ekman equations (or their numerical solution, as in **?**). However, in some situations, which tend to involve horizontal inhomogeneities, $\gamma_0 > 45°$; these include e.g., baroclinity, terrain-induced turning (especially with stability), convective cells, and various persistent storm structures.

The authors note that the `latexdiff` command includes numerous bugs, and thus created more than 1000 LaTeX errors when making the difference file.
We corrected about 100 of them, but were unable to get the file to consistently compile, and discovered error types which were too difficult to surmount. Therefore the 'diff' file ends here.

However, in our responses to the reviewers, we have explicitly shown the changes that we have made.
The current diff file (above) shows most of the 'big' changes, aside from the introduction of the new equation(17).
We appreciate your patience, and direct you to the final PDF file and our responses.