methods were applied, or their application was directed, by the methods' originators. Our objective is to help the reader make a more informed optimization method selection by clearly comparing the pros and cons of a range of methods and presenting results from each method on a shared wind farm layout optimization case study with reasonable complexity and a provided objective function. We also hope to provide a set of high-quality wind farm layout optimization results to serve as benchmarks for other optimization methods. In the remainder of this paper we present our methods, including a description of the case to be optimized, the wind farm simulation approach, and a detailed description of the eight optimization methods included. We then provide results of the case study and optimization method analysis. We conclude with a brief summary and discussion of future work.

2 Methods

As mentioned in the introduction, this paper describes eight different strategies used to optimize the same wind farm layout. The objective of this case study was to maximize the annual energy production (AEP) of a wind farm, based on the Borssele III and IV wind farms, by optimizing the placement of 81 wind turbines. The turbines are 10 MW machines with 198 m rotor diameters based on the IEA 10 MW reference wind turbine (Bortolotti et al., 2018). The wind farm boundary for this case study was split into five discrete regions, shown in Fig. 1. The presence of unconnected regions in the wind farm boundary can be challenging when an algorithm requires a continuous objective function or derivatives. The wind turbines can be placed in any of the five regions of the wind farm, but not between them, making the problem inherently discontinuous and non-differentiable.

We used a simple Gaussian wake model based on Bastankhah's Gaussian wake model (Bastankhah and Porté-Agel, 2016), and presented in the IEA case study 3 and 4 announcement documents (Baker et al., 2021), to calculate wind speeds at each turbine in the wind farm.

$$\frac{\Delta V}{V_\infty} = \begin{cases} \left[1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/d^2}}\right] \exp\left(-0.5\left[\frac{\Delta y}{\sigma_y}\right]^2\right), & \text{if } \Delta x > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where ΔV is the velocity deficit, V_∞ is the wind velocity without wake losses, $C_T = 8/9$ is the constant thrust coefficient; $d = 198$ is the rotor diameter; Δy is the distance from the center of the wake to the point of interest perpendicular to the wind direction; Δx is the distance from the turbine generating the wake to the point of interest in the wind direction; and σ_y controls the width of the wake. The value of σ_y is calculated as

$$\sigma_y = k_y \Delta x + \frac{d}{\sqrt{8}}, \tag{2}$$

where k_y is a tuned variable based on turbulence intensity. We used $k_y = 0.0324555$ based on a turbulence intensity of 0.075 (Niayifar and Porté-Agel, 2016; Baker et al., 2021). The individual wake calculations were combined using the square-root-of-

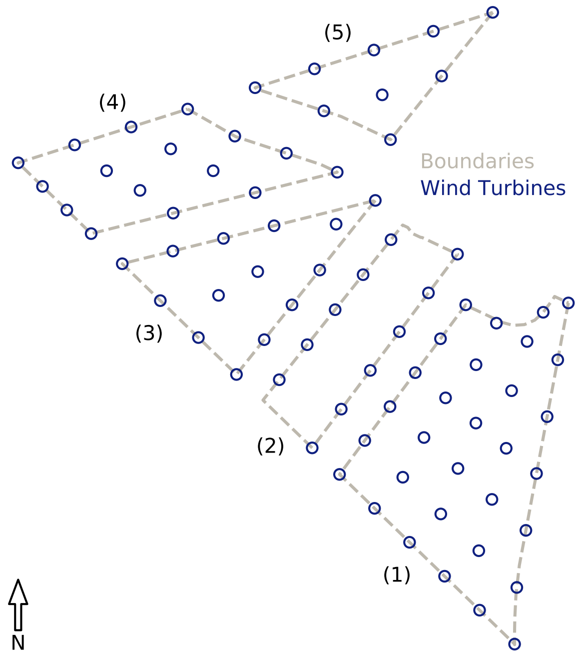

Figure 1. An overhead view of the wind farm used for the case study, including the provided example wind farm layout. Numbers in parentheses indicate region numbers. Wind turbine markers' diameters are the rotor diameters.

the-sum-of-the-squares method (Katic et al., 1986).

$$\left[\frac{\Delta V}{V_\infty}\right]_{total} = \sqrt{\sum_{k=1}^{N_T}\left[\frac{\Delta V_k}{V_\infty}\right]^2}, \tag{3}$$

where N_T is the number of wind turbines.

The objective function code was provided in the Python programming language, but some authors chose to re-implement the code in a different language. The wind resource, shown in Fig. 2, was divided into 360 different wind direction bins, and the wind speeds were assumed to follow a Weibull distribution, with 20 speed samples per wind direction. We increased the number of wind directions from the wind rose given in the original case study documents to make the problem more realistic. A more complete description of the case study can be found in Baker et al. (2021).

We compared the results of the optimization algorithms using a range of metrics in an attempt to capture some of the trade-offs between the algorithms. The simplest comparison was based on the objective function, AEP, calculated as

$$AEP = \left[\frac{\text{hours}}{\text{day}}\right]\left[\frac{\text{days}}{\text{year}}\right]\sum_{i=1}^{N_D}\sum_{j=1}^{N_S} f_{ij}\sum_{k=1}^{N_T} P_{ijk}, \tag{4}$$

where the power of each turbine k for each wind direction i and speed j is represented by P_{ijk}; the probability of a given wind speed and wind direction combination is given by f_{ij}; N_D and N_S represent the number of wind directions and wind

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