Review of *Evolution of Eddy Viscosity in the Wake of a Wind Turbine* by Ryan Scott et al.

Reviewer: M. Paul van der Laan, DTU Wind Energy

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The authors propose an analytical expression for the streamwise evolution of the eddy viscosity in a wind turbine wake using large eddy simulation (LES) cases and wind tunnel measurements. The proposed model is implemented in an engineering wake model developed for deflected wakes and its performance is tested against LES.

The article is relatively well written and include novel ideas. I enjoyed reading about the analytic form of the eddy viscosity of a single wake; I think this is useful to the wind energy community because it could be employed in other engineering wake models as well.

There are a number of major issues with the equations and assumptions, which are listed below; they need to be addressed before the article can be considered for publication in Wind Energy Science.

Main comments

1. There are a number of methods to determine an eddy viscosity from (LES) data. For example, one could also use the direct definition of the eddy viscosity from a two equation model (e.g. k-ε: $\nu_T = C_\mu \frac{k^2}{\varepsilon}$), or a more complex relation following higher order turbulence models, see for example [1]. Here, the difficulty is to determine ε, which some authors obtain from solving an ε transport equation on the reference data (so as a post processing step), see for example [2]. I think it make sense to add such a discussion to the introduction.

2. Your chosen method for obtaining an eddy viscosity from a data set is not well motivated and tested. If you use the Boussinesq approximation, then there are 5 equations but you only have one unknown: the scalar eddy viscosity. Hence, the system is over determined. You have chosen to only look at $\overline{u'w'}$ and then neglect the other equations and you motivate this in Lines 129-132: *Eddy viscosity values are obtained at each downstream location from the slope of a least squares linear regression between $S_{13}|_w$ and $\overline{u'w'}|_w$. The streamwise-vertical components of the Reynolds stress and rate of strain tensors are selected as they are of the greatest magnitude in the wake and are responsible for the majority of energy flux into a wind plant (Porté-Agel et al., 2020; Scott et al., 2020).* I agree that the shear-stresses are the main contributors to the wake recovery, however, $S_{13}|_w$ does not have to the largest component, it can also be the lateral Reynolds-stress $\overline{u'v'}|_w$, depending on the inter wind turbine spacing in the wind farm, see for example [3]. In a recent work of my own [4], I showed/visualized that the peaks of the shear stresses indicate how much wake recovery takes places in the lateral and vertical directions. You could look at your own LES and wind tunnel data to see which is dominant one. Alternatively, you could also write the Boussinesq hypothesis in polar coordinates and then you could choose to only use the equation for the shear stress in the radial direction for calculating the eddy viscosity from the data. This also makes sense for your application where the employed engineering wake models uses axi-symmetry. If you choose to stick with the current method, you should at least show how large the errors are from the unused equations of the Boussinesq hypothesis, especially the one for the lateral shear stress.

3. If you want make general conclusions about the proposed analytical model of the eddy viscosity then I would recommend to investigate aligned single wake wind turbines first using a range of thrust coefficients and turbulence intensities, since these two parameters are the main ones. I am aware that misaligned turbines will typically have lower thrust coefficients, but you will also have a lateral thrust component that changes the wake shape. (Atmospheric stability is also important and could be added in future work.)

4. What is a Hybrid wake model? In the introduction you write that a Hybrid wake model has the computational efficiency of an analytic/engineering wake model. To me that sounds like a Hybrid wake model falls into the class of engineering wake models. In my personal opinion, any model that does not solve a momentum equation iteratively (i.e. including pressure gradients) is a low fidelity / engineering wake model.

5. Lines 34-36: You write *Alternatively, constant eddy viscosities can be modeled with a scalar function tuned to the turbulent production and dissipation of calibration flow (Van Der Laan et al., 2015).* Note that we are not modeling a constant eddy-viscosity in the work that you refer to. We actually use a three-dimensional scalar eddy viscosity similar to the standard k-ε model. The model is then improved by multiplying the eddy viscosity with a scalar function that is dependent on the local flow. Effectively, this scalar function limits the eddy viscosity in regions where the flow is far from its equilibrium, i.e. the near wake. The model can also be interpreted a turbulence length scale limiter, see for example [5]. In theory, you could use the k-ε-f_P definition to determine the eddy viscosity, which should lead to better prediction of ν_T in the near wake, but it could be future work to test other forms of ν_T. (A small note: my is spelled as *van der Laan.*)

6. Equations (2), (5)-(9) are not complete and should include a term with the turbulent kinetic energy, k:

$$\overline{u_i'u_j'} = \frac{2}{3}k\delta_{ij} - 2\nu_T S_{ij} \tag{1}$$

   If you aim to neglect k you should mention this and motivate reader that this makes sense. Later on you calculate the eddy-viscosity from only $\overline{u'w'}$, so I understand where omitting k comes from. You could move the motivation from that part (Section 4.1) to theoretical part (Section 2). In addition, I can think equations (7) and (8) can be removed, because the previous equations already define everything.

7. Line 77: You mention that $S_{ij}|_B$ is assumed to be constant in the fully developed ABL. However, even for a simple neutral surface layer following a logarithmic law we get that $S_{ij}|_B$ is far from constant and scales with $1/z$: $S_{ij}|_B = 1/2u_*/(\kappa z)$. If you aim to only look at one height, e.g. hub height, then I can follow your assumption but this does seem to be case in the remaining part of the article (i.e. Figure 4).

8. Line 81: You mention *Thus our efforts focus on modeling $\nu_{T,w}$ in the range where $S_{ij}|_w > 0$.* However, $S_{ij}|_w > 0$ can also be negative in the wake. (That is why $\overline{u'v'}$ has both positive and negative values in the wake.)

9. Section 3.2, LES setup. It is not clear to me what kind of neutral ABL is simulated. Is this a pressure-driven ABL or is this an ABL including an inversion and Coriolis forces? The first type of ABL should be Reynolds number independent for a fixed ratio of the domain height L_z and roughness length, z_0, i.e. z_0/L_z is constant. In that case, the two LES cases mainly differ in wind turbine model if you have scaled the ABL inflow such z_H/L_z is constant. I expect the thrust coefficient to be main difference between the two LES cases, it would be helpful to report the mean thrust coefficient value for both of them. The thrust force distribution could also play a role in the near wake but is not expected to have a large impact on the far wake; a similar argument can be applied for the ground clearance, z_H/D and other design parameters.

10. Line 160: you mention *At extreme downstream distances, x/D, the eddy viscosity hypothesis no longer holds as both $S_{13}|_w$ and $\overline{u'w'}|_w$ are near zero.* I do not agree, the eddy viscosity hypothesis holds better for a flow that is in near equilibrium (the far-far wake or the background flow) with respect to the (near) wake). I guess your main point refers to the regression method not being able to calculate an eddy-viscosity with the proposed method. Hence this part needs some rewriting.

11. Equation (11) and (12): I do not understand where Equation (11) comes from, it is not the Boussinesq hypothesis (without k) because you lack the gradients and the minus sign. In addition, you could arrive at Equation (12) in a more simply way by dimensional analysis since we have that the unit of ν_T is m^2/s thus one can write $\nu_T = l_s U_s$. In addition, your chosen U_s is based on the far wake of 1D momentum theory, which I think you should mention.

12. Lines 258-259: You conclude to have investigated multiple inflow conditions, but you mainly used one LES ABL inflow as the wind tunnel wake data was not sufficient to capture the entire wake.

13. Conflict of interest: I guess this can be removed as Jens Sørensen is the editor. It should not be a problem if a coauthor is also an editor in the journal, as long as he or she is not the handling editor.

14. I think you author contribution statement is too vague. Please clarify who did what.

Minor comments

1. You sometimes write *LES simulation*. This means that you write the word *simulation* twice so I would suggest to remove the second word.

2. Equation (1): You forgot to mention that you assume an incompressible flow.

References

[1] M. Baungaard, S. Wallin, M. P. van der Laan, and M. Kelly, "Wind turbine wake simulation with explicit algebraic reynolds stress modeling," *Wind Energy Science*, vol. 7, no. 5, pp. 1975–2002, 2022. [Online]. Available: https://wes.copernicus.org/articles/7/1975/2022/

[2] M. Schmelzer, R. P. Dwight, and P. Cinnella, "Discovery of algebraic reynolds-stress models using sparse symbolic regression," *Flow, Turbulence and Combustion*, vol. 104, no. 2, pp. 579–603, 2020. [Online]. Available: https://doi.org/10.1007/s10494-019-00089-x

[3] J. Meyers and C. Meneveau, "Flow visualization using momentum and energy transport tubes and applications to turbulent flow in wind farms," *Journal of Fluid Mechanics*, vol. 715, p. 335–358, 2013. [Online]. Available: https://doi.org/10.1017/jfm.2012.523

[4] M. P. van der Laan, M. Baungaard, and M. Kelly, "Brief communication: A clarification of wake recovery mechanisms," *Wind Energy Science Discussions*, vol. 2022, pp. 1–8, 2022. [Online]. Available: https://wes.copernicus.org/preprints/wes-2022-56/

[5] M. P. van der Laan and S. J. Andersen, "The turbulence scales of a wind turbine wake: A revisit of extended k-epsilon models," *Journal of Physics: Conference Series*, vol. 1037, no. 072001, p. 1, 2018. [Online]. Available: https://doi.org/10.1088/1742-6596/1037/7/072001