Response to Reviewer 1

Many thanks for your detailed comments and questions regarding our Brief Communication. In this response, we have included your comments/questions (framed, black font, italics) followed by our response (red font) to each point raised. When references are used, they can be found in the submitted manuscript or full citations are given in this response.

When preparing our response we noted a typo in Table 2 in the paper. The value of the proportional gain k_p must be changed from 3e-6 s/rad to 3e-2 s/rad.

> *"This paper considers the optimization of wind turbine torque controllers that track an optimal TSR value for below-rated load operation. Due to wear/tear/ fouling the properties of the turbine rotor change over time, possibly leading to a shifted location of the optimal TSR, leading to suboptimal operation when this parameter remains nominal and uncalibrated. A novel LP-PIESC scheme is proposed to calibrate the TSR setpoint value.*
>
> *The motivation of the paper is relevant, although numerous other works have been done in this field, which should be included and acknowledged in the introduction. Furthermore, I think that the major contribution of this paper is the LP-PIESC scheme, which supposedly can provide faster (instantaneous?) convergence. First, the new ESC scheme is not well described, leaving many questions about its working principles. Second, the tuning procedure of the scheme is not explained (only Appendix A elaborates on the excitation frequency). Third, the instantaneous convergence results are questionable: there is no excitation before the convergence, so how is the gradient obtained? Also, please explain how it is possible that there is instantaneous convergence; to me, this seems impossible without prior knowledge of the optimal setpoint value. Overall, I think this work needs significant improvements before it can be considered for publication in WES."*

The aim of our work is to provide numerical evidence of the existence of a class of extremum seeking algorithms (LP-PIESC) that can quickly identify optimal TSR despite changes in average wind speed and turbulence intensity (TI) for situations where TSR values have shifted to unknown values. We heavily rely on existing literature on extremum seeking control without presenting rigorous mathematical proofs.

The LP-PIESC algorithm has two main components: an identifier for parameter estimation and a PI controller using one of the the estimated parameters. Both components have been described in great detail already; see, for example, Guay and Dochain (2017). The parameter estimation in this reference has roots in prior work such as Guay and Dochain (2015), "A time-varying extremum-seeking control approach," Automatica, 51. Thus, there is already available literature to understand the working principles of this algorithm. Note that our WES manuscript is a "Brief communication." Thus, we must rely on the open literature for existing work. Having said that, we are happy to provide some intuition behind the parameter estimation algorithm to facilitate understanding of the working principles of the PIESC. The intuitive arguments given below are based on the work of Guay, M., and Dochain, D. (2015).

Let us consider a simpler static optimization problem, where the goal is to minimize an instantaneous cost $J(u)$ by proper choice of control u. Let y denote the instantaneous measurement of $J(u)$. That is, $y = J(u)$. Now take the time derivative of y to get

$$\dot{y} = \frac{\partial J}{\partial u}^T \dot{u} = \phi^T \theta \quad \text{(a)}$$

where $\phi = \dot{u}$ is the regressor and $\theta = \partial J/\partial u$ the time varying gradient we seek to identify.

Let $\hat{\theta}$ denote the estimate of θ at time t. The algorithm to estimate $\hat{\theta}$ belongs to a class of prediction error methods for problems with time-varying parameters. The proposed predictor $\dot{\hat{y}}$ for the output dynamics $\dot{y}$ is given by

$$\dot{\hat{y}} = \phi^T \hat{\theta} + Ke + c^T \dot{\hat{\theta}} \quad \text{(b)}$$

where $e = y - \hat{y}$ is the output predication error (y is measured), K is a constant (scalar) gain to drive the prediction dynamics $\dot{\hat{y}}$ with the output prediction error e and c^T is a filtered regressor obtained by low pass filtering ϕ; i.e.,

$$\dot{c}^T = -Kc^T + \phi^T \tag{c}$$

where K is the scalar gain introduced in (b). Note that $c^T\dot{\hat{\theta}}$ also drives the predictor dynamics $\dot{\hat{y}}$ to account for the time-varying nature of θ.

The parameter estimate $\hat{\theta}$ we seek is calculated from the following differential equations:

$$\dot{\hat{\theta}} = \text{Proj}(\Sigma^{-1}(c(e-\hat{\eta}) - \sigma\hat{\theta}), \hat{\theta}) \tag{d}$$

$$\dot{\Sigma}^{-1} = -\Sigma^{-1}cc^T\Sigma^{-1} + k_T\Sigma^{-1} - \sigma\Sigma^{-2} \; (\dot{\Sigma} = cc^T - k_T\Sigma + \sigma I) \tag{e}$$

$$\dot{\hat{\eta}} = -K\hat{\eta} \tag{f}$$

Guay and Dochain (2015, 2017), define the auxiliary variable $\eta := e - c^T\tilde{\theta}$, where $\tilde{\theta} = \theta - \hat{\theta}$ is the actual parameter error (with θ unknown). Note that $c^T\tilde{\theta} = e - \eta$. Thus, if we also want to drive the update differential equation (d) with information about the time-varying parameter error $\tilde{\theta}$, then using $e - \hat{\eta}$ in (d) seems reasonable.This argument is based on intuitive interpretations of variables only.

A rigorous proof of convergence to a neighborhood of an optimum using this parameter estimation algortihm coupled with a specific extremum seeking control law for u is given in Theorem 1 of Guay and Dochain (2015). It's proof is based on a Lyapunov argument. Let us now give an idea of the role of the matrix Σ and the projection operator Proj$(\cdot)$.

The matrix Σ^{-1} is the gain matrix for the parameter update law (d) This matrix has an update law (e) similar to the one in continuous-time least-squares with forgetting (Shaferman et al., European Jou. of Control, V. 62, Nov. 2021). In eqn. (d), the operator Proj$(\cdot)$ is a Lipschitz projection operator designed to ensure that the estimates are bounded within a fixed constraint set. This projection algorithm was implemented as discussed in Appendix E of (Krstic et al. (1995), "Nonlinear and Adaptive Control Design," 1st edition, John Wiley & Sons Inc.) and the constraint set adaptation was adopted as per Adetola and Guay (2011) ("Robust adaptive MPC for constrained uncertain nonlinear systems," Int. J. Adapt. Control Signal Process., 25, 155–167). Given the space limitations of a WES Brief Communication, it is not possible to provide all the equations. However, we could provide all the relevant references as done in this reply if a revised version is invited.

This approach from Guay and Dochain (2015) is then extended in Guay and Dochain (2017) using a PI controller for extremun seeking. This is the method we use in our Brief communication. In this case, there is dynamics between the control variable u and the cost function we seek to optimize. Thus, there is an extra parameter to model the dynamics of the instantaneous cost function y, which is parameterized by the differential equation

$$\dot{y} = \theta_0 + \theta_1(u - \hat{u}) = [1 \;\; (u-\hat{u})] * [\theta_0 \;\; \theta_1]^T = \phi^T\theta \tag{g}$$

where u and $\hat{u}$ are defined in equation (2) in our paper. Then we apply back calculation anti-windup to obtain u^s. This is done because we want to maintain the TSR within practical limits. The final TSR set-point λ_{sp} is obtained by rate limiting u^s to smooth the final set point. In equation (g) one may associate θ_0 with the dynamics of the system we seek to optimize and θ_1 with the gradient of the cost function. Note that now the parameter we seek to estimate is $\theta = [\theta_0 \;\; \theta_1]^T$ and the regressor is $\phi = [1 \;\; (u - \hat{u})]^T$. The remaining equations for the identifier are as shown in the text above and in Fig. 5 of the paper.

In a revised version of this paper we could eliminate Fig. 5 and instead give a clear block diagram of the solution with input P and output λ_{sp} (λ_{opt} in Fig. 1). Due to space limits we will also eliminate Fig. 1 and instead write down that our solution provides the TSR set-point value required by the NREL ROSCO controller. In addition, we could provide the equations (with brief explanation) for the parameter estimation algorithm in the text rather than inside the block diagram in Fig. 6.

To conclude the response to your opening comments, time series of key signals are given to enhance the understanding the working principle of the algorithm. First note that the complete algorithm has been implemented in Simulink with a step size of 0.0125 sec for discretization (i.e., we have 80 Hz sampling frequency). In this explanation, we focus on the case of eroded blade, where the TSR set point needs to move from 7.6 (for clean blade) to 8.4 for the eroded blade; i.e., a 0.8 increase in TSR (see Fig. 4 in the paper). As shown in the block diagram of Fig. 5, $\hat{\theta}_1$ is the only estimated parameter used by the PI controller in equation (2). The Simulink scope in Fig. A shows the time series of $\hat{\theta}_1$.[1] Only 1 sec of the simulation leading to Fig. 12 in the paper is shown, which represents about 80 samples. The two terms of PI controller are shown in Fig. A (right-hand side). Note that the proportional term reaches 0.8 in one sample after the LP-PIESC is turned on at 500 sec. Then the proportional term decays to zero and the integral term $\hat{u}$ grows to 8.4 asymptotically after multiple samples. The estimated new TSR is the sum of the proportional and integral terms. We rate limit the result to obtain the final TSR set-point to ROSCO as shown in Fig B. The final value is above 8.4 due to the addition of the sinusoidal dither. Note the accelerating effect of adding the proportional term (entry 1-2 in Fig. A) relative to using the integral term only (entry 2-2 in Fig. A). While we do not have a formal proof, this effect resembles the increase in bandwidth that a PI controller can offer over a P-only controller.

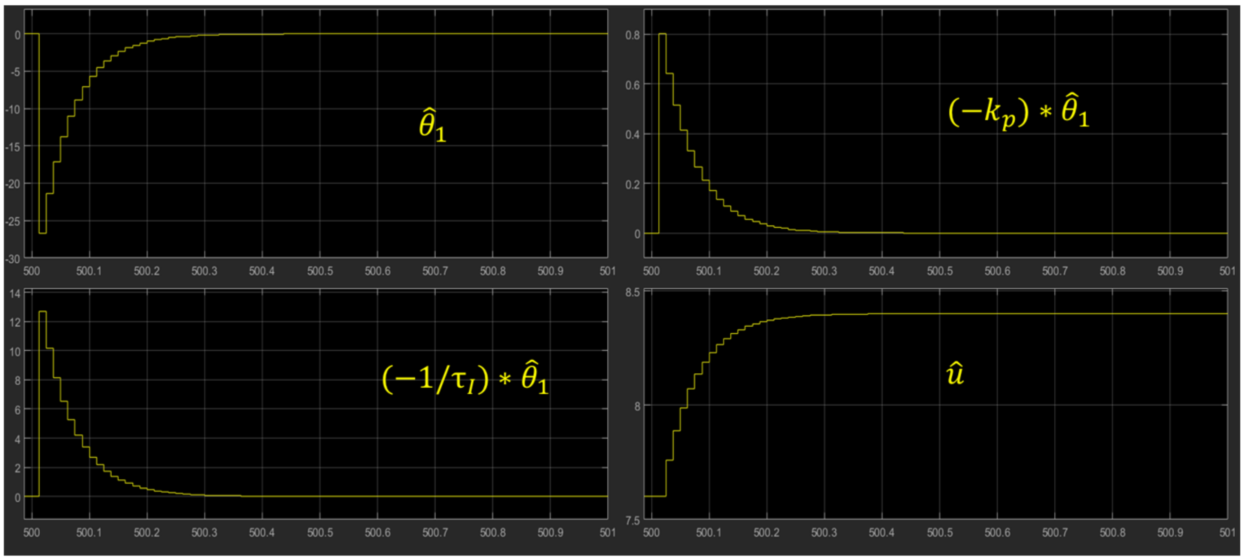


Figure A: Simulink Scope for LP-PIESC signals – Eroded blade (U=8m/s, TI =10%).

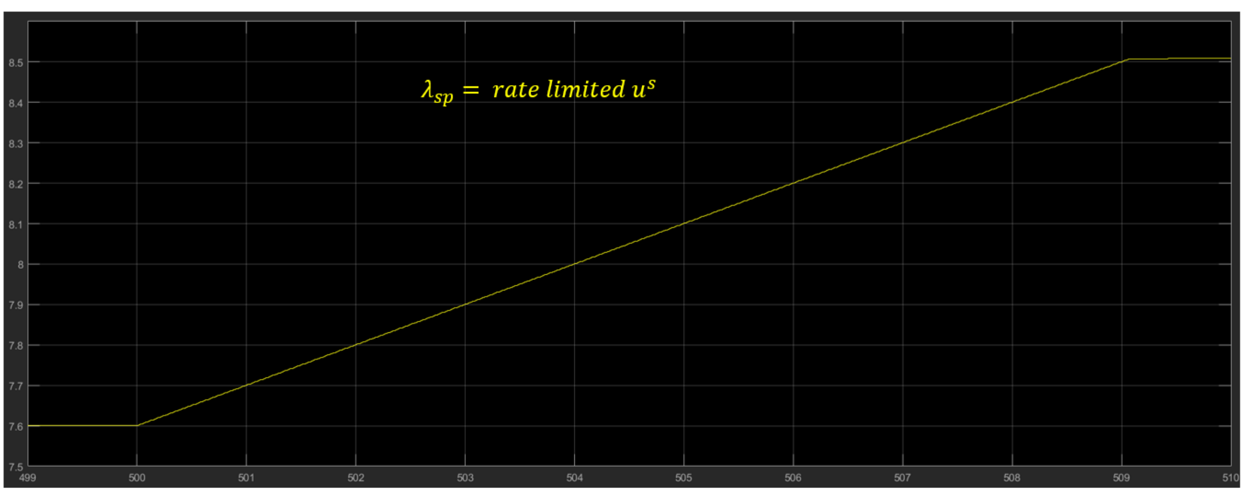


Figure B: Simulink Scope showing TSR command to the ROSCO controller – Eroded blade (U=8m/s, TI =10%).

[1] If one thinks of $\hat{\theta}_1$ as the relevant gradient, note that it is negative because the algorithm actually minimizes the sign inverted performance function. This can be done because the max of a function f can be obtained by minimizing $-f$.

> *"- Announce in the abstract what type of wind turbine controller you are assuming.*
> *- State the significance of using LP-PIESC as compared to regular ESC, as this is the main contribution of the paper."*

- Please note that the type of controller used (ROSCO) is mentioned in the last sentence of the abstract. This turbine controller is as developed by Abbas et al. (2022), which is referenced in the technical brief.
- Your suggestion to "state the significance of using LP-PIESC as compared to regular ESC" can be found in Kumar and Rotea (2022) for a different torque and pitch turbine controller. This reference actually shows a comparison between LP-PIESC and the conventional LP-ESC. Thus, since our technical brief has no comparison, we would prefer to keep this statement in the Introduction, line 52.

> *Introduction:*
> *- You write: "The LP-PIESC has been shown to be a faster variant of the traditional perturbation-based ESC (Kumar and Rotea, 2022)." This might be interpreted that you do not need perturbation using LP-PIESC. You would still need a perturbation to estimate the gradient, right?*
> *- A more elaborate literature study and acknowledgment of the works in the field of set-point/model/controller calibration should be included.*

- The dither signal ensures the persistence of excitation (PE) condition in assumption 5 of Guay and Dochain (2017). This assumption is used to prove the convergence of the PIESC algorithm to a neighborhood of the unknown optimum using a Lyapunov stability argument. While our dither is of low frequency, the PE condition is satisfied after turning the dither on as shown by the following integral
$$\int_{500s}^{501s} c(\tau)c^T(\tau)d\tau = \int_{500s}^{501s} = \begin{bmatrix} 2.0250e-01 & 1.4631e-03 \\ 1.4631e-03 & 1.4856e-05 \end{bmatrix} \geq (4.3e-06) * I \tag{h}$$
where c is the filtered regressor.
- Please note that our paper is a Brief Communication; thus, an elaborate literature study and acknowledgment of the works in the field of setpoint/model/controller calibration would be difficult to include. This is particularly notable in our case because we already need to reduce the length of the original submission. We can, however, include a couple of the top most cited papers in TSR estimation if requested by the associate editor. Of course, we welcome any suggestions you may have concerning salient archived journal papers in this area.

> *Background:*
> *- Is it a valid assumption to have a precise measurement of the rotor effective wind speed?*
> *- Fig 1: In the figure you indicate that $\hat{v}$ comes from a wind speed estimator, while in the text you say something different.*
> *- Fig 3: There seem to be few data points for Cd-curves. Can you increase the resolution?*

- We assume you are referring to the statement after line 85 in the submitted manuscript, where we explain the use of the rotor disk average (RtVAvgxh) wind speed calculated by OpenFAST and not from the wind speed estimator inside ROSCO. We understand that the ROSCO wind speed estimator makes use of the power curve, which changes as the blades degrade. Thus, ROSCO may not calculate the correct rotor speed reference from the TSR set-point unless the power curve is modified to account for degradation/erosion. While our assumption may not be most practical, it is necessary to demonstrate the LP-PIESC by avoiding unknown complexities of ROSCO's wind speed estimator (which is not the main purpose of our study).

- We introduced Fig. 1 (in paper) as a graphical representation of the ROSCO architecture, as per Abbas et al. (2022). The reason for using the rotor disk average (RtVAvgxh) wind speed has been explained already. To reduce the length of the Brief Communication, it is likely that we would need to remove Fig. 1 and add a few sentences with the key aspects of ROSCO used in our study.

- The resolution of lift and drag coefficients for the clean blade as well as the contaminated and eroded blade are shown with markers in Figure C below. The aerodynamic coefficients for clean blade were obtained from the NREL OpenFAST package of the 5MW reference turbine available at `https://github.com/OpenFAST/r-test/blob/main/glue-codes/openfast/5MW_Baseline/Airfoils/NACA64_A17.dat`. These were used to run simulations to obtain C_p for every 0.2 change in TSR (shown in Fig. D). If this resolution of C_p is considered low, we can increase it but we cannot increase the resolution of the lift and drag coefficients as those are the only points for which we have available data for the NACA64 airfoil. We are happy to replace the plots in the Brief Communication with the ones shown below.

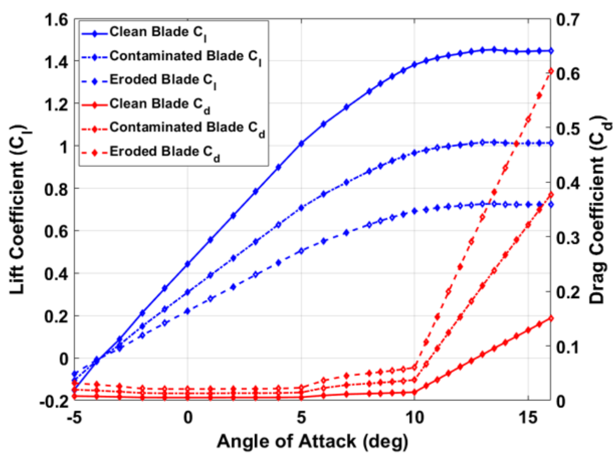


Figure C: Change of lift and drag coefficients for NACA64 airfoil.

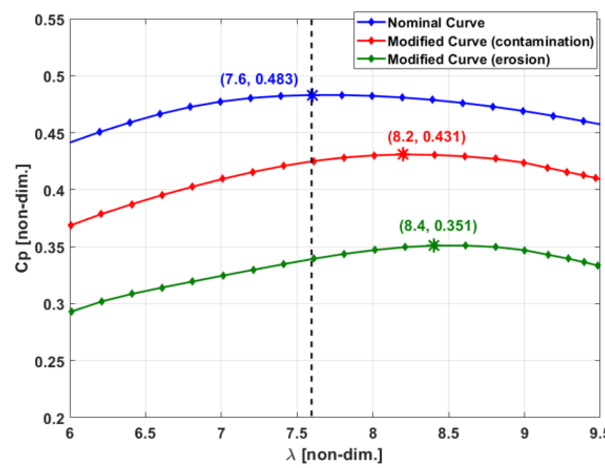


Figure D: Nominal and modified (due to contamination and erosion of blades) $C_\mathrm{P} - \lambda$ curve for NREL 5-MW wind turbine reference model.

Section 2

- You state that θ_1 is proportional to the gradient. It could be better explained that in (2), you can observe that this quantity is subject to a proportional action with kp and an integral action with saturation capabilities.

- What estimation problem are the parameters in θ a result of? What do they represent, and in which context?

- The paper does not describe the rationale behind the gradient estimation scheme. As this is the major contribution of the paper, you should have a proper description of its working principles.

- The statement we made on $\hat{\theta}_1$ is intuitive but not rigorous. Per equation (g), $\hat{\theta}_1$ estimates a parameter used to model the influence of the control increment (i.e., $u - \hat{u}$) on the dynamics of the cost function. From this equation, the "gradient interpretation" can be understood, at least intuitively.

- The actual estimation problem solved is for the 2D vector $\theta^T := [\theta_0, \theta_1]^T$ in equation (g). These parameters are used to parametrize the time derivative of the performance index y (e.g., log of power after moving average filtering) using

$$\dot{y} = \theta_0 + \theta_1(u - \hat{u}) \tag{i}$$

which forms the basis for the parameter estimation algorithm in the box of Fig. 5. Key elements of this algorithm where intuitively explained at the beginning of this reply. Further details can be found in Guay and Dochain (2017).

- We believe the introductory part of this reply cover the essentials of the LP-PIESC algorithm. Space permitting, we would be happy to include the estimation problem being solved, with identifier equations, in a revised Brief communication if requested.

Section 3

- Consider using a more state-of-the-art reference turbine model, like the IEA 15 MW turbine.

- Fig 5: The complete algorithm of finding θ is given in this figure in the large block, without any explanation. It is too complex to understand from a list of relations without explanation and justification!

- All Figures in the paper are given without a proper elaborate caption that allows for interpretation of the figure. Improve on this.

- Table 2: How did you arrive at these ESC parameters? Through trial and error or a systematic tuning procedure?

- In Appendix A, you provide justification for the dither frequency but not for the other values.

- You write: "The LP-PIESC converges to the new optimal tip-speed ratio almost instantaneously for all the cases." How is this possible? As far as I understand, you only estimate the gradient in the form of θ_1. Instantaneous convergence is only possible if you know how far you are from the optimum value, e.g., tuning the proportional gain to precisely the correct value. But this is just guessing, and maybe I am missing something. However, the paper does not clarify this aspect.

- Figure 8: How can the gradient be estimated without perturbation before 500 s? How is it possible to arrive at the optimal value instantaneously?

- It is unclear which variable you excite by dithering, is this λ_{sp}?

- This will be considered for future work. Currently, we do not have the resources to apply the LP-PIESC to the IEA 15 MW as the first author has moved to industry and this project has concluded.

- We agree with you that Fig. 5 is complex to understand. We will have one single figure with input P and output λ_{sp}. The parameter estimation algorithm can be given as separate equations as done in (Kumar and Rotea, 2022), and brief intuition behind these equations provided. Given that this reply can be included with the paper, the beginning of this reply should help readers gain additional insight behind the method. Please note that we cannot include the rigorous proofs given in Guay and Dochain (2017), but we can cite them.

- We feel we already have well detailed captions for all figures. Please let us know if there are any specific figure captions that would need update.

- Table 2: yes, all parameters in Table 2, except the dither frequency, are obtained by trial and error at 8 m/s wind speed and 10% TI for the **clean blade** (see line 250). Parameter tuning is an area of improvement for this algorithm, which is left for future research. Please note that due to the use of the log-of-power, once we calibrate parameters at one wind condition, the same parameters can be used at other wind conditions. We have observed this behavior in simulations (this technical brief, Kumar and Rotea (2022) and in wind tunnel experiments (Kumar et al. (2023), "Wind plant power maximization via extremum seeking yaw control: A wind tunnel experiment," Wind Energy, Vol. 26 (3))

- In a revised version of the paper we will state that all parameters, other than dither frequency, have been obtained by trial an error.

- Please refer to Figs. A and B in this reply, which we believe clarifies the point you raised on convergence.
- As shown in Fig. A, the parameter θ_1 ("the gradient") is being estimated after 500s. It takes around 0.4-0.5s for this parameter to converge to zero.
- The variable that is being excited is u - see eqn. 2 - which is the set point λ_{sp} for the TSR.

Response to Reviewer 2

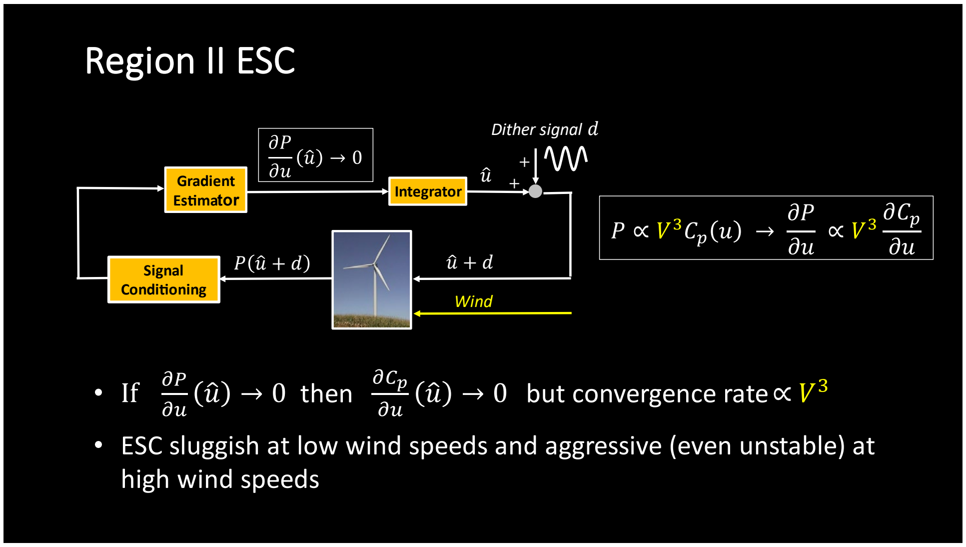
Many thanks for your detailed comments and questions regarding our Brief Communication. In this response, we have included your comments/questions (framed, black font, italics) followed by our response (red font) to each point raised. When references are used, they can be found in the submitted manuscript or full citations are given in this response.

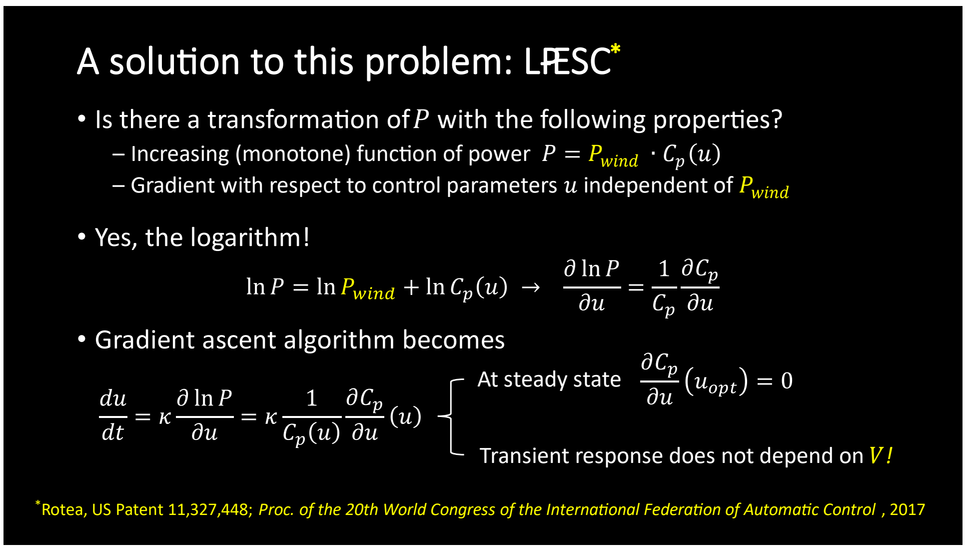
Before responding to your points, please note that in the response to reviewer #1, we provided explanations on the working principle of the LP-PIESC algorithm. There are several details, provided through "the lens of continuous time algorithms for estimating time-varying parameters," which is our case. We hope that the response to reviewer #1 plus specific answers to your points are satisfactory.

In addition, please note that there is a typo in Table 2 of the paper. The proportional gain k_p should be replaced by 3e-2 s/rad.

This paper presents an extremum seeking controller for optimizing a wind turbine controller's tip speed ratio set point. It's nice that the control scheme fits with an existing wind turbine architecture. However, the benefit of using the log of the power is unclear, and the algorithm seems to converge to the optimal solution too quickly without adequate explanation.

Our algorithm uses the log-of-power signal to determine the optimal TSR set-point. The significant advantage is that ln(P) = constant + ln(Cp) + 3*ln(V). Thus, for algorithms based on some form of gradient estimation, like ours, the gradient of log(P) depends only on Cp, which is exactly the quantity we seek to maximize. This use of the "log" transformation has huge implications because it eliminates the need to schedule control parameters on wind speed. That is, once you calibrate the any gradient-based algorithm for one wind speed, the same parameter values work at other wind speeds. Given that between cut-in and rated wind speeds, V^3 could change significantly, the use of log-of-power is highly advisable when adjusting control parameters (e.g., TSR, Torque Gain, Pitch Angle) to improve performance. The use and advantages of log-of-power have been explained and documented earlier in Rotea (2017) and Ciri, et al. (2019). In addition, we are providing two charts below that articulate this point.





The issue of quick convergence is addressed in the response to your major questions below and the reply to reviewer #1.

> *How does the algorithm converge to the optimal solution before a single dither signal cycle can compute the gradient? I think that justification, in wind energy terms, should be provided in this article.*

We do not have rigorous proof of rapid convergence for this specific application of LP-PIESC. Having said that, please see the response to reviewer #1, where we provide empirical evidence of convergence using a persistence of excitation condition like the one required in the main reference of our work Guay and Dochain (2017; eqn. (22)). The method we use for parameter estimation is not the conventional perturbation/demodulation method used in prior ESC algorithms, which is slower and may require a few full dither cycles to estimate unknown parameters. Instead, we use a method that is applicable to estimating and tracking time-varying parameters. This method, developed by Guay and co-authors, has similarities with continuous-time least-squares with forgetting (Shaferman et al., European Jou. Of Control, V. 62, Nov. 2021), which may not require a full cycle of the dither to converge if a persistence of excitation (PE) condition is met. This PE condition involves the positive definitiveness of a matrix obtained by integrating $c(t) * c(t)^T$, where $c(t)$ is the filtered regressor defined by equation (c) in the reply to reviewer #1, with $\phi^T = [1 \;\; u(t) - \hat{u}(t)]$ denoting the actual regressor (see eqn. (g) in the reply to reviewer #1), where $u(t)$ is the commanded TSR with no anti-windup compensation.

While we do not have formal proof, we believe that when the LP-PIESC algorithm is turned on, and the power (i.e., the input to the LP-PIESC) is fluctuating, the PE condition is met rather quickly, which might explain the rapid convergence of the parameter $\widehat{\theta_1}$. See Figs. A and B and eqn. (h) in reply to reviewer #1. In addition, please note that the Cp curves between 7.6 TSR (initial condition for the algorithm) and the new optimal TSR values shown in Fig. 4 in the paper are fairly simple concave segments with well-behaved slopes, which combined with the small TSR increase required to reach optimal values, suggest that rapid convergence may not unrealistic.

How exactly is the gradient estimated over time? What signals from the turbine are needed? The variables in Fig. 5 are not defined in the text. Can you show the gradient estimate over time?

The only signal we need from the turbine is the rotor power P (not the aerodynamic power), which is correlated with the turbine's electrical power output. The instantaneous rotor power is time-averaged with a moving average filter and then the natural log is applied. Fig. 5 is unclear and not consistent with Fig. 6. This situation will be corrected in a revised version of the paper. Now, with some abuse of notation, if we let $y(t) = \ln P(t)$, then this is the input signal to the PI-ESC algorithm in Fig. 5 (note that y is misplaced in the diagram; it should be after the natural log block).

The equations for estimating the parameter $\widehat{\theta_1}$, which can be thought as a gradient, are given in pages 1-3 in the reply to reviewer #1. The time series of $\widehat{\theta_1}$ is shown in Fig. A (reviewer's #1 reply) for the case of eroded blade.

It appears that the TSR set point reaches the "optimal" before the power coefficient or actual tip speed ratio changes in any measurable way. How is this possible? The bandwidth of the torque controller limits the actual TSR; how can this algorithm converge faster than the torque controller?

A change in the TSR or Cp may not be necessary for the algorithm to converge. As mentioned earlier, and in the response to reviewer #1, $\widehat{\theta_1}$ is the key parameter that needs to converge to determine the new optimal TSR. This parameter is used to parametrize the time derivative of the performance index (log-of-power in our case); see equation (g) in response to reviewer #1 and eqn. (8) and in Guay and Dochain (2017). While we do not have formal proof, we believe that when the LP-PIESC algorithm is turned on, and the power (i.e., the input to the LP-PIESC) is fluctuating, the PE condition is met rather quickly, which might explain the rapid convergence of the parameter $\widehat{\theta_1}$. Please note that all simulations in OpenFAST are sampling signals at 80 Hz (0.0125 sec sampling interval). In addition, the accelerating effect of the proportional term in the controller contributes to the fast convergence to the new optimal value for the TSR. One may see this effect by adding the (1,2) entry and the (2,2) entry in Fig. A in the reply to reviewer #1. The effect of adding the proportional term is like the increase in control bandwidth obtained when replacing a pure integral controller (as in several prior ESC algorithms) with a PI control law.

> *From cited work within this article, the authors claim that the log of the power allows the Cp to be maximized directly without requiring the wind speed. \frac{\del J}{\del u} = 1/Cp \frac{\del Cp}{\del u}. Doesn't the Cp in the denominator depend on the wind speed?*

The known approximations of Cp show that the power coefficient is a strong function of TSR and blade pitch angle but not necessarily the incoming mean wind speed V in isolation. See, for example, Carpintero-Renteria *et al*., "Wind turbine power coefficient models based on neural networks and polynomial fitting," IET Renewable Power Generation, Vol. 14, Issue11, August 2020, which contains a complete review of existing Cp models. As explained earlier in this reply, the main advantage of using log-of-power is the removal of V^3 from the gradient of the performance index we seek to maximize. Please refer to Rotea (2017) and Ciri, et al. (2019) to see how calibration of parameters (these papers do not use LP-PIESC) at one wind speed V works at any other wind speed below rated without the need for retuning algorithm parameters – this is a significant benefit of using the logarithm before processing the power signal.

> *In Fig. 8, there is a step change as soon as the algorithm is enabled, and then it seems to converge slowly to another point. How do you account for this behavior? Was an initial guess provided to the algorithm?*

The initial TSR was set at 7.6 (i.e., start with the clean blade optimum). To answer your question, we run one of the simulations in Fig. 8 for 3000 s, instead of the 1500 s in the paper. The result is shown below in Fig. A2 below. Note that the dither is active since the algorithm is turned on at 500 s. The time series of the TSR set point after 1500 s appears to oscillate (dither with amplitude 0.1 or 0.2 peak-to-peak around a mean value between 8.1 and 8.2 (8.2 is the optimum for this case, as it can be seen from Fig. 4). Note also from Fig. 4 (and Fig. D in reply to reviewer #1) that Cp does not change much (less than 10%) between TSR 8.1 and TSR 8.2 for the case of contaminated blade.

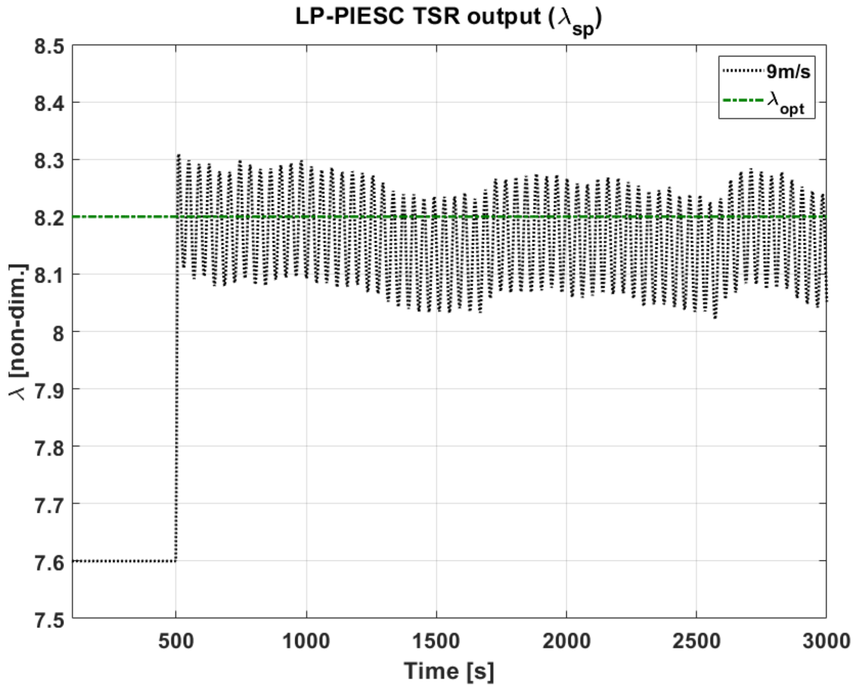


Figure A2: Tip Speed Ratio setpoint (LP-PIESC output) – Contaminated blade (U=9m/s, TI =10%).

Please note that the dynamics of the turbine with ROSCO and LP-PIESC is complex. For the simulation shown in Fig. A2, there are time intervals where the turbine is in above-rated conditions (see pitch signal in Fig. 9 of the paper). Thus, any small fluctuations around the mean value of the commanded TSR, which are not accounted for by dither, would require an analytical investigation with the full nonlinear system. Our paper is only aimed at providing numerical evidence that the LP-PIESC offers an alternative to retune the TSR. In a practical application, one could turn off the dither as soon as the new optimal TSR is reached. A stopping criterion for the dither could be based on the magnitude of the parameter $\widehat{\theta_1}$ (see Fig. A in reply to reviewer #1). In this paper, we have run the dither continuously.