Response to Review:
Predictive and Stochastic Reduced Order Modelling of Wind Turbine Wake Dynamics

Thank you for the detailed review and the positive comments. We have addressed all comments in the updated manuscript and below, where review comments are given in black and response in blue. It should be noted that some minor updates have occurred. We have replaced Welch with a logarithmic-window moving average filter to reduce the spread of the spectra without loosing low frequency information. This improves the spectra, particular at low frequencies because it can better resolve lower frequencies and removes noise at high, which improves interpolation. We have also decided to plot Figure 11 with log-scale on the y-axis to provide clearer comparison of the histograms.

Response to Reviewer 1:

The authors present a predictive and stochastic reduced-order model based on proper orthogonal decomposition for wind turbine wake flow application. The authors did an excellent job explaining the methodology, which I expect will be used by others in the field. The paper should definitely be published. I just have a few minor comments and requests for clarifications.

Thanks your review and positive comments, please see our response below.

1. The introduction section does not provide recent state of art in using POD and/or ROM for wind energy applications. Also, some of the motivations of the current work are already resolved by other studies. For example" Cluster-based probabilistic structure dynamical model of wind turbine wake", and "Clustering sparse sensor placement identification and deep learning-based forecasting for wind turbine wakes". There have been many more studies that should be mentioned here.
We have of course only provided a general overview of what has previously been done in the field, and we are aware of these particular articles. The first does indeed attempt to solve the same challenge as our work, and we have now included it as a reference. We recognize that the Markov Chain and clustering approach yields stochastic realizations of the modal time series, but with a very coarse discretization due to the number of clusters (7). The method we are proposing is equivalent to using infinite clusters for the cluster-based probabilistic ROM. The transition matrix with infinite cluster should contain the same information as the cross-spectral density matrix. Therefore, it is not immediately evident from Ali et al. whether their proposed method is actually able to produce time series with the accurate cross-spectra properties, as such tests are not performed. As discussed and shown in our article, these properties are essential. Additionally, the suggested references does not include the predictive capability beyond the input simulation used to derive the reduced order model. We have also included a reference to Qatramez and Foti, 2022, which attempts to do similar things and have a good discussion on why this problem is difficult.

2. Can you provide details on the time resolution considered in evaluating the POD? How did the authors avoid the correlated features in POD calculation?

The other reviewer also asked about this, and we have now clarified the time resolution of the POD, where every 100th snapshot was used, i.e. every 10sec. The correlation between snapshots is essentially unimportant, because we effectively oversample the flow. We have paid closed attention at having long enough simulations to capture the low frequency components of the flow and enough snapshots in the global POD calculation to have all scales covered by the reduced order model, this can be confirmed by the fact that the last eigenvalues are basically zero. Furthermore, we wish to remind the reader that the important thing here is to obtain a representative and good basis, but the obtained basis will not be "optimal" in the classic POD sense. The proper temporal correlation is obtained afterwards as the flow is projected into this basis.

3. Figure 7: It is helpful to add the energy profile of the POD eigenvalues to the figure to visualize the trend of both.
Thanks for this comment. As addressed to the other reviewer, the eigenvalues are essentially nonsensical, and we believe such a figure would create more confusion than clarification of the method. We acknowledge that this is one of deviations from classical POD analysis.

4. Fig 8: Can you explain why cases CT=0.422 and CT=0.578 showed less correlated Eigenvectors than the other cases?
Figure 8 shows the correlation between the time series obtained from projecting the flow into the global basis. Our interpretation of the reduced correlation is as written in the Discussion that these intermediate flows are neither dominated by the atmospheric background flow nor the turbine generated wake.

5. Fig 13: Can you provide details on the energy content in these 50 modes?
The associated variance effectively yields the energy content of the modes, and hence the relevant information is shown in Figure 7. However, since the modal time series are correlated it is not possible to simply do a linear sum to obtain the energy content.

6. It is very practical of the authors to explain the limitations of the current work. Please explain your insight on how to develop the current approach to capture the non-Gaussian trend and if we have yaw/ tilt mechanisms.
Non-Gaussian time series can in principle be generated using an iso-probabilistic transfer function from the Gaussian to non-Gaussian distributions. However, as the validation shows and as discussed this might be of minor importance for power and load distributions.
The proposed method is completely generic, so it can also be applied for yawing or tilting turbines, where e.g. yaw angle would be the parameter in question for constructing a global basis. Yawing a turbine naturally leads to a change in CT, so it might require a two-parameter implementation. This is ongoing work in our group.