Supplementary Materials for

A New Way to Estimate Maximum Power from Wind Turbines: Linking Newtonian with Action Mechanics

Estimating maximum power from wind turbines

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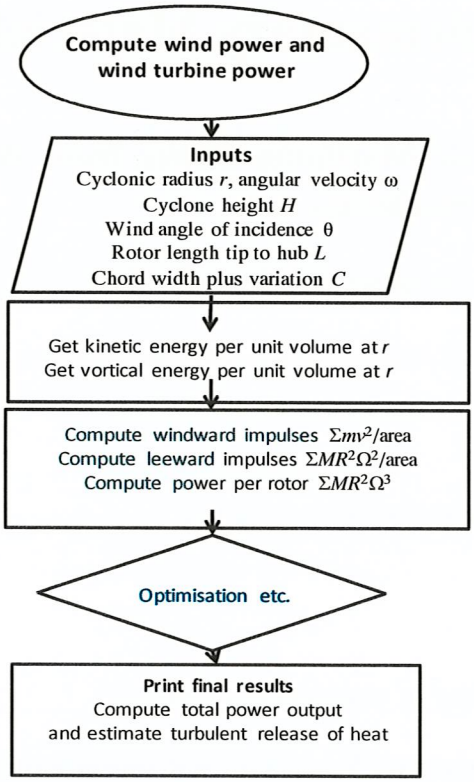
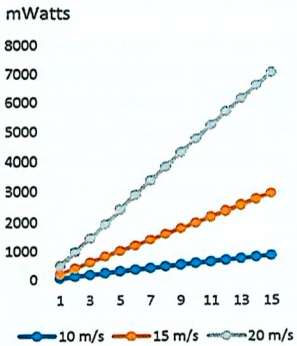


Figure S1. Outline of computer programs.

V=10 m/sec		**Vevor 1.24 m diameter**						**Table S1**										
S=3		Area = 358.8 cm^2				L=52 cm	D=1.225x10^-3		R=10^8 cm			H=10^5 cm						
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Tw x10^7	0.062243	0.244256	0.532217	0.904152	1.33145	1.780834	2.216664	2.603405	2.908089	3.102617	3.165724	3.084494	2.855304	2.48414	1.986262	1.385243	0.711438	0
Tb x10^7	0.080049	0.079134	0.077617	0.075509	0.072826	0.069589	0.065823	0.061555	0.056819	0.051651	0.04609	0.040177	0.033959	0.027483	0.020797	0.013953	0.007003	0
Tw-Tb x10	-0.01781	0.165122	0.454601	0.828644	1.258624	1.711245	2.150841	2.541849	2.85127	3.050966	3.119635	3.044317	2.821345	2.456657	1.965465	1.371289	0.704435	0
Ω	57.6923	57.692	57.69231	57.6923	57.69231	57.69231	57.6924	57.6923	57.69231	57.69231	57.69231	57.69231	57.69231	57.6923	57.69231	57.69231	57.69231	0
P mW	-1.02728	9.526199	26.22695	47.80635	72.61294	98.72566	124.0872	146.6451	164.4963	176.0173	179.9789	175.6337	162.7699	141.7302	113.3922	79.11284	40.64047	0
	-1.02728	9.526199	26.22695	47.80635	72.61294	98.72566	124.0872	146.6451	164.4963	176.0173	179.9789	175.6337	162.7699	141.7302	113.3922	79.11284	40.64047	0
V=15	-1.02728	9.526199	26.22695	47.80635	72.61294	98.72566	124.0872	146.6451	164.4963	176.0173	179.9789	175.6337	162.7699	141.7302	113.3922	79.11284	40.64047	0
S=3																		angle
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Tw x10^7	0.140046	0.549575	1.197489	2.034344	2.995763	4.006876	4.987494	5.857661	6.543201	6.980888	7.122879	6.940113	6.424435	5.589315	4.469089	3.116796	1.600736	0
Tb x10^7	0.120074	0.118701	0.116425	0.113263	0.109239	0.104384	0.098734	0.092333	0.085229	0.077477	0.069134	0.060266	0.050939	0.041224	0.031196	0.02093	0.010505	0
Tw-Tb x10	0.019973	0.430874	1.081064	1.921081	2.886524	3.902492	4.88876	5.765328	6.457972	6.903411	7.053745	6.879847	6.373496	5.54809	4.437893	3.095866	1.590231	0
Ω	86.5384	86.5385	86.5385	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	86.53846	0
P mW	1.728414	37.28721	93.55365	166.2474	249.7953	337.7156	423.0657	498.9226	558.863	597.4106	610.4202	595.3713	551.5525	480.1232	384.0484	267.9115	137.6161	0
	1.728414	37.28721	93.55365	166.2474	249.7953	337.7156	423.0657	498.9226	558.863	597.4106	610.4202	595.3713	551.5525	480.1232	384.0484	267.9115	137.6161	
V=20																		
S=3																		
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Tw x10^7	0.248971	0.977023	2.128869	3.616612	5.325801	7.123335	8.866656	10.41362	11.63236	12.41047	12.6629	12.33798	11.42112	9.93656	7.945048	5.540971	2.845753	0
Tb	0.160098	0.158268	0.155233	0.151018	0.145652	0.139179	0.131646	0.123111	0.113639	0.103302	0.090218	0.080355	0.067919	0.054966	0.041595	0.027907	0.014007	0
Tw-Tb	0.088873	0.818755	1.973636	3.465595	5.180149	6.984156	8.735011	10.29051	11.51872	12.30717	12.57268	12.25762	11.3532	9.881594	7.903453	5.513064	2.831746	0
Ω	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	115.3846	0
P mW	10.25459	94.47168	227.7272	399.8763	597.7094	805.8642	1007.886	1187.366	1329.083	1420.058	1450.694	1414.341	1309.985	1140.184	911.9369	636.1228	326.7399	0
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90 θ
10 m/s	-1.02728	9.526199	26.22695	47.80635	72.61294	98.72566	124.0872	146.6451	164.4963	176.0173	179.9789	175.6337	162.7699	141.7302	113.3922	79.11284	40.64047	0
15 m/s	1.728414	37.28721	93.55365	166.2474	249.7953	337.7156	423.0657	498.9226	558.863	597.4106	610.4202	595.3713	551.5525	480.1232	384.0484	267.9115	137.6161	0
20 m/s	10.25459	94.47168	228	399.8763	597.7094	805.8642	1007.886	1187.366	1329.083	1420.058	1450.694	1414.341	1309.985	1140.184	911.9369	636.1228	326.7399	0

10 m/sec	Vevor 1.24 m diameter					Table S2		Varying tip-speed ratio							
λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tw x10^7	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724	3.165724
Tb x10^7	0.015363	0.030726	0.04609	0.061453	0.076816	0.092179	0.107542	0.122906	0.138269	0.153632	0.168995	0.184358	0.199722	0.215085	0.230448
Tw-Tb x10	3.150361	3.134998	3.119635	3.104271	3.088908	3.073544	3.058182	3.042819	3.027455	3.012092	2.996729	2.981366	2.966003	2.950639	2.935276
Ω rad/sec	19.23077	38.4615	57.69231	76.9231	96.15385	115.3846	134.6154	153.8462	173.0769	192.3076	211.5385	230.7692	250	269.2307	288.4615
P w	60.58386	120.5768	179.9789	238.7901	297.0104	354.6398	411.6783	468.1259	523.9827	579.2485	633.9234	688.0074	741.5006	794.4029	846.7143
15 m/sec															
λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tw x10^7	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288	7.12288
Tb x10^7	0.02305	0.04609	0.06913	0.09218	0.115223	0.138269	0.161314	0.184358	0.207403	0.230448	0.253493	0.276538	0.299582	0.322627	0.345672
Tw-Tb x10	7.09983	7.076789	7.05374	7.0307	7.007655	6.984611	6.961566	6.938521	6.915476	6.89243	6.869387	6.846342	6.823297	6.800252	6.777207
Ω rad/sec	29	57.6923	86.5385	115.3846	144.2307	173.0769	201.923	230.7692	259.6153	288.4614	317.3077	346.1538	375	403.8462	432.6923
P w	204.803	408.2763	610.4202	811.2346	1010.72	1208.875	1405.701	1601.197	1795.364	1988.201	2179.709	2369.888	2558.736	2746.256	2932.446
20 m/sec															
λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tw x10^7	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629	12.6629
Tb x10^7	0.030726	0.061453	0.092179	0.122906	0.153632	0.184358	0.215085	0.245811	0.276538	0.307264	0.33799	0.368717	0.399443	0.43017	0.460896
Tw-Tb x10	12.63217	12.60144	12.57072	12.53999	12.50926	12.47854	12.44781	12.41709	12.38636	12.35563	12.32491	12.29418	12.26345	12.23273	12.202
Ω rad/sec	38.46154	76.92308	115.3846	153.8462	192.3077	230.7692	269.2308	307.6923	346.1538	384.6154	423.0769	461.5385	500	538.4615	576.9231
P w	485.8527	969.3418	1450.467	1929.229	2405.628	28796626	3351.334	3820.642	4287.586	4752.166	5214.383	5674.237	6131.173	6586.853	7039.616

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 λ
10 m/s	60.58386	120.5768	179.9789	238.7901	297.0104	354.6398	411.6783	468.1259	523.9827	579.2485	633.9234	688.0074	741.5006	794.4029	846.7143
15 m/s	204.803	408.2763	610.4202	811.2346	1010.72	1208.875	1405.701	1601.197	1795.364	1988.201	2179.709	2369.888	2558.736	2746.256	2932.446
20 m/s	485.8527	969.3418	1450.467	1929.229	2405.628	2879.663	3351.334	3820.642	4287.586	4752.166	5214.383	5674.237	6131.173	6586.853	7039.616



GE 1.5MW

Table S3

V=m/s TSR=tip speed ratio = RΩ/V

θ = angle of wind incidence

V=5																				
TSR=λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
θ=55																				
Tw J or Nn	6.28E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04	6.29E+04
Tb J	3.99E+03	7.99E+03	1.20E+04	1.60E+04	2.00E+04	2.40E+04	2.80E+04	3.19E+04	3.59E+04	3.99E+04	4.39E+04	4.79E+04	5.19E+04	5.59E+04	5.99E+04	6.39E+04	6.79E+04	7.19E+04	7.59E+04	7.99E+04
Tw-Tb J	5.88E+04	5.50E+04	5.10E+04	4.70E+04	4.29E+04	3.90E+04	3.50E+04	3.10E+04	2.70E+04	2.29E+04	1.90E+04	1.50E+04	1.09E+04	7.03E+03	3.04E+03	-9.54E+02	-4.95E+03	-8.94E+03	-1.29E+04	-1.69E+04
Ω rad/sec	1.29E-01	2.58E-01	3.87E-01	5.16E-01	6.45E-01	7.74E-01	9.03E-01	1.03E+00	1.16E+00	1.29E+00	1.42E+00	1.55E+00	1.68E+00	1.81E+00	1.94E+00	2.06E+00	2.19E+00	2.32E+00	2.45E+00	2.58E+00
P Watts	7.59E+03	1.42E+04	1.97E+04	2.42E+04	2.76E+04	3.02E+04	3.16E+04	3.20E+04	3.14E+04	2.95E+04	2.70E+04	2.33E+04	1.83E+04	1.27E+04	5.88E+03	-1.97E+03	-1.09E+04	-2.08E+04	-3.17E+04	-4.37E+04
λ=8																				
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90		
Tw J	1.37E+03	5.38E+03	1.17E+04	1.99E+04	2.93E+04	3.93E+04	4.89E+04	5.74E+04	6.41E+04	6.84E+04	6.98E+04	6.80E+04	6.29E+04	5.48E+04	4.38E+04	3.05E+04	1.57E+04	0.00E+00		
Tb MJ	7.53E+04	7.44E+04	7.30E+04	7.10E+04	6.85E+04	6.55E+04	6.19E+04	5.79E+04	5.35E+04	4.86E+04	4.34E+04	3.78E+04	3.19E+04	2.59E+04	1.96E+04	1.31E+04	6.59E+03	0.00E+00		
Tw-Tb J	-7.39E+04	-6.91E+04	-6.13E+04	-5.11E+04	-3.92E+04	-2.62E+04	-1.31E+04	-5.21E+02	1.07E+04	1.98E+04	2.64E+04	3.02E+04	3.10E+04	2.89E+04	2.42E+04	1.74E+04	9.09E+03	0.00E+00		
Ω rad/sec	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	0.00E+00		
P Watts	-7.63E+04	-7.13E+04	-6.33E+04	-5.28E+04	-4.04E+04	-2.71E+04	-1.35E+04	-5.37E+02	1.10E+04	2.04E+04	2.73E+04	3.12E+04	3.20E+04	2.98E+04	2.50E+04	1.80E+04	9.39E+03	0.00E+00		
V=10																				
TSR=λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
θ=55																				
Tw J	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05	2.72E+05
Tb J	9.45E+03	1.89E+04	2.83E+04	3.78E+04	4.72E+04	5.67E+04	6.61E+04	7.56E+04	8.50E+04	9.45E+04	1.04E+05	1.13E+05	1.23E+05	1.32E+05	1.42E+05	1.51E+05	1.61E+05	1.89E+05	1.80E+05	1.89E+05
Tw-Tb J	2.63E+05	2.53E+05	2.43E+05	2.34E+05	2.25E+05	2.15E+05	2.06E+05	1.96E+05	1.87E+05	1.77E+05	1.68E+05	1.59E+05	1.49E+05	1.40E+05	1.30E+05	1.21E+05	1.11E+05	8.30E+04	9.24E+04	8.30E+04
Ω	2.58E-01	5.16E-01	7.74E-01	1.03E+00	1.29E+00	1.55E+00	1.81E+00	2.06E+00	2.32E+00	2.58E+00	2.84E+00	3.10E+00	3.35E+00	3.61E+00	3.87E+00	4.13E+00	4.39E+00	5.16E+00	4.90E+00	5.16E+00
P Watts	6.77E+04	1.31E+05	1.88E+05	2.42E+05	2.90E+05	3.33E+05	3.72E+05	4.05E+05	4.34E+05	4.58E+05	4.77E+05	4.91E+05	5.00E+05	5.05E+05	5.04E+05	4.99E+05	4.88E+05	4.28E+05	4.53E+05	4.28E+05
λ=8																				
θ	5.00E+00	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90		
Tw J	5.49E+03	2.15E+04	4.69E+04	7.97E+04	1.17E+05	1.57E+05	1.95E+05	2.30E+05	2.56E+05	2.74E+05	2.79E+05	2.72E+05	2.52E+05	2.19E+05	1.75E+05	1.22E+05	6.27E+04	0.00E+00		
Tb J	1.51E+05	1.49E+05	1.46E+05	1.42E+05	1.37E+05	1.31E+05	1.24E+05	1.16E+05	1.07E+05	9.72E+04	8.67E+04	7.56E+04	6.39E+04	5.17E+04	3.91E+04	2.63E+04	1.32E+04	0.00E+00		
Tw-Tb J	-1.45E+05	-1.27E+05	-9.91E+04	-6.23E+04	-1.96E+04	2.61E+04	7.16E+04	1.14E+05	1.50E+05	1.76E+05	1.92E+05	1.96E+05	1.88E+05	1.67E+05	1.36E+05	9.59E+04	4.96E+04	0.00E+00		
Ω rad/sec	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	0.00E+00		
P Watts	-3.00E+05	-2.63E+05	-2.05E+05	-1.29E+05	-4.05E+04	5.39E+04	1.48E+05	2.35E+05	3.09E+05	3.64E+05	3.97E+05	4.05E+05	3.88E+05	3.45E+05	2.81E+05	1.98E+05	1.02E+05	0.00E+00		
V=15																				
TSR=λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
θ=60																				
Tw J	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05	6.12E+05
Tb J	1.42E+04	2.83E+04	4.25E+04	5.67E+04	7.09E+04	8.50E+04	9.92E+04	1.13E+05	1.28E+05	1.42E+05	1.56E+05	1.70E+05	1.84E+05	1.98E+05	2.13E+05	2.26E+05	2.41E+05	2.55E+05	2.69E+05	2.83E+05
Tw-Tb J	5.98E+05	5.82E+05	5.69E+05	5.55E+05	5.41E+05	5.27E+05	5.13E+05	4.99E+05	4.84E+05	4.70E+05	4.56E+05	4.41E+05	4.28E+05	4.13E+05	3.99E+05	3.85E+05	3.71E+05	3.57E+05	3.43E+05	3.27E+05
Ω	3.87E-01	7.74E-01	1.16E+00	1.55E+00	1.94E+00	2.32E+00	2.71E+00	3.10E+00	3.48E+00	3.87E+00	4.26E+00	4.65E+00	5.03E+00	5.42E+00	5.81E+00	6.19E+00	6.58E+00	6.97E+00	7.35E+00	7.74E+00
P Watts	2.31E+05	4.51E+05	6.61E+05	8.60E+05	1.05E+06	1.22E+06	1.39E+06	1.54E+06	1.69E+06	1.82E+06	1.94E+06	2.05E+06	2.15E+06	2.24E+06	2.32E+06	2.39E+06	2.44E+06	2.49E+06	2.52E+06	2.53E+06
λ=8																				
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90		
Tw J	1.23E+04	4.85E+04	1.06E+05	1.79E+05	2.64E+05	3.53E+05	4.40E+05	5.15E+05	5.77E+05	6.16E+05	6.28E+05	6.12E+05	5.66E+05	4.93E+05	3.94E+05	2.75E+05	1.41E+05	0.00E+00		
Tb J	2.26E+05	2.23E+05	2.19E+05	2.13E+05	2.06E+05	1.96E+05	1.86E+05	1.74E+05	3.08E+06	1.46E+05	1.30E+05	1.13E+05	9.58E+04	7.76E+04	5.87E+04	3.94E+04	1.98E+04	0.00E+00		
Tw-Tb J	-2.14E+05	-1.75E+05	-1.13E+05	-3.37E+04	5.86E+04	1.57E+05	2.54E+05	3.42E+05	9.91E+06	4.70E+05	4.98E+05	4.99E+05	4.71E+05	4.15E+05	3.35E+05	2.35E+05	1.21E+05	0.00E+00		
Ω rad/sec	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	3.10E+00	0.00E+00		
P Watts	-8.13E+06	-5.42E+05	-3.51E+05	-1.04E+05	1.82E+05	4.86E+05	7.87E+05	1.06E+06	1.29E+06	1.45E+06	1.54E+06	1.54E+06	1.46E+06	1.29E+06	1.04E+06	7.29E+05	3.76E+05	0.00E+00		
V=20																				
TSR=λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
θ=55																				
Tw J	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06	1.12E+06
Tb J	2.17E+04	4.34E+04	6.50E+04	8.67E+04	1.08E+05	1.30E+05	1.52E+05	1.73E+05	1.95E+05	2.17E+05	2.38E+05	2.60E+05	2.82E+05	3.04E+05	3.25E+05	3.47E+05	3.69E+05	3.90E+05	4.12E+05	4.34E+05
Tw-Tb J	1.09E+06	1.07E+06	1.05E+06	1.03E+06	1.01E+06	9.86E+05	9.65E+05	9.43E+05	9.21E+05	9.00E+05	8.78E+05	8.56E+05	8.35E+05	8.13E+05	7.9131`e5	7.70E+05	7.48E+05	7.26E+05	7.05E+05	6.83E+05
Ω	5.16E-01	1.03E+00	1.55E+00	2.06E+00	2.58E+00	3.10E+00	3.61E+00	4.13E+00	4.65E+00	5.16E+00	5.68E+00	6.19E+00	6.71E+00	7.23E+00	7.74E+00	8.26E+00	8.77E+00	9.29E+00	9.81E+00	1.03E+01
P Watts	5.65E+05	1.11E+06	1.63E+06	2.13E+06	2.60E+06	3.05E+06	3.49E+06	3.89E+06	4.28E+06	4.64E+06	4.98E+06	5.30E+06	5.60E+06	5.87E+06	6.13E+06	6.36E+06	6.56E+06	6.75E+06	6.91E+06	7.05E+06
λ=8																				
θ	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90		
Tw J	2.20E+04	8.61E+04	1.88E+05	3.19E+05	4.70E+05	6.28E+05	7.82E+05	9.19E+05	1.03E+06	1.09E+06	1.12E+06	1.09E+06	1.01E+06	8.76E+05	6.99E+05	4.89E+05	2.51E+05	0.00E+00		
Tb J	3.01E+05	2.98E+05	2.92E+05	2.84E+05	2.74E+05	2.62E+05	2.48E+05	2.32E+05	2.14E+05	1.99E+05	1.73E+05	1.51E+05	1.28E+05	1.03E+05	7.82E+04	5.25E+04	2.64E+04	0.00E+00		
Tw-Tb J	-2.79E+05	-2.12E+05	-1.04E+05	3.47E+04	1.96E+05	3.66E+05	5.34E+05	6.87E+05	8.12E+05	9.00E+05	9.43E+05	9.37E+05	8.79E+05	7.72E+05	6.21E+05	4.36E+05	2.25E+05	0.00E+00		
Ω rad/sec	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00	4.13E+00		
P Watts	-1.15E+06	-8.74E+05	-4.31E+05	1.43E+05	8.07E+05	1.51E+06	2.21E+06	2.83E+06	3.35E+06	3.72E+06	3.89E+06	3.87E+06	3.63E+06	3.19E+06	2.56E+06	1.80E+10	9.27E+05	0.00E+00		

```
3647    046      .
3648    046      .
3649    046      .
3650    046      .
3651    046      .
3652    046      .
3653    046      .
3654    046      .
3655    046      .
3656    046      .
3657    046      .
3658    046      .
3659    046      .
Turbine5/cal  Vevor Turbine cgs
KE = IW^2/2 I=MR^2/2 28/08/21 08/21
Revised 14/9/21 e^0.67 TbD
Estimate [Tw-Tb]xRadian/sec=Power
Radius of air cell say 1000 km 10^8
Tapered blade 2.3 to 11.5 cm
Radius is R+10 cm + 1.0 cm up to 52 cm
Chord from 2.30 to 11.504 +0.1769/cm
Get KE and Vortical energy throughput
Revised 3/10/21 mv^2 for kT in vort en
0400    096    LABEL
0401    082    R
0402    107    STO MEM
0403    048      0
0404    049      1
0405    119    print
0406    002    HALT
Cell height H say 5000 m 5.10^5 cm
0447    096    LABEL
0448    072    H
0449    107    STO MEM
0450    048      0
0451    050      2
0452    119    print
0453    002    HALT
Angular velocity W radians/sec
0494    096    LABEL
0495    087    W
0496    107    STO MEM
0497    048      0
0498    051      3
0499    119    print
0500    002    HALT
Mean air density g/cm^3
0541    096    LABEL
0542    068    D
0543    107    STO MEM
0544    048      0
0545    052      4
0546    119    print
0547    002    HALT
Area of wind blades A=LxC cm^2
0588    096    LABEL
0589    065    A
0590    107    STO MEM
0591    048      0
0592    053      5
3660    046      .
3661    046      .
3662    046      .
3663    046      .
3664    046      .
3665    046      .
3666    046      .
3667    046      .
3668    046      .
3669    046      .
3670    046      .
3671    046      .
0593    119    print
0594    002    HALT
Chord width of rotor blade cm
0635    096    LABEL
0636    067    C
0637    107    STO MEM
0638    048      0
0639    054      6
0640    119    print
0641    002    HALT
Angle of attack degrees
0682    096    LABEL
0683    079    O
0684    107    STO MEM
0685    048      0
0686    055      7
0687    119    print
0688    002    HALT
Length of rotor input L cm
0729    096    LABEL
0730    076    L
0731    107    STO MEM
0732    048      0
0733    056      8
0734    119    print
0735    002    HALT
Tip speed/wind ratio lambda
0776    096    LABEL
0777    083    S
0778    107    STO MEM
0779    048      0
0780    057      9
0781    119    print
0782    002    HALT
0783    096    LABEL
0784    088    X
Calculate max wind speed rw cm/sec
0825    040      (
0826    109    RCL MEM
0827    048      0
0828    049      1
0829    042      x
0830    109    RCL MEM
0831    048      0
0832    051      3
0833    041      )
Print max wind speed cm/sec
```

```
0782    002     HALT
0783    096     LABEL
0784    088     X
Calculate max wind speed rw cm/sec
0825    040       (
0826    109     RCL MEM
0827    048       0
0828    049       1
0829    042       x
0830    109     RCL MEM
0831    048       0
0832    051       3
0833    041       )
Print max wind speed cm/sec
0874    119     print
0875    107     STO MEM
0876    049       1
0877    048       0
Caculate KE
Calculate volume cm^3
0958    040       (
0959    112       pi
0960    042       x
0961    109     RCL MEM
0962    048       0
0963    049       1
0964    113     Square
0965    042       x
0966    109     RCL MEM
0967    048       0
0968    050       2
0969    041       )
Print vol   cm^3 of air cell, stow m11
1010    119     print
1011    107     STO MEM
1012    049       1
1013    049       1
1014    040       (
1015    109     RCL MEM
1016    049       1
1017    049       1
1018    042       x
1019    109     RCL MEM
1020    048       0
1021    052       4
1022    041       )
Print mass of air cell, stow m12 g
1063    119     print
1064    107     STO MEM
1065    049       1
1066    050       2
1067    040       (
1068    109     RCL MEM
1069    049       1
1070    050       2
1071    042       x
1072    109     RCL MEM
1073    048       0
1074    049       1
1075    113     Square
1076    047       /
1077    052       4
1078    041       )
Print I and stow in m13
1119    119     print
1120    107     STO MEM
1121    049       1
1122    051       3
1123    040       (
1124    109     RCL MEM
1125    049       1
1126    051       3
1127    042       x
1128    109     RCL MEM
1129    048       0
1130    051       3
1131    113     Square
1132    041       )
Print KE IW^2 of air cell, stow m14
1173    119     print
1174    107     STO MEM
1175    049       1
1176    052       4
Calculate KE, power of turbine volume
1217    040       (
1218    040       (
1219    040       (
1220    040       (
1221    109     RCL MEM
1222    049       1
1223    048       0
1224    042       x
1225    112       pi
1226    042       x
1227    109     RCL MEM
1228    048       0
1229    056       8
1230    113     Square
1231    041       )
1232    042       x
1233    040       (
1234    109     RCL MEM
1235    048       0
1236    052       4
1237    042       x
1238    109     RCL MEM
1239    049       1
1240    048       0
1241    113     Square
1242    041       )
1243    041       )
1244    047       /
1245    050       2
1246    041       )
Print rate kinetic energy P=DAv^30.6
1287    119     print
1288    042       x
1289    048       0
1290    046       .
1291    054       6
```

```
1292    041      )
Print adjustment for Cp
1333    119    print
Get vortical energy
1374    107    STO MEM
1375    049      1
1376    052      4
1377    040      (
1378    040      (
1379    040      (
1380    112      pi
1381    042      x
1382    109    RCL MEM
1383    048      0
1384    056      8
1385    113    Square
1386    041      )
1387    047      /
1388    040      (
1389    051      3
1390    042      x
1391    109    RCL MEM
1392    048      0
1393    053      5
1394    041      )
1395    041      )
1396    119    print
1397    114      1/x
1398    042      x
1399    109    RCL MEM
1400    049      1
1401    052      4
1402    041      )
1403    119    print
1404    040      (
1405    040      (
1406    040      (
1407    109    RCL MEM
1408    048      0
1409    049      1
1410    113    Square
1411    042      x
1412    109    RCL MEM
1413    048      0
1414    051      3
1415    041      )
1416    119    print
1417    042      x
1418    040      (
1419    050      2
1420    057      9
1421    042      x
1422    049      1
1423    046      .
1424    054      6
1425    055      7
1426    101      EE
1427    050      2
1428    052      4
1429    111      +/-
1430    041      )
1431    041      )
Print action ∂v
1472    119    print
1473    047      /
1474    049      1
1475    046      .
1476    048      0
1477    053      5
1478    052      4
1479    101      EE
1480    050      2
1481    055      7
1482    111      +/-
1483    041      )
Print nv
1524    119    print
1525    107    STO MEM
1526    056      8
1527    048      0
Get mv^2lnvv
1568    040      (
1569    040      (
1570    109    RCL MEM
1571    056      8
1572    048      0
1573    110    LOG e
1574    119    print
1575    042      x
1576    040      (
1577    040      (
1578    050      2
1579    057      9
1580    042      x
1581    049      1
1582    046      .
1583    054      6
1584    055      7
1585    101      EE
1586    050      2
1587    052      4
1588    111      +/-
1589    041      )
1590    042      x
1591    109    RCL MEM
1592    049      1
1593    048      0
1594    113    Square
1595    041      )
1596    107    STO MEM
1597    057      9
1598    049      1
Print mv^2
1639    119    print
1640    041      )
Print mv^2lnnv
1681    119    print
1682    041      )
1683    107    STO MEM
1684    056      8
```

```
1685    049      1
1686    040      (
1687    109    RCL MEM
1688    056      8
1689    049      1
1690    047      /
1691    109    RCL MEM
1692    056      8
1693    048      0
1694    041      )
Print hvv vortical quantum
1735    119    print
1736    047      /
1737    054      6
1738    046      .
1739    054      6
1740    050      2
1741    054      6
1742    049      1
1743    101      EE
1744    050      2
1745    055      7
1746    111      +/-
1747    041      )
Print v frequency
1788    119    print
1789    114      1/x
1790    042      x
1791    050      2
1792    046      .
1793    057      9
1794    057      9
1795    055      7
1796    057      9
1797    101      EE
1798    049      1
1799    048      0
1800    041      )
Print Y wavelength
1841    119    print
1842    040      (
1843    040      (
1844    049      1
1845    046      .
1846    050      2
1847    050      2
1848    053      5
1849    101      EE
1850    051      3
1851    111      +/-
1852    047      /
1853    040      (
1854    050      2
1855    057      9
1856    042      x
1857    049      1
1858    046      .
1859    054      6
1860    055      7
1861    101      EE
1862    050      2
1863    052      4
1864    111      +/-
1865    041      )
1866    041      )
Print N per cm^3
1907    119    print
1908    042      x
1909    109    RCL MEM
1910    056      8
1911    049      1
1912    041      )
1913    119    print
Print ergs/cm^3 or vortical pressure
1954    107    STO MEM
1955    056      8
1956    049      1
1957    040      (
1958    109    RCL MEM
1959    056      8
1960    049      1
1961    042      x
1962    109    RCL MEM
1963    049      1
1964    048      0
1965    041      )
Print power pe/cm^2
2006    119    print
2007    107    STO MEM
2008    056      8
2009    050      2
Get power for 3 blades and area
2050    040      (
2051    109    RCL MEM
2052    056      8
2053    050      2
2054    042      x
2055    109    RCL MEM
2056    048      0
2057    053      5
2058    042      x
2059    051      3
2060    041      )
Print vortical power total blade area
2101    119    print
2102    107    STO MEM
2103    056      8
2104    051      3
Get T
2145    040      (
2146    109    RCL MEM
2147    057      9
2148    049      1
2149    047      /
2150    040      (
2151    049      1
2152    046      .
2153    051      3
2154    056      8
2155    048      0
```

```
2156    054      6
2157    101      EE
2158    049      1
2159    054      6
2160    111      +/-
2161    042      x
2162    051      3
2163    041      )
2164    041      )
Print Torque/Temperature
2205    119    print
Get thrust per rotor blade @ o degrees
2246    040      (
2247    109    RCL MEM
2248    048      0
2249    053      5
2250    042      x
2251    040      (
2252    109    RCL MEM
2253    048      0
2254    055      7
2255    115    Sine
2256    041      )
2257    042      x
2258    109    RCL MEM
2259    049      1
2260    048      0
2261    041      )
Print AsinO x v = volume per sec
2302    119    print
2303    107    STO MEM
2304    049      1
2305    053      5
2306    040      (
2307    109    RCL MEM
2308    049      1
2309    053      5
2310    042      x
2311    109    RCL MEM
2312    048      0
2313    052      4
2314    042      x
2315    109    RCL MEM
2316    049      1
2317    048      0
2318    113    Square
2319    047      /
2320    050      2
2321    041      )
Print MAsinOv^2/2 KE impact on blade
2362    119    print
2363    107    STO MEM
2364    049      1
2365    054      6
Calculate SumMvsinO/sec force/blade
2406    040      (
2407    109    RCL MEM
2408    049      1
2409    053      5
2410    042      x
2411    109    RCL MEM
2412    048      0
2413    052      4
2414    042      x
2415    109    RCL MEM
2416    049      1
2417    048      0
2418    041      )
Print, stow total impact SumMvsinO/sec
2459    119    print
2460    107    STO MEM
2461    049      1
2462    055      7
Calculate Torque and anti-Torque-heat
2503    040      (
2504    040      (
2505    109    RCL MEM
2506    049      1
2507    055      7
2508    047      /
2509    109    RCL MEM
2510    048      0
2511    053      5
2512    041      )
2513    107    STO MEM
2514    049      1
2515    055      7
2516    041      )
2517    119    print
2518    107    STO MEM
2519    049      1
2520    055      7
2521    040      (
2522    050      2
2523    046      .
2524    051      3
2525    048      0
Print tip chord
2566    119    print
2567    107    STO MEM
2568    048      0
2569    054      6
Print tip width
2610    119    print
2611    109    RCL MEM
2612    048      0
2613    056      8
2614    107    STO MEM
2615    048      0
2616    048      0
2617    119    print
Get Tw and Tb
2658    096    LABEL
2659    084    T
2660    040      (
2661    040      (
2662    048      0
2663    046      .
2664    049      1
2665    055      7
```

```
2666    054      6
2667    057      9
2668    117    SUM MEM
2669    048      0
2670    054      6
2671    041      )
2672    040      (
2673    109    RCL MEM
2674    049      1
2675    055      7
2676    042      x
2677    109    RCL MEM
2678    048      0
2679    054      6
2680    041      )
2681    041      )
Print MvsinO/sec force/metre blade
2722    107    STO MEM
2723    049      1
2724    056      8
2725    040      (
2726    040      (
2727    050      2
2728    042      x
2729    109    RCL MEM
2730    048      0
2731    055      7
2732    041      )
2733    115    Sine
2734    042      x
2735    109    RCL MEM
2736    049      1
2737    056      8
2738    041      )
SMvsinOcos2O/metresec, action/area/sec
2779    107    STO MEM
2780    049      1
2781    056      8
Label Torque or rate of Action
Get SumRotor Windward Torque Tw
2862    040      (
2863    109    RCL MEM
2864    049      1
2865    056      8
2866    042      x
2867    040      (
2868    109    RCL MEM
2869    048      0
2870    048      0
2871    043      +
2872    049      1
2873    048      0
2874    041      )
2875    042      x
2876    051      3
2877    041      )
2878    041      )
Get cumulative Tw
2919    117    SUM MEM
2920    049      1
2921    057      9
Get integrated torque or rate action
SumMvsinOcos2O/metre sec;SumFxR=Tb
Get rotor reverse torque, action rate
3042    040      (
3043    040      (
3044    109    RCL MEM
3045    049      1
3046    048      0
3047    042      x
3048    109    RCL MEM
3049    048      0
3050    057      9
3051    041      )
3052    047      /
3053    109    RCL MEM
3054    048      0
3055    056      8
3056    041      )
Stow rotor W
3097    107    STO MEM
3098    050      2
3099    048      0
3100    040      (
3101    109    RCL MEM
3102    050      2
3103    048      0
3104    042      x
3105    040      (
3106    109    RCL MEM
3107    048      0
3108    048      0
3109    043      +
3110    049      1
3111    048      0
3112    041      )
3113    041      )
Get RW of blade at R and stow
3154    113    Square
3155    041      )
RVdO/sec
3196    107    STO MEM
3197    051      3
3198    048      0
3199    040      (
3200    040      (
3201    040      (
Get blade area dRcost segment at R
3242    109    RCL MEM
3243    048      0
3244    055      7
3245    099    Cosine
3246    042      x
3247    109    RCL MEM
3248    048      0
3249    054      6
3250    041      )
3251    042      x
Get inertia MR/sec, power MR^2Om^2
Use 2-D density D^2/3 for Tw output
```

```
3332    040      (
3333    109    RCL MEM
3334    048      0
3335    052      4
3336    091    Exponent
3337    048      0
3338    046      .
3339    054      6
3340    055      7
3341    042      x
3342    109    RCL MEM
3343    051      3
3344    048      0
3345    041      )
3346    041      )
3347    047      /
3348    109    RCL MEM
3349    050      2
3350    048      0
3351    041      )
3352    042      x
3353    051      3
3354    041      )
Get SumMV^2cosO=MRVdO/dt=MV^2 = HEAT
3395    117    SUM MEM
3396    050      2
3397    049      1
Print total back torque, action/sec
3438    039    Dec JPNZ
3439    084    T
3440    040      (
3441    109    RCL MEM
3442    049      1
3443    057      9
3444    119    print
3445    045      -
3446    109    RCL MEM
3447    050      2
3448    049      1
3449    119    print
3450    041      )
Print and stow 3-rotor net Torque
3491    119    print
3492    107    STO MEM
3493    050      2
3494    050      2
3495    040      (
3496    109    RCL MEM
3497    050      2
3498    050      2
3499    042      x
3500    109    RCL MEM
3501    050      2
3502    048      0
3503    119    print
3504    041      )
Print and stow net Power per windmill
3545    119    print
3546    107    STO MEM
3547    050      2
3548    051      3
3549    109    RCL MEM
3550    048      0
3551    054      6
3552    119    print
Clear cumulative memories for reruns
3593    048      0
3594    107    STO MEM
3595    048      0
3596    054      6
3597    107    STO MEM
3598    049      1
3599    057      9
3600    107    STO MEM
3601    050      2
3602    049      1
3603    107    STO MEM
3604    050      2
3605    050      2
3606    002    HALT
3607    046      .
3608    046      .
3609    046      .
3610    046      .
3611    046      .
3612    046      .
3613    046      .
3614    046      .
3615    046      .
3616    046      .
3617    046      .
3618    046      .
3619    046      .
3620    046      .
3621    046      .
3622    046      .
3623    046      .
3624    046      .
3625    046      .
3626    046      .
3627    046      .
3628    046      .
3629    046      .
3630    046      .
3631    046      .
3632    046      .
3633    046      .
3634    046      .
3635    046      .
3636    046      .
3637    046      .
3638    046      .
3639    046      .
3640    046      .
3641    046      .
3642    046      .
3643    046      .
3644    046      .
3645    046      .
3646    046      .
```

Python coding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}</style>\"))"
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    "import scipy"
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    "%matplotlib inline\n",
    "import matplotlib\n",
    "\n",
    "import matplotlib.pyplot as plt\n",
    "from matplotlib import cm\n",
    "import seaborn as sns\n",
    "matplotlib.rcParams['figure.figsize'] = (8, 8)"
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    "---\n",
    "## Some Functional Background"
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```

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    "### Anonymous Functions/Lambdas"
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    "In python, the keyword `lambda` creates an \"anonymous function\". This is a function (a subroutine) that doesn't have a name, but can be executed like any other function. Since it doesn't have a name, the only way to keep track of it is to assign it to a variable.\n",
    "\n",
    "Historically this comes from the lambda-calculus (Alonso Church in the 1930s), and it is fascinating and very expressive. Unfortunately, it isn't used as a fundamental design unit for code in many languages...\n",
    "\n",
    "Mathematica is an exception: it uses these so extensively that they have quite a compact shorthand (&, #1, #2, ...):  \n",
    "```lambda x, y, z: x + y * z``` in python can be written as ```#1+#2*#3&```, where `#1` corresponds to `x`, `#2` to `y`, etc.\n",
    "\n",
    "These lambdas are functions in the sense meant by mathematicians. They calculate an output from only the arguments:  \n",
    "$f(x, y, z) = x + y z$  \n",
    "is written as..."
   ]
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    "f(2, 3, 4)"
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    "Using a lambda to pass an integrand to an integration algo:  \n",
    "$$\\int_1^3 x^2 dx = \\left[\\frac{x^3}{3}\\right]_1^3 = \\frac{26}{3}$$"
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```

```
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    "scipy.integrate.quad(sq, 1, 3)"
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    "An important feature of anonymous functions in python is that they
can capture values from where they are defined."
   ]
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    "three_plus = lambda x: three + x\n",
    "three_plus(4)"
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    "It is considered very poor form to modify a captured value.\n",
    "Modifying captured values changes the meaning of an anonymous
function without redefining it, which makes code very difficult to read
and manage. Lots of lanugages prohibit this:"
   ]
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    "three = 5\n",
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```

```
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    "### Vectorising\n",
    "This example shows the `numpy.vectorize` function, which transforms one function to another function... It takes a scalar function as an argument and produces another function that does the same operation over a vector/array, elementwise:"
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    "sq_vec(np.array([1, 2, 3, 4, 1.414214]))"
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    "The `numpy.vectorize` function actually does this to all the arguments in a function, so that they behave according to the conventions of numpy arrays (there is a lot of detail about all that). I use this feature when I create the grid-based results for the 3D plot later on."
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    "f_vec = np.vectorize(f) # the function f from above: f(x, y, z) = x + y * z\n",
    "f_vec(x = np.array([1, 2, 3]),\n",
    "      y = np.array([4, 5, 6]),\n",
    "      z = np.array([7, 8, 9]))"
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```

```
    "---\n",
    "# The Equations\n",
    "The integrand functions take numerical values in all arguments.\n",
    "\n",
    "The torque functions require `c_fn` and `theta_fn` as inputs. These functions calculate the chord width and the angle of incidence at a given radius.\n",
    "\n",
    "For example, this `c_fn` defines a constant chord width of 1.3m (the radius argument is unused):  \n",
    "```lambda r: 1.3```  \n",
    "\n",
    "And this `theta_fn` defines the angle of incidence on a twisted blade that goes from `theta0` at the base, to `theta0` plus 10 degrees at the 8m tip:  \n",
    "```lambda theta0, r: theta0 + 10 * deg_to_rad * r / 8.0```"
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    "deg_to_rad = 2.0 * np.pi / 360.0\n",
    "rad_to_deg = 360.0 / (2 * np.pi)\n",
    "radsec_to_revmin = 60/(2*np.pi)\n",
    "\n",
    "rho = 1.225"
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    "### Integrals"
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```

```
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    "    return (rho\n",
    "            * c\n",
    "            * np.sin(theta)\n",
    "            * np.sin(2*theta)\n",
    "            * np.power(v, 2)\n",
    "            * r)\n",
    "\n",
    "def tb_integrand(r, omega, c, theta):\n",
    "    return (np.power(rho, 2/3)\n",
    "            * c\n",
    "            * np.cos(theta)\n",
    "            * np.power(r, 2)\n",
    "            * omega)\n",
    "\n",
    "def torque_windward(v, c_fn, theta_fn, R0, RL, theta0):\n",
    "    integrand = lambda r: tw_integrand(r, v, c_fn(r),
theta_fn(theta0, r))\n",
    "    res, _ = scipy.integrate.quad(integrand, R0, RL)\n",
    "    return res\n",
    "\n",
    "def torque_backward(omega, c_fn, theta_fn, R0, RL, theta0):\n",
    "    integrand = lambda r: tb_integrand(r, omega, c_fn(r),
theta_fn(theta0, r))\n",
    "    res, _ = scipy.integrate.quad(integrand, R0, RL)\n",
    "    return res"
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    "### Power"
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    "def power(v, omega, c_fn, theta_fn, R0, RL, theta0):\n",
    "    tw = torque_windward(v, c_fn, theta_fn, R0, RL, theta0)\n",
    "    tb = torque_backward(omega, c_fn, theta_fn, R0, RL, theta0)\n",
    "    return (tw - tb) * omega\n",
    "    \n",
    "def maximise_power(v, omega, c_fn, theta_fn, R0, RL):\n",
```

```
    "    ''' Maximises power output over theta0.\n",
    "    \n",
    "        Returns the maximum power output of the blade and the theta
that produces it\n",
    "    '''\n",
    "    # scipy implements only minimisation\n",
    "    # the max of a function's values occurs at the min of the
negative of the function's values\n",
    "    def minimisation_objective(theta0):\n",
    "        return -power(v, omega, c_fn, theta_fn, R0, RL, theta0)\n",
    "    res = scipy.optimize.minimize_scalar(minimisation_objective)\n",
    "    max_power = -res.fun # negate the minimum to recover the maximum
value\n",
    "    max_theta = res.x\n",
    "    return max_power, max_theta"
   ]
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    "# The Blades"
   ]
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    "# Rectangular blade definition\n",
    "L = 38.75\n",
    "r0 = 0.00\n",
    "rL = L\n",
    "c_fn = lambda r: 1.0\n",
    "theta_fn = lambda theta0, r: theta0"
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    "# Vevor blade definition\n",
    "#   \"Tapering from 2.30cm at the tip 62.0cm from the hub to 11.50cm
at the base,\n",
    "#    supported on a 10cm stalk to the hub\"\n",
    "\n",
    "L = 0.620\n",
```

```
    "r0 = 0.100\n",
    "rL = L\n",
    "c0 = 0.115\n",
    "cL = 0.023\n",
    "\n",
    "c_fn = lambda r: (r - r0) * (cL - c0) / (rL - r0) + c0\n",
    "theta_fn = lambda theta0, r: theta0"
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    "L = 38.750\n",
    "r0 = 0.000\n",
    "rL = L\n",
    "c0 = 3.025\n",
    "cL = 0.100\n",
    "\n",
    "c_fn = lambda r: r * (cL - c0) / L + c0\n",
    "theta_fn = lambda theta0, r: theta0"
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    "# GE twisted blade definition\n",
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    "r0 = 0.000\n",
    "rL = L\n",
    "c0 = 3.025\n",
    "cL = 0.100\n",
    "phiL = 15 * deg_to_rad # The twist at the tip of the blade, relative
to the base\n",
    "\n",
    "c_fn = lambda r: r * (cL - c0) / L + c0\n",
    "theta_fn = lambda theta0, r: theta0 + phiL * r / L"
   ]
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    "# turbine configuration\n",
    "λ = 8.0\n",
    "v = 15.0\n",
    "Ω = v*λ/L\n",
    "display(f'Ω = {Ω} rad/s')\n",
    "display(f'Ω = {Ω*radsec_to_revmin} rev/min')\n",
    "\n",
    "# varying the angle of incidence\n",
    "N = 80\n",
    "θ = np.linspace(5, 90, N) * deg_to_rad\n",
    "\n",
    "power_vec = np.vectorize(power)\n",
    "torque_windward_vec = np.vectorize(torque_windward)\n",
    "torque_backward_vec = np.vectorize(torque_backward)\n",
    "\n",
    "p = power_vec(v, Ω, c_fn, theta_fn, r0, rL, theta0=θ) # using an
array in the theta0 argument, so that the function produces an array
result\n",
    "tw = torque_windward_vec(v, c_fn, theta_fn, r0, rL, theta0=θ)\n",
    "tb = torque_backward_vec(Ω, c_fn, theta_fn, r0, rL, theta0=θ)\n",
    "\n",
    "max_p, θ_max_p = maximise_power(v, Ω, c_fn, theta_fn, r0, rL)"
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    "display(f'θ_max_p: {θ_max_p*rad_to_deg} deg')\n",
    "display(f'max_p: {3*max_p*1e-6} MW')"
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  },
```

```
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  "\n",
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  "ax.plot(θ*rad_to_deg, 3*tw*1e-6, linewidth=1.0)\n",
  "ax.plot(θ*rad_to_deg, 3*tb*1e-6, linewidth=1.0)\n",
  "\n",
  "ax.axvline(θ_max_p*rad_to_deg, linewidth=0.5, color='k')\n",
  "ax.axhline(3*max_p*1e-6, linewidth=0.5, color='k')\n",
  "\n",
  "ax.set_xlabel(r'Angle of incidence, $\\theta$ (deg)')\n",
  "ax.set_ylabel(r'Power, $p$ (MW)')\n",
  "ax.grid()"
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  "λ = 8\n",
  "idx_v = np.linspace(5, 20, N)\n",
  "idx_θ = np.linspace(5, 90, N)*deg_to_rad\n",
  "\n",
  "grid_v, grid_θ = np.meshgrid(idx_v, idx_θ)\n",
  "grid_Ω = grid_v*λ/L"
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     "fig, ax = plt.subplots(subplot_kw={\"projection\": \"3d\"})\n",
     "surf = ax.plot_surface(grid_v, grid_θ*rad_to_deg, 3*lambda8*1e-6,
cmap=cm.coolwarm, alpha=0.8,\n",
     "                       linewidth=0, antialiased=True)\n",
     "\n",
     "ax.view_init(15, 220)\n",
     "\n",
     "ax.set_xlabel(r'Wind speed, $v$ (m/s)')\n",
     "ax.set_ylabel(r'Angle of incidence, $\\theta$ (deg)')\n",
     "ax.set_zlabel(r'Power, $p$ (MW)')\n",
     "ax.grid()"
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In[ ]:=
```
Canvas[ □ ]
```

In[ ]:=
```
Radial action estimation of windward and leeward torques for wind turbines;
Tw and Tb and potential power output (P) in Watts
```

In[ ]:=
```
vV = 20; (* Wind velocity m/sec *)
dD = 1.225 (* Air density kg/m^3 *)
θ = 50 Degree (* Wind angle of incidence *)
cC = 5 (* Chord width m *)
lL = 80 (*  Blade length m *)
λ = 10 (* Tip speeed ratio RΩ/vV *)
Ω = λ * vV / lL (* Rotor angular velocity rad/sec *)
Nu  = 3 (* Number rotors *)
Tw =  dD cC Sin[θ] Sin[2 θ] vV^2 lL^2 / 2 (* Windward torque *)
Tb = dD cC Cos[θ] Ω lL^3 / 3 (* Leeward or Back torque *)
Pt = (Tw - Tb) Ω Nu
```

Out[ ]= 1.225

Out[ ]= 50 °

Out[ ]= 5

Out[ ]= 80

Out[ ]= 10

Out[ ]= $\frac{5}{2}$

Out[ ]= 3

Out[ ]= 5.91455×10^{6}

Out[ ]= 1.67982×10^{6}

Out[ ]= 3.17605×10^{7}

In[ ]:=

In[ ]:=

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