

University of British Columbia

UBCO-UL NSERC Alliance Grant “Reduced-Order Models of Wind
Farm Induction and Far-Field Wake Recovery”

Response to Reviewer 1

Exec. S. Stipa - March 7, 2024

We would like to thank the reviewer for the time dedicated to revising the paper. We proceed with answering and clarifying, where possible, their comments.

Our response, denoted in black, is shown below, while the reviewer's comments are denoted in blue. Please refer to the track changes document for a detailed overview of the changes made to the manuscript.

Abstract: the authors mention that this new technique demands less than 15% of the computational resources than traditional methods, which is an impressive achievement. However, this technique also reduces the accuracy of an LES to the one of the MSC model. I think this is quite an important limitation, which should be mentioned.

This aspect has been added to the abstract (see line 15 of the track changes document).

Line 34: the authors mention that AGWs have “extremely large spatial scales”. I think the reader would benefit if a number in terms of km would be provided, as the word “extremely” can be subjective. I suggest trying to limit the use of this word throughout the manuscript (e.g. see lines 35 and 51).

The word "extremely" has been omitted and more specific details have been added (see lines 40-42 and 171-172 of the track changes manuscript).

Line 37: I would also refer to some earlier works about AGW excited by hills/mountains (e.g. Klemp and Lilly 1977, Teixeira 2014).

The reference Teixeira (2014) has been added at line 37 of the track changes manuscript, while the work from Klemp and Lilly (1978) has been cited later at line 40 to introduce the problem of wave reflection.

Lines 54-67: the authors describe the Rayleigh damping layer and fringe region technique in this paragraph. However, the descriptions are mixed so that it is easy to confuse the role of the two techniques. I would propose to re-write this paragraph explaining first the Rayleigh damping technique, for instance, and afterwards the working principle and use of the fringe region technique.

The paragraph has been almost completely rewritten following the reviewer's comment to ensure that the Rayleigh damping layer and fringe layer are distinctly described. Please refer to the track changes document.

Line 78: the authors define the rigid-lid approximation as a case with strong free atmosphere stability. However, this could also be seen as a case with a very strong (practically infinite) capping-inversion strength. To avoid confusion, I would define the rigid-lid case as an approximation for a case with very strong thermal stratification above the atmospheric boundary layer.

The comment has been implemented in the revised manuscript. Please see lines 24 and 122 of the track changes document.

Introduction: I suggest to include the general aim of the article in the first paragraph of the introduction. This will give the reader a hint about why AGW numerical models and boundary conditions are described in this section.

The comment has been implemented in the revised manuscript. Please see lines 45-47 of the track changes document.

Line 107: I would mention this in the introduction, i.e. that the article only deals with CNBLs.

The comment has been implemented in the revised manuscript. Please see lines 143-144 of the track changes document.

Section 2.2: the proposed technique uses the MSC model to predict the capping-inversion vertical displacement (η) and only simulate the flow within the ABL with LES. Hence, it is implicitly assumed that $H > H_1$, with $H_1 = 2z_{hub}$. However, H_1 values are getting closer to heights at which the capping inversion is typically located (for instance, the Vestas V172-7.2MW turbine has a hub height of 199 m, a figure that may increase in the coming years) so that cases where the ratio $H/H_1 \approx 1$ are realistic. In the current work, the authors fix H/H_1 to 2.77 and do not discuss this further. However, I believe that the performance of the new technique is sensitive to the H/H_1 ratio. Would it be possible to add a few comments and/or simulations on what would happen when $H/H_1 \approx 1$? Also, how would this technique deal with cases where $H/H_1 < 1$? If this is a limitation, it should be reported in the text.

This comment has been now addressed in the manuscript as follows, between lines 349 and 363 of the track changes document). The case where $H/H_1 \approx 1$ "corresponds to a situation where the turbine top tip almost pierces the inversion layer, with consequent disappearance of the upper layer. Devesse et al. (2023) developed an alternative strategy to the one used in the MSC model to couple the 3LM of Allaerts and Meyers (2019) and the Bastankhah and Porté-Agel (2014) wake model, which also uses the 3LM to address AGW effects. When validating this new model against wind farm LES characterized by $H = 150, 300, 500$ and 1000 m and $h_{hub} = 119$ (Lanzilao and Meyers, 2023), the authors excluded those LES cases with $H/H_1 = 0.63$ ($H = 150$ m). Among the remaining cases, the model showed the highest deviation from the LES when $H/H_1 = 1.26$ ($H = 300$ m). As also the MSC model uses the 3LM to model AGW effects, these results suggest that the MSC model will loose accuracy when $H/H_1 \lesssim 1.5$. In the present manuscript, the dependency of the proposed technique to the ratio H/H_1 is not investigated and this number is fixed to 2.78. We highlight that this is a limitation of the MSC model used to compute η . If η could be evaluated by different means (e.g. with a coarser AGW-resolved LES employing a simple canopy model) at a height located above the inversion layer, the AGW modeling approach could be used for small H/H_1 ratios by placing the upper boundary a few hundreds meters into the free atmosphere and by including the potential temperature transport equation."

Line 184: the authors mention that "Then, the vertical displacement is linearly distributed to the underlying cells, deforming the mesh before starting the simulation". This passage is not clear to me. Would it be possible to explain in more detail?

This aspect has been clarified in the manuscript. Please see lines 322 to 324 of the track changes document. Specifically, the upper boundary initially located at H is vertically displaced according to η before starting the simulation, following which it remains fixed, as the applied η corresponds to the steady state solution obtained with the MSC. Then, the vertical displacement applied to the top boundary is linearly distributed to the underlying cells. This means that, at each horizontal location, the first cell away from the bottom wall is not displaced at all, while the top cell moves vertically by η . In between, the cells are vertically displaced by a distance Δd that is calculated based on their distance from the wall as $\Delta d(\mathbf{x}) = z/L_z \cdot \eta(x, y)$, where L_z is the local vertical domain size and Δd is the vertical displacement at \mathbf{x} .

Line 193: in the LES framework, the vertical displacement of the capping inversion generates a cold anomaly, which in turn results in pressure perturbation. However, the authors mention that in CNBLs, the potential-temperature equation can be left out in the AGW-modelled simulations. Hence, are the pressure perturbations solely caused by the flow convergence/divergence caused by the irregular top edge of the main domain? How is the buoyancy term computed in the vertical momentum equation? Or is it neglected? I would appreciate a more detailed explanation.

On lines 140-144 and 341-342 of the track changes document, we state that under CNBLs there is no need to solve the potential temperature advection equation, as the flow is neutral everywhere except close to the top boundary. For those conditions where $H/H_1 \approx 1$, the upper boundary should be moved a few hundreds meters into the free atmosphere and so potential temperature must be solved and η cannot be calculated with the MSC model anymore (though a coarse LES using a canopy model might be used). This aspect is addressed at the end of Section 2.2.

Moreover, a more detailed explanation on the relation between flow convergence/divergence and AGWs is provided between lines 328-333 of the track changes manuscript, when talking about the rigid lid. To summarize, buoyancy is not required to capture AGW effects inside the ABL, as these are given by flow divergence/convergence of the ABL top. In fact, the pressure disturbance produced within the ABL by AGWs in the free atmosphere has to coincide with the pressure produced by flow convergence/divergence, otherwise the governing equations (Equations 9 and 10 of the revised manuscript) are not satisfied. In particular, there is a unique η that satisfies this condition, which is the one that we impose using the MSC model. This whole reasoning is the backbone of Section 2.2 and it is shown using the simple model derived by simplifying the 3LM model of Allaerts and Meyers (2019).

Line 194: the authors mention that “This condition is only violated very close to the top boundary, where discrepancies in turbulent fluctuations produced by the absence of stability and by the physical boundary are deemed acceptable as they happen away from the wind farm”. I would note that this affirmation does not hold for low H/H_1 ratios (as mentioned in comment 8).

We agree with the reviewer, but in this case the main problem would be not being able to use the MSC model to compute η . To extend our approach to low H/H_1 ratios, we would advocate using a coarse LES that employs a canopy model to run a computationally cheaper AGW-resolved simulation. Under those conditions, the domain in the AGW-modeling method can be truncated a few hundred meters into the free atmosphere, instead of at H , as the streamline displacement is available here from the AGW-resolved LES. Of course, potential temperature transport has to be retained in this case even if a CNBL is simulated. This suggestion is given at lines 359-363 of the track changes document.

Section 3.2: if I understand correctly, the inflow data used for the AGW resolved and modelled simulations are computed with two different precursor techniques. Hence, differences in the wind-farm simulation results between the two techniques could be also attributed to this difference in inflow conditions. Why the authors do not drive the AGW-modelled simulations with inflow sections taken from the precursor simulations used for the AGW-resolved simulations?

We agree with the reviewer that this would have been the best approach. However, the inflow data used for the AGW-resolved cases is not available as it was generated at runtime during the simulations (these employed a concurrent precursor method) and not saved to slices. Hence, the approach followed in the manuscript is arguably the best alternative. A supporting rationale has been added to the paper (lines 431-434 of the track changes document).

Line 264: the authors mention that “we used the velocity inflow data of the subcritical case to prescribe an inlet for the rigid lid”. Is the velocity profile in the precursor domain for sub- and supercritical cases equal? Showing some vertical profiles of the precursor simulation would be beneficial.

The analysis required by the reviewer has been added to the revised manuscript in Appendix A.

Figure 2: I suggest to center the colorbar around the zero value (so that the background color is white).

The reviewer's comment has been implemented in the revised manuscript.

Line 274: the authors mention that “the AGW-modelling technique requires a domain that is more than 85% smaller compared to the AGW-resolving approach”. In which terms? Number of cells? Does this number also consider the precursor domain (for instance, the AGW-modeled simulations use a precursor domain 7 times bigger in the y-direction)?

This comment has been addressed in more detail in the revised manuscript (see lines 479-482 of the track changes document).

Figure 3: How would you explain the differences between AGW-modelled simulations and the MSC model? Are those due to the simplifications made in the MSC model (for instance, linearity, absence of resolved turbulence, etc.)?

We explain the differences as follows. The AGW-modeled and MSC model feature the exact same η , but a different level of fidelity inside the boundary layer. Hence, the same η does not lead to identical pressure and velocity perturbations. Conversely, the AGW-modeled and AGW-resolved cases use the same model inside the ABL, but η is slightly different, as it comes from the MSC model in the former and it is resolved in the latter. As a consequence, mass and momentum conservation show some differences in the perturbation velocity and pressure. This explanation has been added to the revised manuscript (see lines 518-529 of the track changes document).

Figure 3: I suggest extending the caption of this figure, mentioning for instance that the profiles shown are averaged in height between H1 and H and along the wind-farm width. This comment extends to the whole manuscript since I feel that the combination of figure and caption is often not self-explanatory.

The reviewer's comment has been addressed in the revised manuscript and further extended to all figure captions. Please refer to the track changes document.

Line 304: the domain in Figure 3 is too small to appreciate this behavior.

We have rephrased by pointing at Figure 3 in the revised manuscript, which corresponds to Figure 2 of the original manuscript (i.e. the entire η fields for the subcritical and supercritical conditions).

Line 313: at which height are the profiles taken? I suggest mentioning it in the manuscript.

They are taken at the hub height. The reviewer's comment has been implemented in the revised manuscript (see line 537 of the track changes document).

Figure 4: the match in terms of velocity is really good. This makes it hard to spot differences between AGW-modelled and resolved simulations in Figure 4, which obviously is a good sign. Could be an idea to also plot the relative error for both cases? This will allow the reader to easily understand where the two methods differ the most.

The reviewer's suggestion has been implemented in the revised manuscript.

Table 3: Would it be possible to include the non-local, wake and farm efficiency values in this table (as defined in Lanzilao and Meyers (2024), for instance)? The total farm power (and farm efficiency) in the supercritical case is almost identical in the AGW-M and AGW-R cases. However, I expect some differences in non-local and wake efficiencies.

The reviewer's comment has been addressed. However, since the inflow data relative to the AGW-resolved simulations is not available, it is impossible to conduct isolated with turbine simulations to compute P_∞ , as done in Lanzilao and Meyers (2023). For this reason, in order to compute the efficiency, we used the data from Appendix B of Stipa et al. (2023), which are evaluated with uniform inflow and in absence of turbulence. We agree that this would lead to values of η_{nml} and η_{tot} that are different from the actual figures, but since the same procedure to compute P_∞ has been used for all the entries of Table 4 of the revised manuscript, comparisons can still be drawn (lines 561-586 of the track changes document). In particular, the ability of the model to capture the underlying physics is confirmed by noticing that the AGW-modeled and AGW-resolved simulations lead to the same conclusions regarding which case is characterized by the highest wake efficiency, blockage effect and total wind far power.

Section 4.2: I'm assuming that the rigid lid is located at $H = 500$ m. Is this correct? I suggest to explicitly mention the vertical extent of the domain in the text.

Correct. This information has been added (see line 595 of the track changes document).

Section 4.2: the pressure build-up in the rigid-lid case is solely attributed to flow confinement. However, even in neutral conditions, the cumulative turbine induction generates a pressure build-up and consequently, a flow slow down (typically much lower in values than the one observed in the presence of thermal stratification). Therefore, I would rephrase this sentence and/or find an alternative method to evaluate the flow blockage solely induced by the flow confinement.

This section has been heavily modified (please see the track changes document). To specifically address the reviewer's comment, we would like to highlight the statement added at lines 605-609 of the track changes document. In particular, global blockage effects is always due to flow confinement, which is an alternative way of referring to the AGW-induced pressure gradient. In fact, the two are uniquely related, as explained in Section 2.2. Hence, in the rigid-lid global blockage is generated in the exact same manner as in the full AGW solution, with the only difference being that flow confinement is approximated to that produced when $\eta = 0$. This implies that the flow is horizontally divergence free in the rigid lid (i.e. on wall-parallel planes), while continuity is satisfied on pliant surfaces defined by η in the full AGW solution (i.e. surfaces coincident with the local vertical streamline displacement). Notably, both induce global blockage due to flow confinement or, alternatively, to stability effects above H , but the rigid lid corresponds to the limiting case where $\Delta\theta \rightarrow \infty$ and/or $\gamma \rightarrow \infty$.

Line 350: Which is the MSC setup used to generate Figure 8? In the text, only the capping-inversion strength and free lapse rate are mentioned. In general, I would appreciate more details on the simulation setup, so that it would get easier and more intuitive to reproduce the results.

The reviewer's request has been implemented throughout the revised paper (see for instance Table 3 of the revised manuscript).

Figure 8: it would be interesting to split this figure into three panels, reporting the sensitivity of the relative error based on non-local, wake and farm efficiency to the capping-inversion strength and free lapse rate. I suggest this because at times the total farm power of two simulations can be almost identical, even though the power trends are very different (two behaviors that cancel out).

The reviewer's request has been addressed in the revised manuscript. In particular, instead of computing the error between the different models, 1D parametric analyses have been conducted (Figures 9 and 10), where the different approaches are directly (and more visually) compared. Moreover, Figure 8 of the old manuscript has been removed and substituted with the sensitivity of η_{nml} , η_w and η_{tot} to the parameters $\Delta\theta$ and γ . The error when these are estimated using the rigid lid approximation can still be well appreciated

from Figures 9 and 10.

Table 4: the relative error remains positive for all cases when the MSC model is used. However, in the LES case, the error becomes negative for the subcritical case. Any idea about why this occurs?

The sensitivity study has been extended, and there are indeed conditions where the rigid lid approximation performs slightly worse than the full AGW solution. In fact, it appears that η_{tot} approaches the rigid lid solution from above instead of below. The cross-over point occurs, according to the MSC model, around $\Delta\theta = 10$ K. This is higher than the value used in the subcritical LES conditions, where η_{tot} for the subcritical case is already higher than the rigid lid at 7.312 K. Unfortunately, while the crossover of the full AGW solution over the rigid lid seems to be predicted by both the MSC model and the LES, we do not have a clear explanation regarding the difference in the value of $\Delta\theta$ at which such crossover is observed.

Line 372-374: the fringe region is adopted in pseudo-spectral (or fully spectral) flow solvers to impose the inflow conditions. The presence of gravity waves does not imply the use of a fringe region, as inflow-outflow boundary conditions can be adopted (although the implementation is not trivial).

This aspect has been addressed in Section 2.1. And the sentence mentioned by the reviewer has been corrected by specifically referring to finite volume codes (see line 713).

Line 385: As a future work, I would also suggest further validation of this technique, as it has been only applied to two idealized cases. For instance, Lanzilao and Meyers (2024) performed 40 LESs in different atmospheric conditions, for which the displacement of the capping inversion is computed. This comparison could offer further insights into the performance of the proposed method.

The reviewer's suggestion has been added to the revised manuscript (see lines 727-729 of the track changes document).

Appendix A: this construction looks quite artificial to me. From my point of view, it would be easier to drive the main domain using the same precursor simulation for both the AGW-modelled and AGW-resolved simulations. Is there a particular reason why the authors decided to not pursue this option? It would eliminate the need for the process described in this appendix together with ensuring equal inflow conditions in both cases.

We totally agree with the reviewer. Unfortunately, as previously mentioned, the inflow data for the AGW-resolved simulations (which have been conducted some time ago and already presented in Stipa et al. (2023)) have been generated at runtime and have not been saved to slices in the disk. In this regard, the approach followed in the paper was the only one that allowed us to avoid re-rerunning the AGW-resolved simulations, which we consider an unnecessary use of computational resources in light of the results presented in the paper.

Line 113: replace "If one wishes to resolve AGW within LES" with "When simulating AGWs in an LES framework" or similar.

Corrected (see line 168 of the track changes document).

line 205: used -> use.

Rephrased (see line 370 of the track changes document).

Line 368: conventionally neutral boundary layers -> CNBLs.

Corrected (see line 709 of the track changes document).

References

- Allaerts, D. and Meyers, J.: Sensitivity and feedback of wind-farm-induced gravity waves, *Journal of Fluid Mechanics*, 862, 990–1028, <https://doi.org/10.1017/jfm.2018.969>, 2019.
- Bastankhah, M. and Porté-Agel, F.: A new analytical model for wind-turbine wakes, *Renewable Energy*, 70, 116–123, <https://doi.org/https://doi.org/10.1016/j.renene.2014.01.002>, special issue on aerodynamics of offshore wind energy systems and wakes, 2014.
- Devesse, K., Lanzilao, L., and Meyers, J.: A meso-micro atmospheric perturbation model for wind farm blockage, Submitted to *Wind Energy Science Journal*, URL <https://arxiv.org/abs/2310.18748>, 2023.
- Klemp, J. and Lilly, D.: Numerical Simulation of Hydrostatic Mountain Waves, *J. Atmos. Sci.*, 35, 78–107, [https://doi.org/10.1175/1520-0469\(1978\)035<0078:NSOHEMW>2.0.CO;2](https://doi.org/10.1175/1520-0469(1978)035<0078:NSOHEMW>2.0.CO;2), 1978.
- Lanzilao, L. and Meyers, J.: A parametric large-eddy simulation study of wind-farm blockage and gravity waves in conventionally neutral boundary layers, 2023.
- Stipa, S., Ajay, A., Allaerts, D., and Brinkerhoff, J.: The Multi-Scale Coupled Model: a New Framework Capturing Wind Farm-Atmosphere Interaction and Global Blockage Effects, *Wind Energy Science Discussions*, 2023, 1–44, <https://doi.org/10.5194/wes-2023-75>, 2023.
- Teixeira, M. A. C.: The physics of orographic gravity wave drag, *Frontiers in Physics*, 2, <https://doi.org/10.3389/fphy.2014.00043>, 2014.

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Response to Reviewer 2

Exec. S. Stipa - March 7, 2024

We would like to thank the reviewer for the time dedicated to revising the paper. We proceed with answering and clarifying, where possible, the proposed comments.

Our response, denoted in black, is shown below, while the reviewer's comments are denoted in blue. Please refer to the track changes document for a detailed overview of the changes made to the manuscript.

Global blockage effects: The authors are drawing strong conclusions on the mechanisms that cause the global blockage effect. They attribute the velocity deceleration upstream of the wind farm either to a gravity wave-induced pressure gradient or to flow confinement (e.g., Lines 41-43, Lines 185-187, Lines 289-291, Lines 344-345, Lines 388-389). I agree that flow confinement and gravity waves may play a role in these cases; however, the deceleration of the wind upstream of the wind farm can also be due to other mechanisms that are likely present in these simulations but that are not discussed here (Bleeg and Montavon, 2022; Sanchez Gomez et al., 2023). In fact, the authors clearly show that other mechanisms (i.e., not gravity wave-induced velocity deceleration) are responsible for more than 50% of the velocity deceleration upstream of the wind farm and gravity-wave-induced blockage is secondary (Figure 6).

Our statements on the mechanism causing global blockage effects are based on the findings from a number of recent studies that emphasize the critical role played by atmospheric gravity waves (see for example Devesse et al., 2023; Lanzilao and Meyers, 2023; Stipa et al., 2023). However, we acknowledge that these may have not been presented in the clearest way and the manuscript has been heavily revised. Moreover, some additional comments can be made to clarify the specific points addressed by the reviewer.

First, we do not fully agree with the definition of local and global blockage given by Bleeg and Montavon (2022). In particular, they define local blockage as the blockage from immediate neighbors, while global blockage is given by wind-farm-scale blockage effects. In our opinion, this definition does not allow to clearly distinguish between individual turbine induction (and its cumulative effect) and effects related to atmospheric stability. In fact, while local induction acts mainly at the turbine scale, it also has an impact — albeit small — at the wind farm scale (this is clearly shown in Stipa et al., 2023). Conversely, stability effects are only observable at the wind farm scale. For this reason, we refer to global blockage as the flow deceleration produced by the presence of stability above the ABL. When also individual turbine induction and its cumulative effect are considered, i.e. local blockage, the entirety of the upstream flow deceleration can be captured Devesse et al. (2023); Stipa et al. (2023). It is not clear to us what are the other physical mechanisms beyond gravity-wave-induced blockage the reviewer is referring to. We certainly acknowledge that global blockage will show a dependency on the wind farm geometry, wind shear, wind veer, and stability inside the ABL, but we argue that the physical mechanism still remains that described by, e.g. Devesse et al. (2023); Lanzilao and Meyers (2023); Stipa et al. (2023) and in the present paper.

Another important point is that focusing solely on global blockage only covers half of the underlying physics. In fact, the higher the blockage the more favorable the pressure gradient is inside the wind farm. As shown in Figure 11 of the revised paper, those conditions characterized by higher blockage are far from experiencing the lowest overall wind farm efficiency, a testament of the importance of unfavorable and favorable pressure gradients produced by stability both upstream and inside the wind farm.

To specifically address the reviewer's comment on flow confinement and AGWs contribution to blockage, we highlight the statement added at lines 605-609 of the track changes document. In particular, global blockage effects is always due to flow confinement, which is an alternative way of referring to the AGW-induced pressure gradient. The two are in fact uniquely related, as mathematically shown in Section 2.2 (this section has been heavily modified, please see the track changes document). To further expand on this, in the rigid lid cases, global blockage is generated in the exact same manner as in the full AGW solution, with the only difference being that flow confinement is restricted to that produced when $\eta = 0$. This implies that the flow is horizontally divergence free in the rigid lid case (i.e. on wall-parallel planes),

while continuity is satisfied on pliant surfaces defined by η in the full AGW solution (i.e. curved surfaces, locally coincident with the vertical streamline displacement). Notably, both induce global blockage due to flow confinement or, alternatively, to stability effects above H , with the rigid lid being a limiting case for $\Delta\theta \rightarrow \infty$ and/or $\gamma \rightarrow \infty$. Additions to the revised manuscript regarding these aspect can be found at lines 328-333, 605-609 and 730-737 of the track changes document.

Finally, Figure 6 (which became Figure 7 in the revised manuscript) indicates that global blockage corresponding to $\eta = 0$ (rigid lid) yields the majority of the blockage observed when the flow confinement accounts for the AGW solution in the free atmosphere. In both cases, global blockage is produced by flow confinement.

To better elucidate the relation between fully neutral conditions (no stratification) in which blockage is only produced by turbine induction, the rigid lid condition which only considers the effect of H , and the full AGW solution, which also considers the effect of $\Delta\theta$ and γ , we have calculated the non-local, wake and wind farm efficiencies for each of these cases. These are defined by Lanzilao and Meyers (2023) and are reported in Section 4.2 of the revised manuscript. Besides enhancing our understanding of detrimental (global blockage) and beneficial (turbine wake recovery) effects produced by stability, we arrive to the same conclusion of Section 4.2 of Bleeg and Montavon (2022), i.e. that the rigid lid approximation might overestimate wind farm power even more than fully neutral conditions, even though it captures some of the global blockage effects. This emphasizes the importance of modeling the entirety of free atmosphere stability effects and not only global blockage.

Rigid-lid approximation: The authors use the rigid-lid approximation throughout the manuscript; however, it is not clear what is the purpose of using such a simplified and unrealistic modeling approach. In Lines 83-85, the authors suggest the rigid-lid approximation may be useful for use in engineering parameterizations. What do the authors mean by engineering parameterizations? Also, the rigid-lid approximation is tested here neutral boundary layer flow, which is unrealistic compared to the atmospheric boundary layer. For example, Bleeg and Montavon (2022) show that neglecting the temperature stratification in the capping inversion and troposphere misrepresents the blockage effect.

By engineering parametrizations we refer to low-cost reduced order models such as the 3LM Allaerts and Meyers (2019) or the MSC model (Stipa et al., 2023). Reduced order models based on the rigid lid approximation are currently being used in industry tools to model global blockage effects. This aspect is also reported in Section 4.2 of Bleeg and Montavon (2022), where it is referred to as the symmetry plane method. The two things are in principle equivalent.

The last comment made by the reviewer implies that the basic idea of the approach described in the manuscript is not clear. To rectify this, a clarification about the purpose of investigating the rigid lid approximation has been added to the revised manuscript and can be found at lines 591-593 of the track changes document. In particular, the fact of imposing a certain height and displacement of the upper boundary in the LES is automatically equivalent to consider a certain inversion strength and free atmosphere lapse rate (see Section 2.2 of the revised manuscript, where this is explained using a simple analytical model). This means that the effect of stability above the ABL can be implicitly modeled within the ABL if the vertical boundary layer displacement η corresponding to the specific conditions under investigation (wind farm geometry, and unperturbed velocity and potential temperature profiles) are known. This is because the heterogeneous pressure gradient produced by flow confinement due to η and arising from the AGW solution in the free atmosphere have to be coincident. As a consequence, the rigid lid approximation is not equivalent to a case without thermal stratification, where the upper boundary should be placed ideally very far from the ground. Instead, it is a limiting solution corresponding to very large stratification above the ABL. The purpose of studying this approximation is to understand how it compares with current industry practice (i.e. fully neutral conditions) and with the full AGW solution. The same limiting solution has been studied by Bleeg and Montavon (2022) (cases 3 vs 5 and 3b vs 5b).

Line 153-154: Why are the wind farm and upper layer characterized by the same background velocity? This assumption virtually discards the effect from shear and the large gradients associated with the atmospheric surface layer.

The analytical model presented in Section 2.2 is only used to explain the unique relation that exists between the pressure p^* and the inversion displacement η . The assumption of constant velocity inside the boundary layer allows the original 3LM equations to be easily rewritten in terms of η instead of η_1 and η_2 by summing up the continuity equations in the wind farm and upper layer. Doing the same within the original 3LM equations would only be possible in Fourier space and the conclusions would be more difficult to see. However, the generality of our reasoning can be readily proved by noticing that, once η is known, Equation 10 is not required anymore and pressure can be obtained by solving Equation 9.

Figure 2: The divergent color map is not centered at 0, making it very difficult to distinguish between positive and negative inversion displacements.

The reviewer's comment has been implemented in the revised manuscript.

Lines 299-300: I would argue that the AGW-modeled and AGW-resolved approaches do not predict almost the same pressure perturbation for the subcritical case (Figure 3a). Differences in the pressure perturbation field between the AGW resolved and modeled approaches are at least on the order of 10% upstream of the wind farm.

We agree with the reviewer's comment and the paragraph has been rephrased.

Lines 307-312: The differences upstream of the wind farm are just as large (or larger) than the differences at the domain outflow. However, the hypothesis presented by the authors does not address these differences. The flow upstream of the wind farm is outside and downstream of the fringe region and these differences are still large.

We agree with the reviewer's comment and added additional explanation in the revised manuscript. In particular, the following considerations can be made. The AGW-modeled and MSC model feature the exact same η , but a different level of fidelity inside the boundary layer. Hence, the same η does not lead to identical pressure and velocity perturbations. Conversely, the AGW-modeled and AGW-resolved cases use the same model inside the ABL, but η is slightly different, as it comes from the MSC model in the former and it is resolved in the latter. As a consequence, mass and momentum conservation show some differences in the perturbation velocity and pressure. This aspects have been added to the revised manuscript (see lines 518-529 of the track changes document).

Lines 344-345: The authors conclude that flow confinement is responsible for blockage to a lesser extent than gravity waves. However, Figure 6 clearly shows that the velocity deceleration with gravity waves is less than twice as large as the deceleration in the rigid-lid simulations. Thus, it seems flow deceleration from gravity wave-induced pressure gradients is not the main cause for blockage in these simulations. Also, I would argue that flow confinement is not the only cause for blockage in the rigid-lid case.

We realized that the way the sentence was written was misleading. In particular, blockage is in both cases given by flow confinement effects. In the AGW-modeled cases, the flow confinement is produced by an inversion displacement calculated with linear theory using the MSC model (thus accounting for the full AGW solution). In the rigid lid case, the value of η has been set to zero according to an infinitely high stability above the boundary layer (which is an approximation). Figure 6 (corresponding to Figure 7 in the revised manuscript), shows that assuming an infinitely strong stability accounts for the majority of the

global blockage effect observed when modeling also the inversion displacement. The sentence has been rephrased according to these considerations (see lines 605-609 of the track changes document).

The authors mention that the LES domain should extend to one or more wavelengths in each direction (Line 113). However, extending the LES above 10-12 km in the atmosphere means you are performing simulations above the tropopause, where the temperature stratification is very different from the constant lapse rate assumed within the troposphere. Is gravity wave propagation sensitive to having multiple thermally stratified layers like in the atmosphere compared to a single constant lapse rate? This might be out of the scope of the paper but is something to consider.

This is correct. While the validity of the Boussinesq approximation for such tall domains has been demonstrated (see "Response to Reviewer 2" from <https://wes.copernicus.org/preprints/wes-2023-40/wes-2023-40-AR2.pdf>), the assumption of linear lapse rate might be somewhat strong for certain LES setups where λ_z is very large. However, although undoubtedly worthwhile to be kept in mind for the future, this aspect has not been addressed yet in the context of wind farm LES. The only study that looked at non-uniform lapse rate in the free atmosphere has been performed by Devesse et al. (2022), who extended the 3LM to model these types of conditions. The authors state that a non-uniform lapse rate can play a big role in some cases. However, the 3LM model is based on linear theory and cannot account for other important physical phenomena such as gravity wave break-up and non-linear interaction.

References

- Allaerts, D. and Meyers, J.: Sensitivity and feedback of wind-farm-induced gravity waves, *Journal of Fluid Mechanics*, 862, 990–1028, <https://doi.org/10.1017/jfm.2018.969>, 2019.
- Bleeg, J. and Montavon, C.: Blockage effects in a single row of wind turbines, *Journal of Physics: Conference Series*, 2265, 022 001, <https://doi.org/10.1088/1742-6596/2265/2/022001>, 2022.
- Devesse, K., Lanzilao, L., Jamaer, S., van Lipzig, N., and Meyers, J.: Including realistic upper atmospheres in a wind-farm gravity-wave model, *Wind Energy Science*, 7, 1367–1382, <https://doi.org/10.5194/wes-7-1367-2022>, 2022.
- Devesse, K., Lanzilao, L., and Meyers, J.: A meso-micro atmospheric perturbation model for wind farm blockage, Submitted to *Wind Energy Science Journal*, URL <https://arxiv.org/abs/2310.18748>, 2023.
- Lanzilao, L. and Meyers, J.: A parametric large-eddy simulation study of wind-farm blockage and gravity waves in conventionally neutral boundary layers, 2023.
- Stipa, S., Ajay, A., Allaerts, D., and Brinkerhoff, J.: The Multi-Scale Coupled Model: a New Framework Capturing Wind Farm-Atmosphere Interaction and Global Blockage Effects, *Wind Energy Science Discussions*, 2023, 1–44, <https://doi.org/10.5194/wes-2023-75>, 2023.

An LES Model for Wind Farm-Induced Atmospheric Gravity Wave Effects Inside Conventionally Neutral Boundary Layers.

Sebastiano Stipa¹, Mehtab Ahmed Khan², Dries Allaerts², and Joshua Brinkerhoff¹

¹University of British Columbia, Okanagan Campus, CA

²Delft University of Technology, NL

Correspondence: Sebastiano Stipa (sebstipa@mail.ubc.ca)

Abstract. The interaction of large wind farm clusters with the thermally-stratified atmosphere has emerged as an important physical process that impacts the productivity of wind farms. Under stable conditions, this interaction triggers ~~the creation of~~ atmospheric gravity waves (AGWs) in the free atmosphere due to the vertical displacement of the ~~boundary-layer atmospheric boundary layer (ABL)~~ boundary layer (ABL) by the wind farm. AGWs induce horizontal pressure gradients within the ~~boundary-layer ABL~~ boundary layer ABL that alter the wind speed distribution within the farm, influencing both wind farm power generation and wake development. Additional factors, such as the growth of an internal boundary layer originating from the wind farm entrance and increased turbulence due to the wind turbines, further contribute to wake evolution. Recent studies have highlighted the considerable computational cost associated with simulating gravity wave effects within large eddy simulations (LES), as a significant portion of the free atmosphere must be resolved due to the large vertical spatial scales involved. Additionally, specialized boundary conditions are required to prevent wave reflections from contaminating the solution. In this study, we introduce a novel methodology to model the effects of AGWs without extending the LES computational domain into the free atmosphere. The proposed approach addresses the wave reflection problem inherently, eliminating the need for these specialized boundary conditions. We utilize the recently developed multi-scale coupled (MSC) model of Stipa et al. (2023b) to estimate the vertical ~~boundary-layer ABL~~ ABL displacement triggered by the wind farm, and apply the deformation to the domain of an LES that extends only to the inversion layer. ~~We validate our AGW modeling technique. The accuracy in predicting the AGW induced pressure gradients is equivalent to the MSC model. The AGW modeling technique is verified~~ for two distinct free atmosphere stability conditions ; by comparing it to the traditional approach in which AGWs are fully resolved using a domain extending that extends several kilometers into the free atmosphere. The proposed approach accurately captures AGW effects on the row-averaged thrust and power distribution of wind farms while demanding less than 15.7% of the computational resources compared to needed for traditional methods. ~~This can be further reduced in cases of~~ Moreover, when conventionally neutral boundary layers ~~since are studied~~ there is no longer a need for solving the potential temperature equation-, as stability is neutral within the boundary layer. The developed approach is subsequently used to compare global blockage and pressure disturbances obtained from the simulated cases against a solution characterized by zero boundary layer displacement, which represents ~~a the~~ the limiting case of very strong ~~free atmosphere stratification-. Finally, we discuss the implications of making such~~ stratification above the boundary ~~layer. This approximation, sometimes referred to as the "rigid-lid rigid lid" approximation, instead of considering the full gravity wave solution-, when predicting wind farm power~~ is compared against the full AGW solution using the MSC model. This is

done for different values of inversion strength and free atmosphere lapse rate, evaluating the ability of the "rigid lid" to predict blockage, wake effects and overall wind farm performance.

1 Introduction

30 Wind farms, especially those situated offshore, are increasing both in number and size, interacting with the atmosphere well beyond their physical boundaries. Such interactions play an important role both in the evolution of cluster wakes and on the amount of flow deceleration experienced upstream, also known as blockage. On the one hand, wind farm wake recovery is greatly influenced by surface stability within the atmospheric boundary layer (ABL). On the other hand, the stably stratified free atmosphere leads to the generation of atmospheric gravity waves (AGWs) when mean flow streamlines are vertically perturbed
35 by the wind farm. These waves exist in the form of interface waves within the capping inversion layer and internal gravity waves aloft, introducing a pressure feedback mechanism at the wind farm scale that ultimately impacts the flow dynamics inside the ABL (Smith, 2010). In contrast to terrain-generated gravity waves (~~Smith, 1980, 2007~~)(Smith, 1980, 2007; Teixeira, 2014), gravity waves triggered by wind farms yield smaller pressure and velocity perturbations inside the ABL ~~as compared to when these are compared against~~ turbulent fluctuations. ~~This, combined with the extremely large spatial scales of AGWs and their dependence on the~~ Moreover, similarly to mountain waves, wind farm-induced AGWs are characterized by horizontal and
40 vertical wavelengths of $\mathcal{O}(10)$ km, depending on the wind farm length, specific potential temperature structure ~~-, makes their and geostrophic wind. These aspects make AGW~~ observation and experimental measurement ~~extremely~~ difficult to achieve. Because of such complexities, ~~AGWs have only been studied so far~~ wind farm-induced AGWs have mainly been studied by means of high-fidelity models such as large eddy simulations (LES) or using linear gravity-wave theory (Nappo, 2012; Lin,
45 2007). LESs of AGWs suffer from high computational cost and AGW reflections at the numerical boundaries. The aim of this study is to overcome these difficulties by using a reduced order model based on linear theory to construct an LES methodology that significantly simplifies the inclusion of AGW effects within the ABL flow.

Using LES a two-dimensional numerical model, the impact of gravity waves on the flow around ~~wind farms has been subject of investigation by Allaerts and Meyers (2017); Lanzilao and Meyers (2022b); Stipa et al. (2023b); Lanzilao and Meyers (2023)~~
50 ~~-, who all assumed a~~ hills and complex terrain was first studied by Klemp and Lilly (1978), who addressed the problem of wave reflection from the upper boundary. Later, wind farm-induced AGWs were investigated using LES for conventionally neutral boundary ~~layer layers~~ (CNBL) ~~-This is~~ Allaerts and Meyers (2017); Lanzilao and Meyers (2022b); Stipa et al. (2023b); Lanzilao and Meyers (2023). CNBLs are characterized by a neutral stratification within the ABL, followed by a positive potential temperature jump $\Delta\theta$ across the inversion layer and ~~by a stable free atmosphere with a~~ linear lapse rate γ aloft. ~~The studies mentioned above in the~~
55 free atmosphere aloft. These studies showed that the presence of AGWs ~~have has~~ two main implications, namely an adverse pressure gradient upwind of the wind farm, which is responsible for global blockage, and a favorable pressure gradient inside the wind farm that is beneficial for wake recovery. Moreover, Centurelli et al. (2021) and Maas (2023) showed that LES results strongly differ from reduced-order wake models when thermal stratification is considered. To assess the impact of inversion height, strength and lapse rate on wind farm blockage and wind farm efficiency in general, Lanzilao and Meyers (2023) con-

60 ducted an LES parametric study, concluding that the overall effect of AGWs on wind farm ~~operation is~~ performance can be either beneficial or detrimental depending on the specific structure of the potential temperature profile. This result highlights the importance of including AGWs when modeling wind farms, both in high-fidelity and low-fidelity models.

An important aspect that emerges from the above studies, highlighted by Lanzilao and Meyers (2023), is that ~~LES-LESs~~ of wind farms including AGWs ~~is a challenging endeavor~~ are challenging. Firstly, they are rendered ~~extremely~~ computation-
65 ally intense by the domain size ~~that is required to resolve the large spatial scales associated with~~ required to spatially resolve AGWs. Furthermore, special boundary conditions should be used to damp out AGWs before they reach the domain bound-
aries and reflect, contaminating the solution. ~~In this regard~~ To overcome this issue, different approaches have been proposed ~~in literature, such as~~ so far. Béland and Warn (1975) and Bennett (1976) constructed transient radiation boundary conditions ~~(Béland and Warn, 1975; Bennett, 1976), Rayleigh damping layers (Klemp and Lilly, 1978), or the fringe region technique~~
70 ~~(Inoue et al., 2014). The latter is a~~ using the Laplace transform, but these require storing the entire flow history at each reflecting boundary. Klemp and Durran (1983) overcame this limitation by deriving a radiation condition for the top boundary that is local in time, using the linear, hydrostatic, Boussinesq equations. The authors also showed that low AGW reflectivity is still observed when these hypotheses are not strictly met, a result that was later confirmed by Lanzilao and Meyers (2022a). Another approach that avoids AGW reflections at the top boundary is the so-called Rayleigh damping layer ~~where~~ (Klemp and Lilly, 1978). This
75 ~~is a region of the domain where the momentum equation features an extra source term, proportional to the reference velocity~~ used to compute the damping action depends on time, so that a concurrently resolved turbulent flow can be prescribed within the ABL while simultaneously damping wave reflections aloft. ~~However, these approaches require ad-hoc tuning, usually accomplished by trial and error, which depends on the specific CNBL conditions and further raises computational costs.~~ difference between the perturbed and unperturbed ABL states. In theory, this eliminates AGWs before they can reach the
80 boundary but the proportionality coefficient, which increases with height, should be properly tuned. Reflections may still be observed both when damping is too strong or when it is too weak. In the first case, the Rayleigh damping region behaves as a physical boundary, while in the latter the damping is insufficient to cancel out perturbations before they reach the physical boundary.

Some guidelines on how to choose the Rayleigh damping parameters ~~were~~ have been provided by Lanzilao and Mey-
85 ers (2022a) and Klemp and Lilly (1978). Moreover, ~~Lanzilao and Meyers (2022a) also noted that the fringe region itself~~ may trigger spurious gravity waves, necessitating an additional layer in which horizontal advection of vertical momentum is suppressed to prevent these spurious perturbations to be transported downstream. Notably, many studies seem to many other studies (see Allaerts and Meyers, 2017, 2018, among others) agree that the Rayleigh damping layer located at the top should be larger than the expected vertical wavelength of the AGWs, estimated as $\lambda_z = 2\pi U_g / N$ $\lambda_z = 2\pi G / N$, where N is the Brunt-
90 Väisälä frequency and U_g G is the geostrophic wind (~~Klemp and Lilly, 1978; Allaerts and Meyers, 2017, 2018; Lanzilao and Meyers, 2022~~ . The same rule holds for the vertical height . Similarly, the vertical extent of free atmosphere ~~to be~~ included within the physical portion of the domain ~~such that simulation domain must allow~~ at least one vertical wavelength ~~should to~~ be resolved.

The description of AGWs Non-reflecting boundaries are also required in the horizontal directions, but their implementation is complicated by the fact that they should not alter the incoming ABL turbulence. In this case, two options are possible.

95 The first is the so-called fringe region technique (Inoue et al., 2014), which is essentially a Rayleigh damping layer where the unperturbed state used to compute the momentum source term is local in both time and space. This requires a separate simulation of the unperturbed flow, referred to as the concurrent precursor, to run concurrently with the main simulation, i.e. the successor. This ensures that a time- and spatially-resolved reference turbulent flow is available within the fringe region to compute the damping source at each iteration. As the concurrent precursor naturally contains the incoming turbulence, this
100 technique eliminates AGW while simultaneously prescribing the unperturbed turbulent inflow to the successor simulation. Similarly to the Rayleigh damping layer, the fringe region requires *ad-hoc* tuning of the proportionality coefficient that controls the amount of damping, which is usually accomplished by trial and error, further raising computational costs. Notably, Lanzilao and Meyers (2022a) observed that the fringe region itself may trigger spurious gravity waves while attempting to restore the unperturbed state, requiring an additional layer in which horizontal advection of vertical momentum is suppressed
105 to prevent these spurious perturbations to be transported downstream. Another possibility to avoid wave reflections at the inlet and outlet boundaries is to use Rayleigh damping regions above the boundary layer (Mehtab Khan, personal communication), so that turbulence remains unaffected below. However, this technique requires to accurately chose the horizontal unperturbed flow in the free atmosphere and poses issues when a capping inversion layer is present.

Regarding the description of AGW by means of reduced-order models, this was first achieved by Smith (1980, 2007) for
110 the flow around terrain features, in what is referred to as the two-layer model (2LM). The 2LM exploits the linear theory for interacting gravity waves and boundary layers, ~~later extended by Smith (2010) and was later extended~~ to wind farms immersed in CNBLs (Smith, 2010). Building on his work, Allaerts and Meyers (2019) developed the three-layer model (3LM), a substantial improvement of the 2LM characterized by extra features such as the Coriolis force, the additional wind farm layer that relaxes Smith's homogeneous vertical mixing assumption, and the wind farm/gravity wave coupling mechanism. Although the
115 3LM was the first study to incorporate AGW effects into predictions of ~~the~~ wind farm power losses, it lacked a local coupling between the mesoscale and turbine scales, failing to address the effects of gravity wave induced pressure gradients inside the wind farm and in the wake. Recently, ~~Stipa et al. (2023b) proposed a~~ Devesse et al. (2023) and Stipa et al. (2023b) proposed
120 new localized coupling ~~strategy~~ strategies between the 3LM and conventional wake models that capture all features of the wind farm interaction with AGWs under CNBLs. The latter, referred to as the multi-scale coupled model (MSC), capturing all features of the wind farm interaction with atmospheric gravity waves for CNBLs is characterized by a lower computational cost and its formulation is independent of the adopted wake model.

~~Under strong free atmosphere stability~~ When thermal stratification above the ABL is very strong, lapse rate and inversion strength lose importance and the background pressure gradient is mainly determined by the height of the inversion. Such boundary layer height. In this case, the flow cannot be perturbed vertically because thermal stratification acts as a lid located
125 at the ABL top. Such idealized limiting case is commonly referred to as the ~~rigid-lid~~ rigid lid approximation (Smith, 2023). ~~Specifically, as~~ As the lid imposes zero mean vertical mass flux, the solution is characterized by ~~an~~ a harmonic perturbation pressure that renders the mean flow horizontally divergence-free, with maximum and minimum pressure at the wind farm start and exit, respectively. ~~In particular, the rigid-lid~~ The rigid lid approximation maintains some properties of the full gravity wave solution, such as the presence of global blockage and flow acceleration within the wind farm. This, combined to its inherently

130 simpler formulation than the full ~~gravity-wave~~ AGW solution, makes the ~~rigid-lid~~ rigid lid approximation worth investigating for its potential use in engineering parametrizations.

~~In the present study, we propose a novel approach that models~~ The methodology proposed in this study allows to model AGW effects within a wind farm LES while eliminating the computational burden associated with resolving internal and interfacial waves above the ABL. In fact, while ~~the proposed methodology uses LES~~ LES is used below the inversion layer, 135 AGWs in the free atmosphere and within the inversion layer are modeled through the MSC model (Stipa et al., 2023b) using the vertical ABL displacement as the coupling variable. As a consequence, the developed approach only requires a vertical domain size that is equal to the height of the inversion layer. ~~Moreover, if CNBLs are considered, the solution of a potential temperature transport equation is not required as the flow is neutral below the capping inversion. Finally, assumed to coincide with the ABL height. Moreover,~~ no damping regions are needed as the large-scale pressure gradient ~~gradients~~ produced by 140 AGWs ~~is~~ are modeled without resolving the actual waves. Finally, when dealing with CNBLs, the flow is neutral within the boundary layer and the solution of a potential temperature equation can be omitted. Although the proposed method could be applied in principle to internally stable ABLs by solving for the potential temperature equation, the accuracy of the MSC model — to which the LES solution depends — has not been tested in this condition yet. Hence, this manuscript solely focuses on CNBLs, leaving internal ABL stability as an object for future investigation.

145 The present paper is organized as follows. Section 2 describes the proposed LES methodology, pointing out its differences with respect to the conventional approach used to simulate the wind farm/gravity wave interaction. Section 3 describes the set-up of the LES cases used to verify the proposed methodology. Model verification is presented in Section 4, together with ~~some considerations on an analysis regarding~~ the implications of using the ~~rigid-lid approximation to model the inversion layer~~ rigid lid approximation. Finally, Section 5 highlights the conclusions of the present study.

150 2 LES Methodology

For the LES simulations presented in this paper, we use the open-source finite volume code TOSCA (Toolbox fOr Stratified Convective Atmospheres) developed at the University of British Columbia and extensively validated in Stipa et al. (2023a). In order to distinguish between ~~AGW-resolving~~ AGW-resolved simulations and the proposed approach, we first describe, in Section 2.1, the characteristics of a simulation that naturally resolves ~~atmospheric gravity waves~~ AGWs and their effects within 155 the boundary layer. Then, in Section 2.2, we present the proposed modeling strategy with guidelines on its application. ~~Only~~ Here, only the turbulent part of the CNBL is included in the LES domain, while the steady-state solution in the free atmosphere is obtained from the MSC model of Stipa et al. (2023b).

2.1 ~~AGW-Resolving~~ AGW-Resolved Approach

~~As AGWs in the free atmosphere can only exist under stable conditions (Lin, 2007; Nappo, 2012)~~ Large wind farms may 160 trigger interface waves when an inversion layer is present as well as internal waves in the stably stratified free atmosphere (Lin, 2007; Nappo, 2012) by steadily perturbing the boundary layer height vertically. Although the extent of such perturbation

also depends on the level of stability experienced inside the boundary layer, the present study only focuses on CNBLs. ~~This excludes those cases featuring a stable or unstable stratification within the ABL, which represent a subject for future investigation.~~ Governing equations correspond to mass and momentum conservation for an incompressible flow with Coriolis forces and Boussinesq approximation for the buoyancy term. The latter is calculated using the modified density ρ_k , evaluated by solving a transport equation for the potential temperature. The exact form of the equations implemented in TOSCA and used in the present study is reported in Stipa et al. (2023a).

~~If one wishes to resolve AGW within LES. When simulating AGWs in an LES framework,~~ the simulation domain should extend to one or more wavelengths in each direction (Klemp and Lilly, 1978; Allaerts and Meyers, 2019; Stipa et al., 2023b; Lanzilao and Meyers, 2023). ~~This greatly increases the computational cost, as AGWs are characterized by extremely large spatial scales, both vertically and horizontally.~~ Under conditions that are representative of normal wind farm operation, i.e. lapse rates ranging between 1–10 K/km and geostrophic winds of 5–20 m/s, λ_z can be between 2–20 km. In addition, waves inherently reflect if they do not decay before reaching boundaries, ~~requiring an even higher domain size either requiring the computational domain to span several wavelengths~~ or the use of damping regions where ~~they~~ AGWs are artificially damped ~~before exiting the domain. Another source of.~~ With reference to the different boundary conditions listed in Section 1 at the domain top, the Rayleigh damping region represents the best solution in terms of wave reflectivity (Lanzilao and Meyers, 2022a). This is prescribed by applying a source term in the vertical momentum balance calculated as

$$s^r(\mathbf{x}) = \nu_r(z) [\bar{w} - w(\mathbf{x}, t)], \quad (1)$$

where \bar{w} is the prescribed unperturbed vertical velocity that the source term tries to attain, $w(\mathbf{x})$ is the vertical velocity at a given point \mathbf{x} and $\nu_r(z)$ is an activation function, defined as

$$\nu_r(z) = \alpha_r \left[1 - \cos \left(\frac{\pi}{2} \frac{z - z_s}{z_e - z_s} \right) \right] \quad (2)$$

with z_s and z_e the start and end heights of the Rayleigh damping region, and α_r the proportionality coefficient to be chosen depending on the specific problem. Note that only the vertical velocity is damped, as this is the only source of reflection for the upper boundary. In particular, $w(\mathbf{x})$ should be practically zero at the boundary, hence $\bar{w} = 0$. Notably, Lanzilao and Meyers (2022a) also apply the damping to the horizontal velocity components. In the present study such operation is not performed, as horizontal fluctuations are not reflected. This also limits the possible counteraction given by the source terms in those cells where the Rayleigh damping overlaps with the fringe region. However, we follow their approach in prescribing α_r , which is set to three times the Brunt-Väisälä frequency.

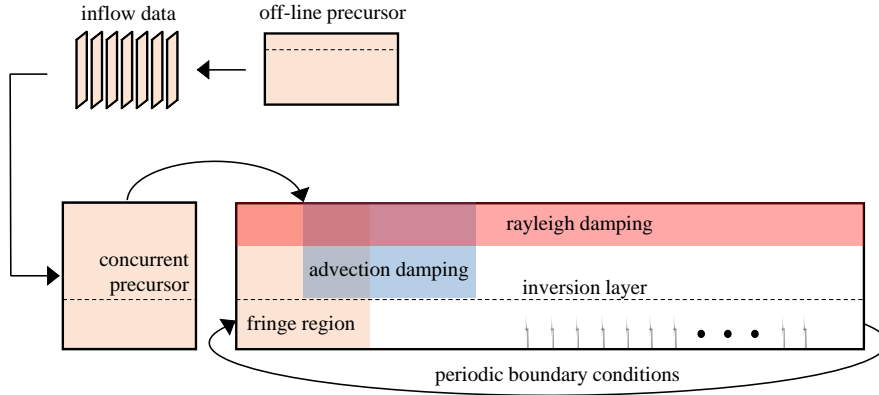


Figure 1. Methodological sketch of the AGW-resolved method employing streamwise periodic boundary conditions in the successor domain and a fringe region located at the inlet. The relative location of the Rayleigh damping layer and advection damping region are also shown. The figure is not to scale.

Special care should be paid to the lateral boundaries as well. In the spanwise direction, periodic boundary conditions are used. This ensures no reflections, but in essence renders the solution periodic, allowing waves leaving the domain from one side to re-enter at the opposite side. This is not an issue as long as the domain width is sufficient to ensure that waves reach the spanwise sides far downstream of the wind farm.

In the streamwise direction, the use of periodic boundary conditions implies that the wind farm wake is re-advected at the inlet. Moreover, Smith (1980) showed that the propagation of wave energy is aligned to the wind direction close to the wave source, i.e. the wind farm. This means that, for conditions practical to large wind farms, energy is radiated almost perpendicularly to the inlet and outlet boundaries, making it impossible to avoid reflections without using damping regions or by massively increasing the domain length. Among these two solutions, the former is usually preferred as it drastically reduces the cell count. For instance, the domain length that allowed to avoid AGW reflections in Lanzilao and Meyers (2022a) was of 40 km with a fringe region and 200 km without. In the present study, an inlet fringe region is applied. This stems from the method used in pseudo-spectral codes to enforce an arbitrary inlet boundary condition while still using periodic boundaries in the spectral directions (a requirement imposed by the Fourier transform). The fringe region method is essentially a Rayleigh damping layer where the unperturbed field is heterogeneous in both space and time. While employing periodic boundaries, the flow is slowly brought to an unperturbed state as it transits through the fringe region. This is achieved by applying a source term on all components of the momentum equation, calculated as

$$s_i^f(\mathbf{x}) = \nu_f(x) [\bar{u}_i(\mathbf{x}, t) - u_i(\mathbf{x}, t)] \quad (3)$$

where $u_i(\mathbf{x}, t)$ are the velocity components at every cell and $\bar{u}_i(\mathbf{x}, t)$ are the temporally- and spatially-resolved unperturbed flow components, whose calculation will be explained later. The activation function $\nu_f(x)$ only depends on the streamwise

coordinate and, following Inoue et al. (2014), it is given by

$$\nu_f(x) = \alpha_f \left[F \left(\frac{x - x_s^f}{\Delta_s^f} \right) - F \left(\frac{x - x_e^f}{\Delta_e^f} + 1 \right) \right], \quad (4)$$

210 with

$$F(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \left[1 + \exp \left(\frac{1}{x-1} + \frac{1}{x} \right) \right]^{-1}, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \geq 1. \end{cases} \quad (5)$$

The parameters x_s^f and x_e^f are the start and end of the fringe region, respectively, while Δ_s^f and Δ_e^f are the distances required to transition from zero to a damping equal to α_f , and from α_f back to zero at the fringe start and exit, respectively. The parameter α_f is the fringe coefficient, which has to be tuned depending on the specific case. For instance, potential temperature should also be damped according to Equation (3), where $u_i(\mathbf{x}, t)$ and $\bar{u}_i(\mathbf{x}, t)$ are replaced with $\theta(\mathbf{x}, t)$ and $\bar{\theta}(\mathbf{x}, t)$, respectively.

215 The unperturbed state required to compute the momentum and potential temperature source terms in the fringe region is evaluated by conducting a second simulation, referred to as the concurrent precursor, without wind turbines, in a domain coincident to or larger than the fringe region. This has to advance simultaneously with the wind farm simulation, so that the unperturbed state is available at each iteration. The need to solve for a concurrent precursor and the higher cell count due to the inclusion of damping regions represent the major increase in computational cost for the AGW-resolved method. Notably, streamwise periodic boundary conditions in the wind turbine domain allow to use a single fringe region located at the inlet (Stipa et al., 2023a, present) or at the outlet (Lanzilao and Meyers, 2023).

220 An additional source of contamination of the physical solution is represented by spurious gravity waves that are generated by ~~damping regions themselves as they try to force physical~~ the fringe region as it tries to force the wave perturbations to zero. This issue has been addressed by Lanzilao and Meyers (2022a), who developed the ~~so-called-so-called~~ advection damping region, where horizontal advection of vertical velocity is brought to zero to prevent these spurious oscillations to be advected downstream. ~~Furthermore, special care must be paid at the domain inlet in order to provide a time-dependent turbulent inflow to the simulation. For instance, it proves extremely difficult to damp out gravity waves and ensuring at the same time that turbulence is not distorted by the damping action if non-periodic boundary conditions are used in the streamwise direction.~~ Moreover, two damping regions have to be applied in this case, namely at the inlet and outlet, respectively. Conversely, periodic boundary conditions allow to use. Specifically, the term $\partial(uw)/\partial x$ is multiplied by

$$\nu_a(x, z) = 1 - \left[F \left(\frac{x - x_s^a}{\Delta_s^a} \right) - F \left(\frac{x - x_e^a}{\Delta_e^a} + 1 \right) \right] \mathcal{H}(z - H), \quad (6)$$

where x_s^a and x_e^a are the start and end of the fringe region, respectively, while Δ_s^a and Δ_e^a are the distances required to transition from unity to null magnitude of the advection term and from null back to unity at the region start and exit, respectively;

235 $\mathcal{H}(z - H)$ is the Heaviside function which ensures that this operation is only performed above the boundary layer, in order to leave turbulence unaffected.

In the present paper, the hybrid off-line/concurrent method described in Stipa et al. (2023a) is used. This essentially reduces the computational cost associated with turbulence initialization in the concurrent precursor domain. In fact, while the size of the concurrent precursor has to be equal to or larger than the fringe region technique (Inoue et al., 2014), a damping layer in which the reference flow used, the latter is required to compute the damping source term is unsteady. This more easily allows to provide a realistic turbulent inflow while simultaneously damping wave reflections by using a single damping region located at the inlet (Stipa et al., 2023a) or at the outlet Lanzilao and Meyers (2022a,b).

These considerations highlight that the design of an LES simulation in which AGW effects are not altered by spurious or numerical phenomena is challenging. Such studies require trial and error to obtain a suitable source terms only when the wind turbine simulation is started. Hence, turbulence spin up can be achieved by first running a separate precursor in a reduced domain, referred to as the off-line precursor. In particular, since no gravity waves are expected during this phase, the vertical domain size is such that only a small portion of the free atmosphere is resolved. Moreover, if the ratio between the spanwise size of the damping layers and their damping coefficients. Guidelines on selecting these quantities have only recently been provided by Lanzilao and Meyers (2022a). In light of such difficulties, a technique that eliminates the need for solving the governing equations in the free atmosphere and avoids damping regions will be a valuable tool to enable larger wind farm simulations at the same computational cost—or the same simulations at a reduced cost—while avoiding the *ad-hoc* tuning of the LES damping parameters concurrent and offline precursor is an integer, off-line precursor data can be prescribed at the inlet of the concurrent precursor by tiling them in the spanwise direction and extrapolating in the vertical. The concurrent precursor is then evolved using inlet-outlet boundary conditions for one flow turnover time, i.e. until it is filled with such pre-calculated turbulent flow. Boundary conditions in the concurrent precursor are then switched to periodic and the simulation becomes self-sustained. The hybrid off-line/concurrent precursor method allows to reach a fully developed ABL within a domain that is sufficient to decorrelate turbulent fluctuations but whose size is not dictated by the wind farm and AGW scales, thus allowing to save computational resources (see Stipa et al., 2023a for more details). In Figure 1, a methodological sketch of the AGW-resolved approach employing the concurrent precursor method is displayed.

260 2.2 AGW-Modeling AGW-Modeled Approach

As pointed out by Allaerts and Meyers (2017, 2018, 2019) and Lanzilao and Meyers (2022b), AGWs in the free atmosphere induce large-scale pressure gradients inside the ABL. The MSC model developed by Stipa et al. (2023b) is based on the concept that the effect of AGW on the wind farm is given by the change in mean velocity produced by this horizontally-varying horizontally heterogeneous pressure field, here referred as p^* . Unfortunately, this idea cannot be directly applied to wind farm LES by prescribing p^* as a separate source term. In particular, for reasons that will be clarified later, the presence of the top-upper boundary automatically prescribes a certain pressure gradient that satisfies horizontal pressure gradient inside the ABL such that mass and momentum conservation. In fact, are satisfied. Specifically, we show below that the vertical

streamline displacement η prescribed by the presence of the top boundary and the large-scale pressure field p^* cannot be imposed simultaneously, but are rather interdependent.

270 ~~To explain the~~ The relationship between these two variables ~~, we propose here can be explained by constructing~~ a simple model ~~obtained through~~ based on a perturbation analysis applied to the depth-averaged linearized Navier-Stokes equations. ~~This leads to~~ Although this leads to a consistent simplification of the equations proposed by Allaerts and Meyers (2019) ~~; while it still provides the required insight regarding the physics. In particular, we assume and cannot provide an accurate description of the boundary layer flow, this simple model still provides a level of physical insight that is sufficient to elucidate~~ the relationship between the boundary layer displacement η and the pressure p^* . First, an infinitely wide wind farm is assumed in the spanwise direction, so that quantities can only change along the streamwise direction ~~;~~ (i.e. $\partial/\partial y = 0$). Furthermore, the background flow is assumed to have a null mean spanwise component ~~$V=0$~~ (i.e. Coriolis ~~forcing~~ force is neglected). The structure of the potential temperature profile is that of a CNBL characterized by lapse rate γ , inversion strength $\Delta\theta$ and inversion height H . The bulk velocity within the boundary layer and the geostrophic wind are referred to as U and $\underline{U}_g \underline{G}$,
275 respectively. Similarly to Allaerts and Meyers (2019), the region below H is divided into two layers, namely the wind farm layer, characterized by a height H_1 , and the upper layer, of depth $H - H_1$. The depth of the wind farm layer is chosen as twice the hub height, i.e. $H_1 = 2h_{\text{hub}}$. Finally, ~~we assume~~ it is assumed that wind farm and upper layer are characterized by the same background velocity U , but at the same time admit different perturbation velocities u_1 and u_2 . With the above simplifications, the 3LM equations derived by Allaerts and Meyers (2019) become

$$285 \quad \begin{cases} U \frac{\partial u_1}{\partial x} + \frac{1}{\rho} \frac{\partial p^*}{\partial x} = -\frac{C}{H_1} u_1 - \frac{f_x}{H_1} \\ U \frac{\partial \eta_1}{\partial x} + H_1 \frac{\partial u_1}{\partial x} = 0, \end{cases} \quad (7)$$

for the wind farm layer, and

$$\begin{cases} U \frac{\partial u_2}{\partial x} + \frac{1}{\rho} \frac{\partial p^*}{\partial x} = 0 \\ U \frac{\partial \eta_2}{\partial x} + H_2 \frac{\partial u_2}{\partial x} = 0, \end{cases} \quad (8)$$

for the upper layer, where $C = 2u^{*2}/U$. It should be noted that $\eta_1 + \eta_2 = \eta$, i.e. the total vertical displacement of the pliant surface initially located at H . This, at steady state, coincides with the flow streamline through H far upstream, and can be
290 thought as both the inversion layer or ABL vertical displacement.

Rewriting the system in terms of η reads

$$\begin{cases} U \frac{\partial u_1}{\partial x} + \frac{1}{\rho} \frac{\partial p^*}{\partial x} = -\frac{C}{H_1} u_1 - \frac{f_x}{H_1} \\ U \frac{\partial u_2}{\partial x} + \frac{1}{\rho} \frac{\partial p^*}{\partial x} = 0 \\ U \frac{\partial \eta}{\partial x} + H_1 \frac{\partial u_1}{\partial x} + H_2 \frac{\partial u_2}{\partial x} = \underline{0,0}. \end{cases} \quad (9)$$

To complete the system, ~~we add~~ an extra equation is added that relates the vertical inversion displacement to the pressure anomaly ~~that will be~~ felt inside the boundary layer due to the increase or decrease in weight of the air column overtopping a given x location. This can be expressed in Fourier space by means of linear theory (Nappo, 2012; Lin, 2007) as

$$\frac{1}{\rho} \hat{p}^* = \Phi \hat{\eta}, \quad (10)$$

where the hat denotes Fourier coefficients and Φ is the so-called complex stratification coefficient, which accounts for pressure anomalies generated by both the inversion layer displacement (surface waves) and the resulting ~~motion perturbations~~ aloft (internal waves). We refer to Smith (2010) ~~and Allaerts and Meyers (2019)~~ for the definition of such function; in this context it ~~suffices to know that Φ contains~~ is sufficient to notice that all the physics related to AGWs and thermal stratification enters the system through the complex stratification coefficient Φ , while Equation (9) does not contain any stability-related term.

Equation (9) and Equation (10) form a fully determined system, which can be easily solved upon ~~transformation~~ transforming Equation (9) into Fourier space. In particular, Equation (9) describes the flow physics below H , while Equation (10) refers to the flow in the free atmosphere. It can be observed that the pressure field p^* , which satisfies both Equations (9) and (10), is the one ~~that reconciles~~ reconciling momentum and mass conservation inside the boundary layer with pressure anomalies due to overtopping density differences produced by a determined vertical displacement of the pliant surface at H . Specifically, η represents the coupling variable between the ABL and the free atmosphere, i.e. the neutral and stratified regions of the flow, respectively, under CNBL. Now, focusing only on the flow below H , i.e. on Equation (9), it is evident how the pressure gradient induced by ~~AGWs—and AGWs—~~ and its effects on the ~~velocity—could velocity—~~ could be readily obtained without ~~including thermal~~ any knowledge about free atmosphere stratification if the correct inversion displacement η was somehow known ~~and a priori~~. The same reasoning can be applied to the full 3LM equations derived by Allaerts and Meyers (2019) in the first two layers by simply noticing that the number of unknowns is reduced by one. Extending this idea to LES, the knowledge of η can be used to vertically deform the top boundary of the ~~LES domain~~ computational domain when this is initially located at H . Since a slip condition is usually applied ~~to the top boundary, deforming this here, deforming the upper~~ boundary alters the mean flow streamlines in a manner that is consistent with the inversion-layer displacement. ~~Hence, AGWs given, allowing the mean AGW induced pressure gradient to be automatically recovered within the ABL. In summary, AGWs effects produced by different stability conditions can be easily modeled by suitably deforming the top boundary and then conducting the LES simulation of using their corresponding η field to vertically deform the upper boundary. By doing so, the LES can be conducted only in~~ the turbulent part of the flow ~~that is~~ capped below the start of the stable flow region, where the wind farm is located.

In the present study, ~~we place~~ the top boundary is placed at the inversion center and ~~use the full MSC model~~ the MSC model is used to compute η . ~~Then, the~~ The vertical displacement is linearly distributed to the underlying cells, deforming the mesh before starting the simulation. This means that, at each horizontal location, the first cell away from the wall is not displaced, while the top cell is moved vertically by η . The cells in between will be displaced between zero and η depending on their distance from the wall. Notably, the

The case where the top boundary is a flat surface corresponds to the ~~rigid lid~~ rigid lid limiting solution. In particular, while this differs from the actual solution with atmospheric gravity waves, it still ~~allows to model models—~~ to a certain extent—

both global blockage and flow acceleration within the wind farm produced by **strong**-flow confinement inside the boundary layer. We highlight that flow confinement and free atmosphere stability effects are different ways to refer to the same physical manifestation. In fact, in light of the unique relationship between pressure and ABL displacement, stability effects determine an inversion displacement such that pressure perturbations induced by flow confinement and by AGW are equivalent, i.e. they satisfy both Equations (9) and (10). For this reason, the rigid lid assumption models global blockage to a certain degree, as the flow is indeed confined. However, the mechanism under which such confinement happens disregards gravity waves by neglecting the inversion perturbation field that complies with the actual potential temperature structure.

As a further consideration on the AGW-modeled technique, since the overall pressure disturbance is fully determined by the inversion displacement, **we note that** any spatially-varying source term imposed in the form of a pressure gradient will not produce any effect on the simulation results, but rather change the significance of the pressure variable such that the **original** overall pressure disturbance complying with the imposed streamline displacement is retained.

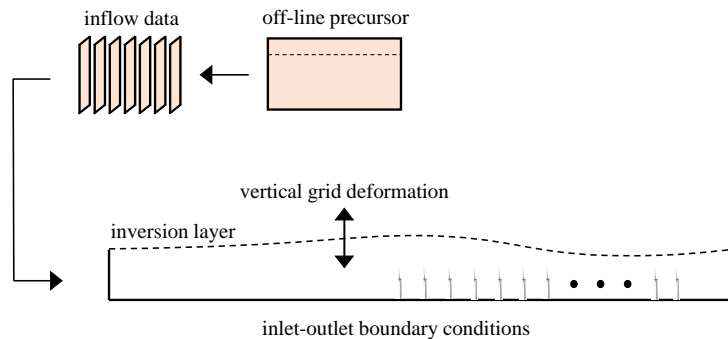


Figure 2. Methodological sketch of the AGW-modeled method. The figure is not to scale.

The developed approach, sketched in Figure 2, is convenient for at least **two-three** reasons. First, it substantially reduces the computational cost by eliminating the need for damping regions and the requirement of a domain that is large enough to vertically resolve AGWs. Secondly, it does not require to run a concurrent precursor simultaneously to the wind farm simulation. Third, under CNBLs it eliminates the need to solve for a potential temperature transport equation as the flow below H is neutral **in CNBLs**. This condition is only violated very close to the top boundary, where discrepancies in turbulent fluctuations produced by the absence of stability and by the physical boundary are deemed acceptable as they happen away from the wind farm. Moreover, at the inversion height fluctuations are naturally close to zero, as this roughly coincides with the top of the boundary layer.

A limitation of the proposed method is that the accuracy of the large-scale pressure gradient produced by displacing the top boundary is dependent on the accuracy of the MSC model in predicting the overall **physics of AGWs. Fortunately,**

AGW physics. Stipa et al. (2023b) showed that the pressure disturbance produced by the MSC model agrees well with a ~~AGW-resolving-AGW-resolved~~ wind farm LES simulations ~~with~~ ~~for~~ different values of capping inversion strength. Another limitation is given by the fact that the MSC model is untested for values of H/H_1 that are less than or equal to one, a realistic condition for modern large turbines. This corresponds to a situation where the turbine top tip almost pierces the inversion layer, with consequent disappearance of the upper layer. Devesse et al. (2023) developed an alternative strategy to the one used in the MSC model to couple the 3LM of Allaerts and Meyers (2019) and the Bastankhah and Porté-Agel (2014) wake model, which also uses the 3LM to address AGW effects. When validating this new model against wind farm LES characterized by $H = 150, 300, 500$ and 1000 m and $h_{hub} = 119$ (Lanzilao and Meyers, 2023), the authors excluded those LES cases with $H/H_1 = 0.63$ ($H = 150$ m). Among the remaining cases, the model showed the highest deviation from the LES when $H/H_1 = 1.26$ ($H = 300$ m). As also the MSC model uses the 3LM to model AGW effects, these results suggest that the MSC model will loose accuracy when $H/H_1 \lesssim 1.5$. In the present manuscript, the dependency of the proposed technique to the ratio H/H_1 is not investigated and this number is fixed to 2.78. We highlight that this is a limitation of the MSC model used to compute η . If η could be evaluated by different means (e.g. with a coarser AGW-resolved LES employing a simple canopy model) at a height located above the inversion layer, the AGW modeling approach could be used for small H/H_1 ratios by placing the upper boundary a few hundreds meters into the free atmosphere and by including the potential temperature transport equation. A related limitation applies to those cases characterized by an unsteady flow in the free atmosphere, as the MSC model assumes steady state conditions.

3 Suite of Simulations

To verify the validity of the proposed approach, we use the two LES simulations available from Stipa et al. (2023b). These correspond to a subcritical and a supercritical regime of ~~the CNBL,~~ interface waves within the inversion layer and are characterized by damping regions and a domain size ~~and damping region~~ that is sufficient to resolve AGWs in the free atmosphere. For this reason, they are referred to as AGW-resolved cases in the present study. Each case is then compared to its AGW-modeled counterpart, where the technique proposed in Section 2.2 is applied. ~~In addition, we used the developed modeling technique~~ Once validated, the AGW-modeled approach is leveraged to simulate a case corresponding to the rigid-lid rigid lid limiting solution, where the top boundary is not associated with any vertical displacement. ~~Given its potential use in future engineering parametrizations, this allows to study the implications of such assumption in the flow solution and in the predicted wind farm power output.~~ This analysis is motivated by the fact that the rigid lid enforces dependency of the inversion layer height while discarding the full AGW solution, making it an appealing approximation in the context of reduced-order engineering parametrizations. As it will be shown, its estimates on blockage are in some cases better than those of a conventional wake model combined with a local induction model, which only account for local turbine induction.

3.1 AGW-Resolved Simulations

The subcritical and supercritical regimes of the AGW-resolved CNBL simulations are obtained by setting the inversion strength to 7.312 K and 4.895 K, respectively. Table 1 summarizes the remaining input parameters, namely the reference velocity u_{ref} at the reference height h_{ref} (chosen as the hub height), the reference potential temperature θ_0 , the lapse rate γ , the inversion height H and the equivalent roughness length z_0 . The Coriolis parameter f_c corresponds to a latitude of 41.33 deg. The simulated wind farm has a rectangular planform, with 20 rows and 5 columns organized in an aligned layout. The first row is located at $x = 0$, and extends from 300 m to 2700 m in the spanwise direction. This determines a lateral spacing of 600 m (4.76 D), while streamwise spacing is set to 630 m (5 D). Wind turbines correspond to the NREL 5-MW reference turbine, and are equipped with [the angular velocity and pitch controllers](#) described in Jonkman et al. (2009). A very simple yaw controller is also added, which rotates wind turbines independently using a constant rotation speed of 0.5 deg/s when flow misalignment exceeds 1 deg. Flow angle [for the yaw controller](#) is calculated by filtering the wind velocity at a sampling point located 1 D upstream of the rotor center, using a time constant of 600 s. Turbines are modeled using the actuator disk model (ADM) described in Stipa et al. (2023a), while the tower and nacelle are not included in the simulation. The ADM force projection width is set to 18.75 m.

u_{ref} [m/s]	h_{ref} [m]	θ_0 [K]	Δh [m]	γ [K/km]	H [m]	f_c [1/s]	z_0 [m]
9.0	90	300	100	1	500	$9.6057 \cdot 10^{-5}$	0.05

Table 1. ~~ABL-parameters~~ [Reference velocity \$u_{\text{ref}}\$ at the reference height \$h_{\text{ref}}\$, reference potential temperature \$\theta_0\$, inversion width \$\Delta h\$, lapse rate \$\gamma\$, inversion center \$H\$, Coriolis parameter \$f_c\$ and equivalent roughness height \$z_0\$ used as input for the finite wind farm simulation simulations](#) presented in this section.

The AGW-resolved simulations employ the hybrid off-line/concurrent precursor method described in Stipa et al. (2023a). For the off-line precursors, the Rampanelli and Zardi (2004) model is used to initialize the potential temperature profile, where ~~we take H is taken~~ as the center of the capping inversion layer. ~~Both off-line precursors~~ [Off-line precursors for both ABL conditions](#) are advanced in time for 10^5 s, after which data are averaged for $2 \cdot 10^4$ s. ~~Their Results from this phase are reported in Appendix A.~~ [The off-line precursor domain size is of \$6 \text{ km} \times 3 \text{ km} \times 1 \text{ km}\$ in the streamwise, spanwise, and vertical directions respectively. The off-line precursor-mesh has a horizontal resolution of 15 m, while in the vertical direction it is graded equally as the concurrent precursor and successor simulations, described later. A driving pressure controller ~~with geostrophic damping that employs the geostrophic damping method~~ is used to fix the average velocity at \$h_{\text{ref}}\$ while \[eliminating inertial oscillations in the free atmosphere. Moreover,\]\(#\) a potential temperature controller is used to fix the average potential temperature profile throughout the simulation \(both controllers and geostrophic damping use the ~~same~~ settings reported in Stipa et al., 2023a\). ~~The concurrent precursor method requires periodic boundary conditions combined with a single~~ \[Inflow slices saved during the off-line precursor phase are then used to feed the concurrent precursor for one flow through time \\(approximately 700 s\\). Then, boundary conditions in the concurrent precursor domain are switched to periodic and the solution becomes self-sustained.\]\(#\)](#)

405 At each successor iteration, velocity and temperature from the concurrent precursor are used to compute the damping terms for the momentum and temperature equations inside the fringe region, which we locate at the domain inlet located at the inlet of the successor domain. This allows a time-varying turbulent flow to be produced at the fringe exit while eliminating the reintroduction of the wind farm wake at the inlet operated by the periodic boundary and damping boundaries. Moreover, the fringe region allows to damp gravity wave reflections. At the upper boundary, we use a Rayleigh damping layer is used
410 with a thickness of 12 km, i.e. slightly more than one expected vertical wavelength λ_z (this parameter can be calculated as $\lambda_z = 2\pi U_g / N$, where N is the Brunt-Väisälä frequency and U_g is the geostrophic wind estimated as explained in Section 1). Lateral boundaries are periodic, implying that gravity waves induced by the wind farm will interact with their periodic images. This dictates that the domain must be sufficiently large for these interactions to happen far from the wind turbines. Moreover, we use the The advection damping technique developed by Lanzilao and Meyers (2022a) described in Section 2.1 is used
415 ensure that interactions between fringe-generated and physical gravity waves are not advected downstream but instead remain trapped inside the advection damping region. Specifically, we set the The Rayleigh damping coefficient to $\nu_{RDL} = 0.05 \alpha_r$ is set to 0.05 s^{-1} and, while the fringe damping coefficient to $\nu_{FR} = 0.03 \alpha_f$ is set to 0.03 s^{-1} . The fringe and advection damping functions are identical to Lanzilao and Meyers (2022a) and given by Equation (4) and Equation (6), respectively, and their parameters are reported in Table 2.

$x_s - x_s^f$ [km]	$x_e - x_e^f$ [km]	$\Delta_s \Delta_s^f$ [km]	$\Delta_e \Delta_e^f$ [km]	$x_s - x_s^a$ [km]	$x_e - x_e^a$ [km]	$\Delta_s \Delta_s^a$ [km]	$\Delta_e \Delta_e^a$ [km]
-20	-15	1	1	-18	-11	1	1

(a) Fringe region parameters.

(b) Advection damping region parameters.

Table 2. Fringe and advection damping region information parameters.

420 Methodological sketch comparing the AGW-resolved method (top) and the AGW-modeled approach (bottom):

The domain size of the AGW-resolved successor cases is $40 \text{ km} \times 21 \text{ km} \times 28 \text{ km}$ in the streamwise, spanwise and vertical direction respectively, discretized with $1554 \times 1194 \times 345$ cells. All directions are graded to reach a mesh resolution of $30 \text{ m} \times 12.5 \text{ m} \times 10 \text{ m}$ around the wind farm, as reported by Stipa et al. (2023b). The concurrent precursor mesh coincides with the portion of the successor domain located inside the fringe region. As a consequence, it extends for $5 \text{ km} \times 21 \text{ km} \times 28 \text{ km}$.
425 Here, the mesh resolution in the streamwise direction is 15 m, while in the spanwise and vertical directions it is is identical to the successor mesh. Additional details of the AGW-resolved simulations are provided in Stipa et al. (2023b).

3.2 AGW-Modeled Simulations

The AGW-modeled simulations feature the same horizontal domain size and discretization as the AGW-resolving AGW-resolved cases. Conversely, the vertical domain size is set to 500 m so that the top boundary, which is coincident with the unperturbed
430 inversion height. Moreover, as layer height. Since the fringe region, Rayleigh and advection damping layers are not required, inlet-outlet boundary conditions are used along x. Specifically, we conducted in the streamwise direction. Due to the fact that

the same inflow data used in the AGW-resolved cases was not available because it has been generated at run time within the concurrent precursor, two additional off-line precursor simulations have been conducted, corresponding to the subcritical and supercritical conditions defined in Section 3.1. These are first run for 10^5 s, after which data are averaged for $4 \cdot 10^4$ s and inflow sections are saved at each iteration to be used as inlet boundary conditions in the wind farm successors. These additional precursor cases are characterized by an enlarged spanwise domain size of 21 km – coincident with the successor cases – to avoid the spanwise periodization of the inflow data that used by Stipa et al. (2023b) in the characterizes the initial condition for the AGW-resolved cases. ~~In fact, as in Stipa et al. (2023b). As~~ reported by the same authors, this led to turbulent streaks that slowed down the convergence of turbulence statistics. ~~Moreover, we applied~~ To further address this issue, a spanwise shift velocity of 1 m/s is applied to the inflow data in the AGW-modeled successor simulations, with the objective of enhancing statistics convergence. ~~Specifically, instead~~ Instead of being added to the inflow velocity field, such shift velocity is used to physically move the inflow data along the spanwise direction so that the average wind direction remains unaffected. Results from these off-line precursor characterized by an enlarged domain are reported in Appendix A, together with their comparison with the off-line precursors conducted for the AGW-resolved simulations. The inflow data is then mapped at the successor inlet patch by means of bi-linear interpolation, further interpolating at the desired time value from the two closest available times. AGW-modeled simulations are progressed in time for $4 \cdot 10^4$ s, using the entire inflow database. ~~As our method does not require the definition of a potential temperature field, nor the solution of the corresponding equation, we used the velocity inflow data of the subcritical case to prescribe an inlet for the rigid lid simulation. A qualitative comparison between the This differs from the~~ AGW-resolved and AGW-modeled methodologies is reported in ?? cases, which have been only progressed for $2 \cdot 10^4$ s. Since the unperturbed flow corresponds to a CNBL, we do not solve for potential temperature, as this is constant throughout the simulation domain. As explained in Section 2.2, stability effects on the ABL flow are embedded in the applied vertical displacement of the inversion layer, which imposes the corresponding pressure perturbation.

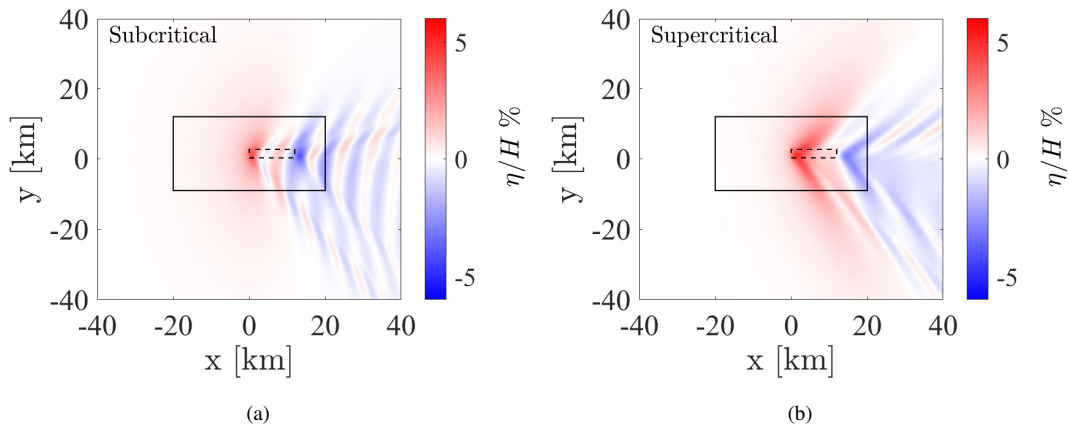


Figure 3. Inversion displacement as a percentage of the boundary layer height for (a): subcritical case and (b): supercritical case. The LES domain is identified by the continuous rectangle, while the wind farm is represented by the dashed rectangle.

Finally, the simulation setup for the AGW-modeled cases is complete upon providing the inversion layer displacement that is necessary to vertically deform the top boundary. As previously mentioned, this is calculated using the MSC model. 455 ~~As the model's input parameters~~ The input parameters for the MSC model are calculated using the fully-developed off-line precursors of the AGW-resolved cases to ensure consistency between these and the AGW-modeled simulations. Although the input parameters required by the MSC model are detailed in Stipa et al. (2023b), ~~we only show the~~ they are also reported in Table 3 for completeness. The resulting inversion displacement ~~corresponding to the subcritical and supercritical cases fields~~ are displayed in Figure 3. For more details about the MSC model setup the reader is referred to Stipa et al. (2023b), where the 460 same atmospheric conditions have been investigated.

4 Results

The proposed modeling approach has two main advantages. First, it allows to cut down the computational cost associated with including AGW effects in wind farm simulations, by requiring a domain that only extends to the capping inversion layer. For the cases presented in this manuscript, the AGW-modeling technique requires a domain that is more than 85% smaller 465 compared to the AGW-resolving approach. Secondly, it does not require numerical artifacts aimed at damping gravity waves reflections at the domain boundaries, as these waves are not physically resolved by the simulation. Regarding the simulation corresponding to the rigid lid approximation, the velocity inflow data from the subcritical case has been used to prescribe the inlet boundary condition, while the top boundary, also located at $H = 500$ m, has not been displaced.

In Section 4.1

<u>input parameter</u>	<u>N1</u>	<u>N2</u>	
g		9.81	[m/s ²]
ρ		1.225	[kg/m ³]
H, H_1, H_2		500, 180, 320	[m]
$\Delta\theta$	<u>7.312</u>	<u>4.895</u>	[K]
θ_0		300	[K]
γ		1	[K/km]
ϕ		41.33	[deg]
z_0		0.05	[m]
u^*		0.43	[m/s]
$\nu_{t,1}, \nu_{t,2}$		9.37, 6.19	[m ² /s]
(U_1, U_2, U_3)	<u>(8.31, 10.07, 9.77)</u>	<u>(8.41, 10.32, 10.16)</u>	[m/s]
(V_1, V_2, V_3)	<u>(-0.05, -0.78, -4.49)</u>	<u>(0.09, -0.2, -4.41)</u>	[m/s]
$\ \tau\ _{z=0}, \ \tau\ _{z=H_1}$		0.19, 0.11	[m ² /s ²]
TI_∞		0.09	[-]

Table 3. MSC model input parameters for the subcritical and supercritical case. The parameters $\nu_{t,1}$ and $\nu_{t,2}$ are the deep-averaged effective viscosities in the wind farm and upper layers, evaluated using the Nieuwstadt (1983) model; U_i and V_i (with $i = 1 : 3$) are the streamwise and spanwise velocity components, respectively, deep-averaged from the AGW-resolved off-line precursors within layer i ; TI_∞ is the hub-height freestream turbulence intensity.

470 4 Results

In the following, the accuracy of the proposed ~~AGW-modeling method is verified against AGW-resolving AGW-modeled method is first verified against AGW-resolved~~ simulations corresponding to Stipa et al. (2023b), for both subcritical and supercritical conditions, in Section 4.1. Then, in Section 4.2, ~~we investigate~~ the implications of employing the ~~rigid-lid rigid lid~~ approximation (Smith, 2023) in terms of our ability to capture global blockage effects are investigated. The latter ~~completely~~
475 ~~neglects~~ corresponds to an infinitely high free atmosphere stability ~~and treats~~, obtained by modeling the inversion layer as a rigid lid ~~that cannot be deformed~~. As a consequence the resulting horizontal pressure gradient solely responds to the requirement of mass conservation inside the boundary layer.

~~In the following analysis, time averaging of the simulation results was performed for~~ For the cases presented in this manuscript, the AGW-modeled technique requires a domain with $\approx 12.7\%$ of the grid cells used for the AGW-resolved
480 simulations. Although the domain used for the off-line precursors is larger for the AGW-modeled cases, this is not a requirement of the developed approach. In fact, a smaller domain with lateral inflow periodization technique is probably sufficient if combined with the spanwise shift used in this manuscript to accelerate statistics convergence.

The AGW-resolved and AGW-modeled cases consist of 15,000 s and 35,000 s of available data, respectively, following the establishment of a statistically-steady flow field. Moreover, in the successor simulations. However, when comparing turbine quantities, time averaging has been performed for 15,000 s in both cases. Specifically, since the AGW-modeled cases used different precursor time histories from the AGW-resolved counterparts, the start of the time averaging window was has been shifted in time until the row-averaged freestream velocity at the first turbine row matched those from the corresponding AGW-resolved simulations. This procedure allows to compare the two approaches eliminating any bias in freestream wind speed produced by the different large-scale turbulent structures in the two cases. More details on this procedure and on its motivation are reported in Appendix B. This approach is only followed when comparing turbine power and thrust between the AGW-resolved and AGW-modeled simulations. Elsewhere, results are always averaged on the entirety of the available time history.

4.1 Model Verification

Global blockage can be seen as the wind While local blockage is given by the combination of individual turbine induction effects, global blockage can be explained as the flow responding to the pressure gradient induced by the mean vertical displacement of the boundary layer due to the vertical perturbation triggered ABL displacement (Stipa et al., 2023b). In turn, this results from the vertical velocity perturbation operated by the wind farm and the corrective response provided by buoyancy forces. Specifically, the The induced perturbation pressure field is heterogeneous in space and features an unfavorable pressure gradient region upstream of the farm and a favorable region that extends through throughout most of the cluster. Downstream and around the wind farm, the perturbation pressure field is strongly dependent on the strength of the inversion, the free atmosphere lapse rate γ , and the geostrophic wind.

In light of the critical role played by the perturbation pressure field, we first compare the mean pressure variations resulting from the AGW-modeled approach with those obtained by resolving gravity waves in the free atmosphere. Figure 4 plots the streamwise distribution of pressure perturbation, spanwise-averaged-averaged over the wind farm width and in the upper layer (from $2h_{\text{turb}}/H_1$ to H), for both the subcritical and supercritical cases, using the AGW-resolved and AGW-modeled approaches. For completeness, we also show the pressure variation resulting from the MSC model is also reported. In both atmospheric cases, the AGW-modeled and the AGW-resolved approach predict almost the same states, all models predict similar trends in the pressure perturbation distribution. This, as As explained in Section 2.2, the latter is a function of the imposed vertical boundary layer displacement for the AGW-modeled simulations, while it naturally arises from the free atmosphere solution in both the AGW-resolved approach As previously reported by Stipa et al., 2023b, data are also in good agreement with results from and the MSC model. Subcritical conditions produce larger pressure gradients if compared to the supercritical ABL state, both unfavorable upstream and favorable inside the wind farm. Moreover, lee waves can be observed in subcritical conditions (also visible in Figure 3), which induce pressure oscillations on a wavelength that is smaller than the wind farm length. These waves As can be appreciated from Figure 4, these oscillations are superimposed on the favorable pressure gradient inside the wind farm and lead to oscillations in the background velocity field that yield, power variations throughout the wind farm

~~as well as and~~ an intermittent wake recovery (Stipa et al., 2023b). ~~In both atmospheric states, the largest difference between downstream.~~

Differences in the pressure disturbance predicted by the AGW-resolved ~~simulations and~~, AGW-modeled and MSC results can be explained as follows. First, with reference to the AGW-modeled and MSC ~~results is near the domain exit. In our opinion,~~
520 ~~this is because the~~ model results, differences in the pressure disturbance are attributable to the simplifications made in the MSC model such as linearity, the simpler parametrization of the wind farm and of the turbulent momentum fluxes, as well as the lack of resolved turbulence. As a consequence, even though η is identical between the two approaches, these aspects inevitably affect the momentum budget, leading to differences in both velocity and pressure. Regarding the AGW-resolved ~~simulations employ a fringe region~~ and AGW-modeled approaches, differences in the pressure field arise from slight differences in the imposed
525 η distribution (AGW-modeled) versus the η distribution that develops naturally (AGW-resolved), although the two methods share the same accuracy inside the ABL. Notably, both the MSC and AGW-modeled approaches are able to capture the different trends in pressure disturbance arising from the potential temperature structure in the subcritical and supercritical cases. This is achieved at a drastically lower computational cost than the AGW-resolved method, in our opinion justifying the small pressure deviations observed in Figure 4. Moreover, the arguments provided in Section 2.2 regarding the role of the pressure variable
530 are confirmed, as its dependence on free atmosphere stability is captured in the AGW-modeled approach without solving for the potential temperature equation. Finally, the mismatch in p' between the AGW-modeled and AGW-resolved results near the end of the domain in Figure 4 is produced by the fringe region employed at the domain inlet in ~~which a source term is applied that the AGW-resolved simulations.~~ As explained in Section 2.2, the fringe region removes the wind farm wake by forcing the flow to adhere to the concurrent-precursor solution ~~by at~~ the fringe exit. In ~~so doing, the fringe region doing so, it~~ modifies the
535 momentum balance, altering the pressure field both inside and immediately upwind of the fringe region, which coincides with the domain exit in Figure 4 owing to the periodic boundary conditions.

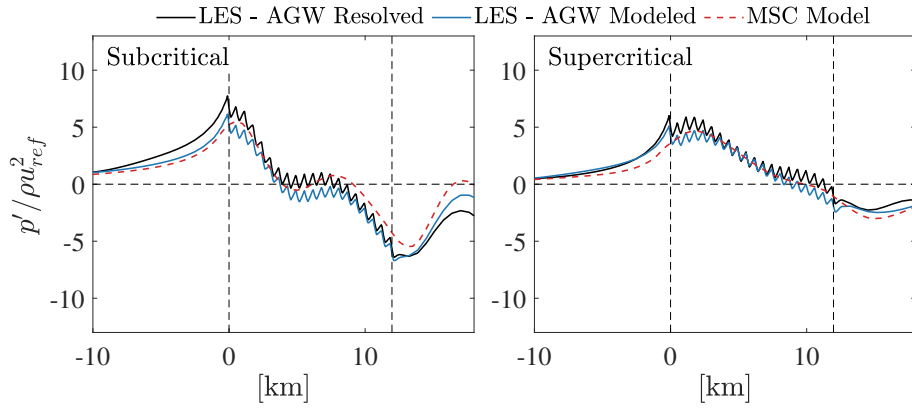


Figure 4. Time- and spanwise-averaged time-averaged streamwise distribution of pressure perturbation, further averaged over the wind farm width (between $y = 0$ and $y = 3000$ m) and in the upper layer (from H_1 to H), for both the subcritical and supercritical cases. For completeness, we also report data the pressure variation resulting from the MSC model, is also shown in dashed red. AGW-resolved and AGW-modeled results are shown in black and blue, respectively.

In Figure 5, we report Figure 5 shows the streamwise evolution of hub-height velocity, averaged over time and over the wind farm width, for both the subcritical and supercritical ABL states. Overall, results from using the AGW-modeled, for the AGW-resolved and AGW-modeled methods. On the right panel, the metric $(u_{AGWM} - u_{AGWR}) / u_{AGWR}$ in percent is also reported, showing the relative percentage error of the velocity predicted by the AGW-modeled with respect to the AGW-resolved methods are in good approach. Results from the two models are in excellent agreement with each other, indicating that the proposed methodology is able to capture not only blockage, but also the entirety of gravity wave effects on the ABL flow. Velocity reductions extending several kilometers upstream of the wind farm can be observed in both cases, indicating the presence of global blockage. Moreover, the mean velocity deficit in the wind farm wake is captured equivalently between both cases methods, where the effect of gravity-wave induced pressure gradients on promoting wake recovery can be observed, especially for the subcritical case. (Stipa et al., 2023a). Regarding the error on velocity, the differences between the two approaches are approximately $\pm 5\%$ inside the wind farm and -1% outside, which is small enough that becomes difficult to state if they are due to the AGW-modeled technique or attributable to the differences in the inflow data. Notably, this seems to be the case for supercritical conditions in the region upstream of the wind farm, where results from the AGW-resolved method depict a higher velocity than the AGW-modeled simulations.

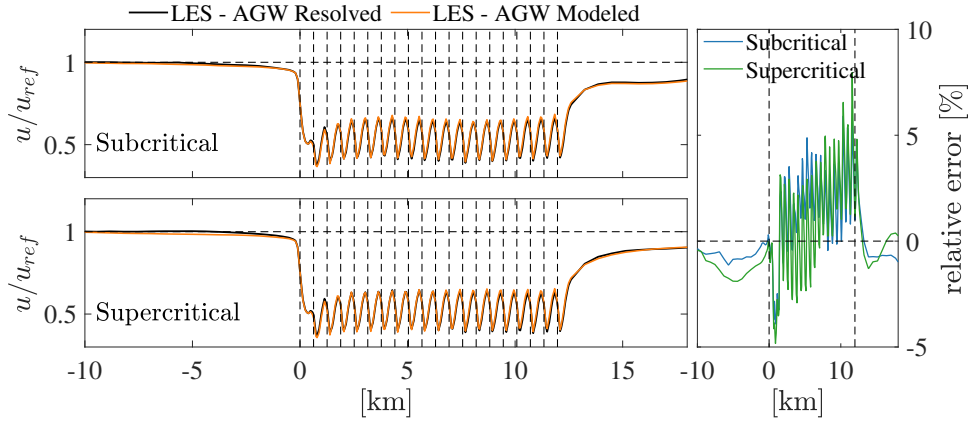


Figure 5. Time and spanwise averaged Time-averaged hub-height velocity, further averaged over the wind farm width (between $y = 0$ and $y = 3000$ m) for subcritical (top) and supercritical (bottom) conditions for both the AGW-resolved (black) and AGW-modeled (orange) approaches. On the right panel, the relative error of the AGW-modeled with respect to the AGW-resolved method, defined as $(u_{AGWM} - u_{AGWR})/u_{AGWR}$ in percent, is also shown.

In order to verify the accuracy of the proposed method in capturing the trend-turbine thrust and power trends along the wind farm length, the time and row-averaged thrust and power at each wind farm row is-are plotted in Figure 6 for both the subcritical and supercritical cases. In order to consistently average in time, the approach described in Appendix B has been used. The effect of lee waves aloft in the subcritical state-case can be appreciated by looking at the large scale oscillations in thrust and power throughout the wind farm. Moreover, the weaker favorable pressure gradient that characterizes supercritical conditions implies lower power towards the wind farm exit. Conversely, the subcritical state is affected by a stronger unfavorable pressure gradient upwind, leading to increased blockage effects. In general, increased blockage also leads to a more-stronger favorable pressure gradient within the wind farm-but. However, as demonstrated by Lanzilao and Meyers (2023), whether the net result is beneficial or detrimental depends on the specific conditions. Overall, the-proposed-AGW-modeling-it can be stated that the proposed AGW-modeled approach captures the effects of gravity waves on the wind farm thrust-and-power(see Table 4)power, which is arguably the most important information obtained from a wind farm LES. Table 4 reports the overall wind farm power, as well as the non-local, wake and total wind farm efficiencies η_{nml} , η_w and η_{tot} , respectively, as defined by Lanzilao and Meyers (2023), namely

$$\eta_{nml} = \frac{P_1}{P_\infty} \quad \eta_w = \frac{P_{tot}}{N_t P_1} \quad \eta_{tot} = \eta_{nml} \eta_w, \quad (11)$$

where P_1 is the average power at the first wind farm row, P_{tot} is the total wind farm power, N_t is the total number of wind turbines and P_∞ is the power that an isolated wind turbine would experience in the same operating conditions. Notably, η_{nml} quantifies blockage effects, η_w provides information on wake effects, while η_{tot} refers to the overall wind farm efficiency. In order to compute η_{nml} , the power P_∞ produced by an isolated wind turbine subject to the same conditions is required. To get this information, Lanzilao and Meyers (2023) conducted additional isolated turbine LES that employed the same inflow time

570 history used in the wind farm simulations. In our case, the inflow data is not available for the AGW-resolved simulations,
as it has been generated at runtime and is not saved to disk. Hence, we use the turbine data reported in Appendix B of
Stipa et al. (2023b) to compute P_∞ by interpolating the turbine power curve using the hub-height freestream velocity experienced
at the domain inlet of each simulation, averaged over the entirety of the available samples. Since the data from Stipa et al. (2023b)
575 are evaluated from LESs characterized by uniform inflow and absence of turbulence, the values of η_{nrl} and η_{tot} likely differ
from the figures that would be obtained the same inflow data as the AGW-resolved and AGW-modeled simulations. However
the differences in each parameter between the two methodologies can still be compared, highlighting the ability of the proposed
method to capture blockage and wake effects.

	subcritical					supercritical				
	P [MW] AGW-R	η_{tot}	η_{nrl}	η_w		P [MW] AGW-M	η_{tot}	η_{nrl}	η_w	
subcritical <u>AGW-resolved</u>	135.0	139.5 <u>139.5</u>	0.40 <u>0.40</u>	0.74 <u>0.74</u>	0.54 <u>0.54</u>	133.5	0.38 <u>0.38</u>	0.75 <u>0.75</u>	0.51 <u>0.51</u>	
<u>AGW-modeled</u>	<u>139.5</u>	<u>0.42</u>	<u>0.78</u>	<u>0.54</u>		133.3	<u>0.39</u>	<u>0.79</u>	<u>0.49</u>	

Table 4. Overall wind farm power obtained from LES ~~simulations~~ in subcritical and supercritical conditions using the ~~AGW-resolving (AGW-R)~~ AGW-resolved and ~~AGW-modeling (AGW-M)~~ AGW-modeled techniques. In addition, the total, non-local and wake wind farm efficiencies η_{tot} , η_{nrl} and η_w , respectively, are reported. The value of P_∞ required to compute η_{nrl} has been obtained from the data reported in Appendix B of Stipa et al. (2023b).

The obtained values of η_w agree better than η_{nrl} between the AGW-resolved and AGW-modeled cases, for both subcritical and supercritical conditions. The AGW-modeled method captures both the increase in blockage effects from supercritical to subcritical conditions (η_{nrl} from 0.79 to 0.78) as well as the efficiency improvement owing to the favorable pressure gradient throughout the wind farm in the subcritical case (η_w from 0.49 to 0.54). The larger differences observed in the values of η_{nrl} between the two methodologies demonstrate that the variations in total wind farm power — and consequently η_{tot} — are mostly due to a power bias at the first row rather than an inconsistency distributed over the entire wind farm. However, we believe that differences in the turbulent inflow data between the AGW-resolved and AGW-modeled simulations are the main cause of such power bias at the first row, considering the extremely good agreement on the hub-height velocity observed in Figure 5.

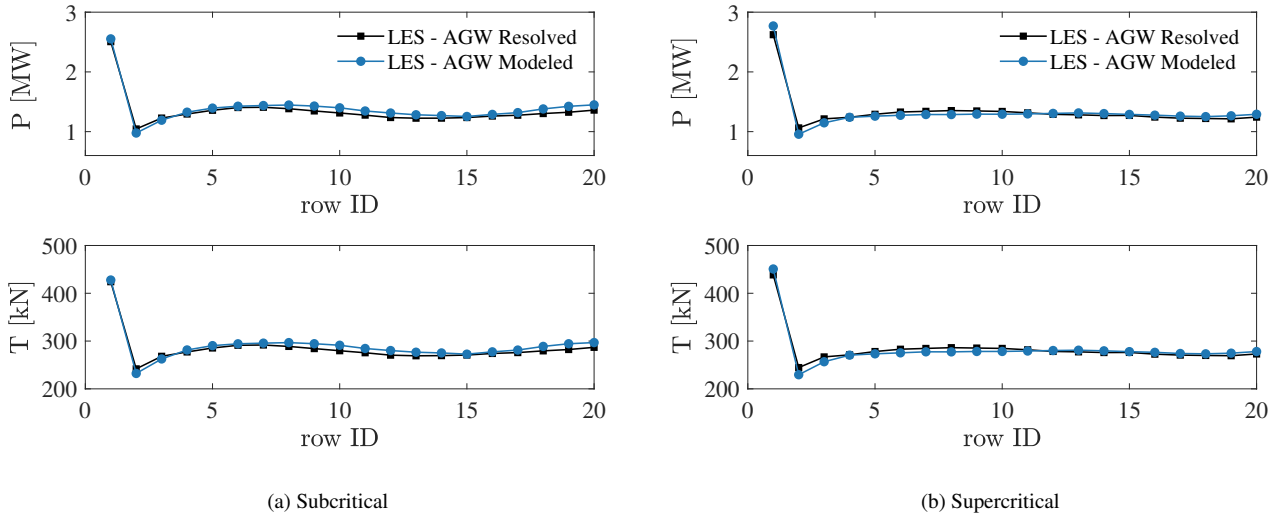


Figure 6. Comparison of row-averaged thrust and power distributions for ~~wind farm the~~ subcritical (a) ~~subcritical~~ and supercritical (b) ~~supercritical~~ cases. Time averaging is performed as described in Appendix B. AGW-resolved data are shown in black, while results from the AGW-modeled simulations are depicted in blue.

4.2 Rigid Lid Approximation

In the present section, ~~we leverage~~ the proposed methodology ~~is leveraged~~ to assess the implications of the ~~rigid-lid-rigid lid~~ approximation in evaluating global blockage effects. ~~Although this is certainly attractive from an engineering standpoint, as~~
590 ~~it allows for a simpler simulation, it fails to account for gravity waves. The impact of this omission on the global blockage is~~
~~therefore assessed~~ The simpler formulation with respect to the full AGW solution renders the rigid lid assumption attractive for
its potential use in future fast engineering models. However, its relation with the full AGW solution has not yet been assessed
in detail, together with its differences when compared with a truly neutral case, where stratification is absent.

To enforce the ~~rigid-lid-rigid lid~~ approximation within LES, we employ the ~~AGW-modeling technique where~~ AGW-modeled
595 ~~technique with~~ no vertical displacement of the top boundary ~~is applied, which is located at 500 m~~. Figure 7 shows ~~the mean~~
streamwise distributions of ~~velocity-hub-height velocity and depth-averaged pressure between H_1 and~~ ~~pressure, averaged in~~
~~time and H , further averaged~~ over the width of the wind farm, ~~i.e. from $y = 0$ m to $y = 3000$ m~~. In particular, ~~we compare~~
it compares the subcritical, supercritical and ~~rigid-lid-rigid lid~~ cases, all obtained using the ~~AGW-modeling-AGW-modeled~~
approach. A close up view of the blockage region within 1 km upstream of the wind farm is also reported. First, by looking at the
600 pressure gradient, it can be noticed how each case is characterized by an anti-symmetric pressure distribution, with maximum
and minimum at the wind farm start and exit, respectively. Moreover, the ~~rigid-lid-rigid lid~~ approximation is characterized
by the smallest values of favorable and unfavorable pressure gradients upstream and inside the wind farm, respectively. As
a consequence, while the ~~rigid-lid-rigid lid~~ approximation features global blockage, ~~it this~~ is less pronounced than both ~~the~~
subcritical or supercritical conditions. Regarding wake recovery, results from the ~~rigid-lid case exhibits-rigid lid case exhibit~~

605 the highest deficit. Overall, while it is clear that the ~~rigid-lid approximation cannot track~~ rigid lid approximation differs from the full gravity-wave induced effects, our results indicate solution in terms of pressure perturbations, wake recovery and farm blockage, our results suggest that flow confinement is responsible for a certain (lesser) extent of the global blockage. associated with a homogeneous — instead of heterogeneous — inversion height may be responsible for the majority of global blockage effects. This concept will be expanded further in this section.

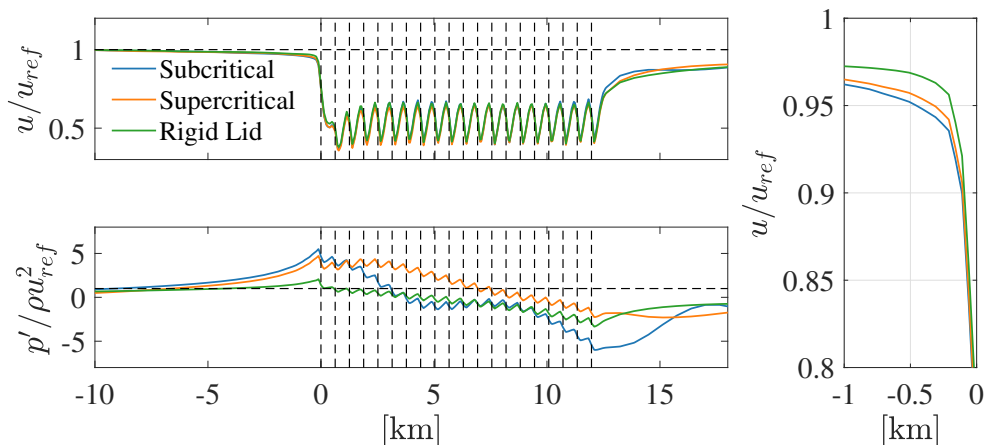


Figure 7. Comparison of velocity magnitude (top) and pressure (bottom) between subcritical, supercritical and rigid lid limiting solution cases. On the right panel, a magnification of the differences in velocity magnitude in the wind farm induction region is reported. Data correspond to the ~~AGW-modeled~~ AGW-modeled simulations and are averaged both in time (from 105,000 s to 140,000 s) and along the wind farm width (from $y = 0$ m to $y = 3000$ m). Pressure data is further averaged vertically between H_1 and H .

610 In particular, referring to the time and Regarding the mean row-averaged power distributions depicted in Figure 8, it can be stated that, for the simulated conditions, results obtained using the rigid-lid rigid lid approximation are not far from the subcritical and supercritical figures. Again, while wind farm gravity wave interaction is completely missing In fact, referring to the quantitative data reported in Table 5, overall wind farm power from the rigid-lid case agrees with the cases that include the effect of gravity waves rigid lid case seems in agreement with the simulations featuring AGW effects. The rigid lid case under-estimates power by 1.5% compared to the subcritical case and overestimates power by 3% with respect to the supercritical case. This trend agrees with the underlying hypotheses of the approximation, where the stronger the potential temperature jump across the inversion layer, the more it behaves as a rigid lid.

615

	P [MW]	rel. difference [%]
subcritical	139.5	-1.5
supercritical	133.3	3.0
rigid lid	137.4	

Table 5. Overall wind farm power obtained from LES simulations in subcritical and supercritical conditions, as well as employing the rigid lid approximation. The relative difference of the latter with respect to the first two cases is also reported. Time averaging has been performed in all cases from 105,000 s to 140,000 s.

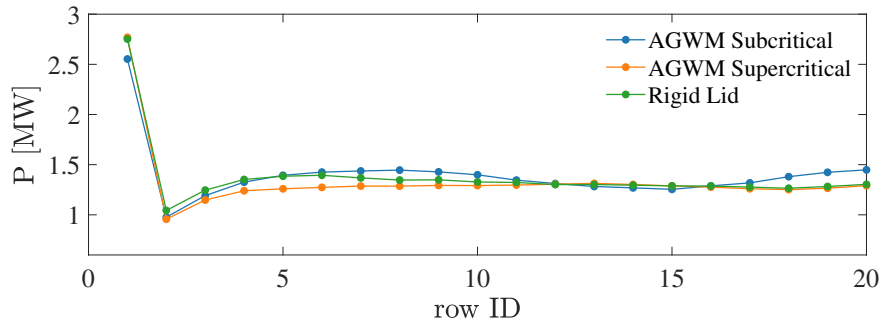


Figure 8. Row-averaged power from the subcritical, supercritical and rigid lid limiting solution cases. Data relative to the subcritical and supercritical conditions correspond to the AGW modeled simulations. Time averaging has been performed in all cases from 105,000 s to 140,000 s.

620 However, the LES results do not provide a clear picture on the relation between the full AGW solution and the rigid lid approximation, where the effect of $\Delta\theta$ and γ is removed. Notably, removing also the effect of H leads to considering a fully neutral boundary layer, where vertical streamline displacement is not constrained in any way. To further investigate this aspect, we used the relation between these three conditions, the MSC model has been used to run a parametric analysis in which lapse rate and inversion jump are systematically where $\Delta\theta$ and γ have been individually varied from 1 to 10-20 K/km and 0 to 10-20 K, respectively. For each of these atmospheric states, we compute the quantity $(P_{RL} - P_{MSC})/P_{MSC}$, where P_{RL} and P_{MSC} are. When varying $\Delta\theta$, γ has been set to 1 K/km to match the LESs conducted in this paper. Similarly, when varying γ , $\Delta\theta$ has been set to 7.312 K and 4.895 K, corresponding to the subcritical and supercritical conditions in this paper, respectively. In addition, two simulations corresponding to the fully neutral and rigid lid cases have been conducted. In the first, $\Delta\theta$ and γ have been set to zero, while in the latter they have been set to 1000 K and 1000 K/Km, respectively. The rest of the input parameters are identical to those reported in Table 3. Notably, all cases feature a local blockage model as described in Stipa et al. (2023b). For each run, the non-local, wake and total wind farm efficiencies are evaluated. The power of an isolated wind turbine in the same conditions has been calculated by running an additional MSC simulation where the local induction model has been

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630

removed from the fully neutral setup. This neglects any kind of blockage effect and all first row turbines produce the same power, taken as P_∞ .

For this analysis, it is worthwhile to recall the definitions of the interface Froude number F_r and of the parameter P_N , previously defined by Smith (2010), which regulate the physics and magnitude of interface and internal waves, respectively.

635 These can be calculated as

$$F_r = \frac{U_b}{\sqrt{g'H}}, \quad (12)$$

$$P_N = \frac{U_b^2}{NHG}, \quad (13)$$

where $g' = g\Delta\theta/\theta_0$ is the reduced gravity, and U_b is the bulk velocity inside the boundary layer. Subcritical and supercritical conditions are identified by $F_r < 1$ and $F_r > 1$ respectively, while the importance of internal waves reduces as P_N increases.

640 In Figure 9, the effect of varying $\Delta\theta$ on the wind farm efficiencies is shown. In the top axis, the inversion strength is converted to F_r using Equation (12), where U_b is calculated according to Allaerts and Meyers (2019). For low values of $\Delta\theta$, the non-local efficiency approaches that of a truly neutral case, which is only affected by the combination of individual turbine induction effects. Conversely, when $\Delta\theta$ is large, η_{nvl} approaches the rigid lid solution. The transition from these two cases is strongly non linear, and has a minimum around $F_r = 1$, identified by the vertical continuous line. Interestingly,

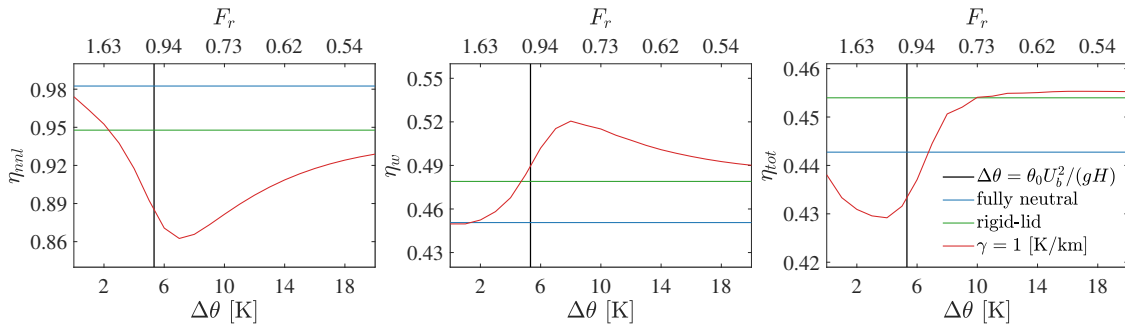
645 the minimum of the η_{nvl} occurs for a value of F_r slightly lower than unity. For instance, by simplifying the depth-averaged linearized Navier-Stokes equations, Allaerts and Meyers (2019) showed that the second derivative of the total vertical ABL displacement along the streamwise direction $\partial^2\eta/\partial x^2$ is multiplied by a factor $(-1 + F_r^{-2} + P_N^{-1}\mathcal{G})$ (the reader is referred to Allaerts and Meyers, 2019 for the definition of the convolutional operator \mathcal{G}), where F_r is defined by Equation (12). On one hand, this shows evidence that using the bulk ABL velocity as the characteristic velocity scale for F_r is based on mathematical

650 grounds. On the other hand, as previously noticed by Smith (2010), the term $(-1 + F_r^{-2} + P_N^{-1}\mathcal{G})$ produces a singularity when $P_N \rightarrow \infty$ (no internal waves). Conversely, when internal waves are present, energy is moved away from the source and the singularity disappears. Hence, referring to a condition where both $\Delta\theta$ and γ are non-zero, our results seem to suggest that the maximum blockage may be observed at $F_r = 1 - P_N^{-1}\mathcal{G}$ (\mathcal{G} is positive upstream the wind farm) instead of $F_r = 1$, thus exhibiting a dependence on γ . This behavior can be also noticed from Smith (2010) but has not been mentioned nor discussed

655 further. For our specific case, the minimum of η_{nvl} corresponds to $\Delta\theta \approx \theta_0 G^2/(gH)$, i.e. $F_r = 1$ when this is evaluated with G instead of U_g . Although this may only hold for the conditions adopted in Figure 9, ~~the values of total wind farm power obtained using the rigid-lid assumption within the MSC and the full MSC calculation that accounts for gravity waves drift in the minimum of η_{nvl} to lower values of F_r as γ increases seems to be a general conclusion, as shown later in Figure 11.~~ Still referring to Figure 9, the truly neutral case is characterized by the highest overall non-local efficiency, while the rigid lid approximation seems to represents a limiting solution for $\Delta\theta \rightarrow \infty$. In fact, both higher and lower values of η_{nvl} are obtained for different values of $\Delta\theta$ when considering the full AGW solution. The wake efficiency depicts a reversed behavior, with fully neutral conditions characterized by the lowest overall η_w . Regarding the total wind farm efficiency, this decreases from a value of ≈ 0.44 in the truly neutral case to ≈ 0.43 around $F_r = 1$. For increasing values of $\Delta\theta$, ~~respectively~~ η_{tot} increases

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again, overshooting the value corresponding to the rigid lid case by a small amount and slowly approaching it from above when $\Delta\theta \rightarrow \infty$. However, it is interesting to note that while fully neutral conditions represent the best and worst case scenario for η_{wvl} and η_w , respectively, the rigid lid approximation yields a value of η_{tot} that is close to the maximum achievable by the wind farm, making it far less conservative.



Error

relative to the AGW solution on wind farm power produced by the rigid-lid approximation, as predicted by the MSC model.

Figure 9. Non-local (left), wake (center) and total (right) wind farm efficiency as a function of $\Delta\theta$, when $\gamma = 1$ K/km (red line). Blue and green lines refer to the fully neutral and rigid lid cases, respectively. Continuous vertical black line refers to $F_r = 1$. The value of F_r , as defined by Equation (12) is reported on the top axis.

Our results, reported in ??, show that the full gravity wave solution approaches the rigid-lid approximation for high values of inversion strength and free atmosphere lapse rate. This is consistent with the grounds upon which the approximation is based. Moreover, the behavior of η_{wvl} , η_w and η_{tot} when varying γ with $\Delta\theta$ fixed is somewhat simpler. This is depicted in Figure 10, where the corresponding value of P_N is also shown on the top axis by converting each γ using Equation (13). First, it can be noticed how the approximation always over-predicts wind farm power, with greater errors (around 7%) observed for supercritical conditions (that efficiencies are less sensitive to γ than $\Delta\theta$ approximately less than 6 K for this specific background wind profile) and low values of the free atmosphere and their behavior is simpler than that observed in Figure 9. Interestingly, the subcritical case shows little dependency of η_{tot} on γ . In supercritical conditions, the wind farm produces less power than truly neutral conditions for low values of γ (also confirmed by our LES simulations), while the efficiency is superior when $\gamma \geq 10$ K/km.

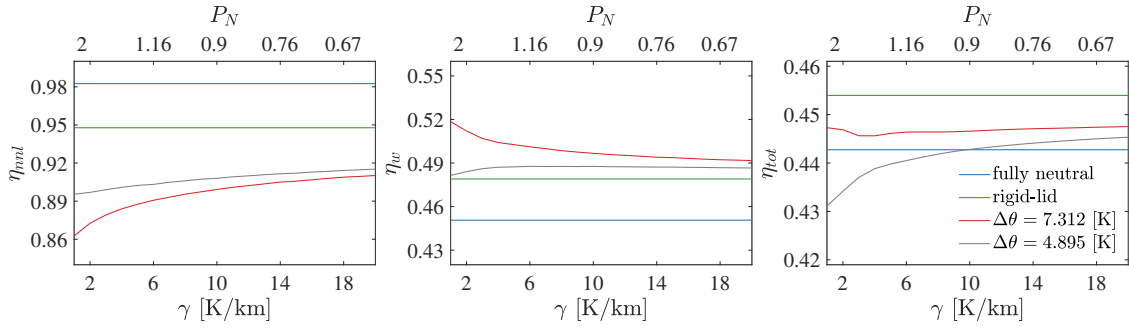


Figure 10. Non-local (left), wake (center) and total (right) wind farm efficiency as a function of γ , with $\Delta\theta = 7.312$ K (red line) and $\Delta\theta = 4.895$ K (gray line). Blue and green lines refer to the fully neutral and rigid lid cases, respectively. The value of P_N as defined by Equation (13) is reported on the top axis.

The parametric study has been then expanded by systematically computing the wind farm efficiencies for all combinations of $\Delta\theta$ and γ between 0 and 10 K and 1 and 10 K/km, respectively, with unitary step. The result of this analysis, reported in
680 Figure 11, show that η_{tot} increases when both $\Delta\theta$ and γ increase, with higher sensitivity to $\Delta\theta$. Notably, conditions where the wind farm extracts more power are also characterized by a large amount of blockage, as testified by the contours of η_{nml} . In fact, the decrease in non-local efficiency is compensated by the effect of the favorable pressure gradient inside the wind farm, which increases η_w . The location in terms of values of $\Delta\theta$ and γ where the minimum η_{nml} is experienced is shown in the η_{nml} contour. This supports the earlier observation that the minimum of η_{nml} corresponds to a F_r slightly lower than one, further
685 decreasing for increasing lapse rate. ~~This trend is generally confirmed by the LES results.~~ These results have been obtained for an aligned wind farm layout with a fixed number of turbine rows and columns, but they are likely to also depend on the wind farm geometry. However, this is outside of the scope of this paper and represents a subject for future investigation.

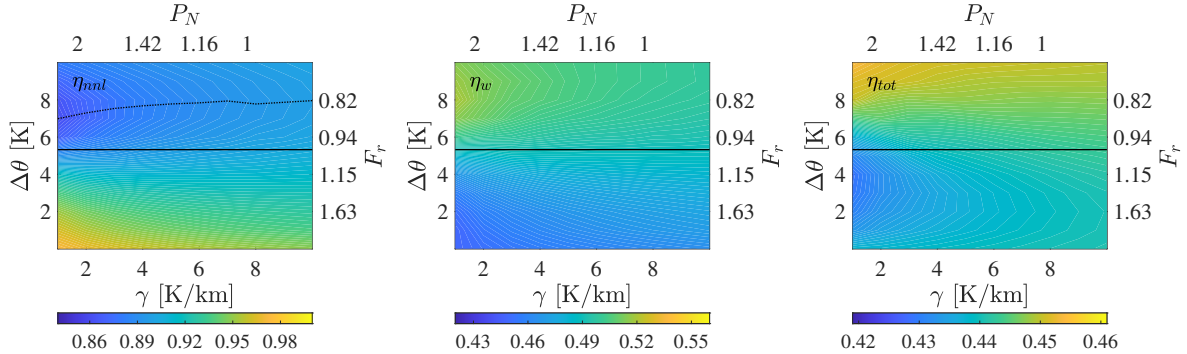


Figure 11. Contours of non-local (left), wake (center) and total (right) wind farm efficiency as a function of $\Delta\theta$ and γ . Continuous horizontal black line refers to $F_r = 1$. The value of F_r as defined by Equation (12) is reported on the right axis, while the value of P_N as defined by Equation (13) is reported on the top axis. The dashed line on the plot of η_{nml} indicates the locus of minima for the non-local efficiency.

690 To summarize, our results show that the rigid lid approximation yields a total wind farm efficiency that is close to the maximum achievable by the wind farm, while results obtained under a fully neutral ABL are in the middle of the analyzed conditions. This highlights that AGWs play a crucial role in determining the actual value of η_{tot} , which is in general lower than that observed when only the effect of H is considered. As a consequence, models employing the rigid lid approximation likely overestimate wind farm power, and it is reported in Table 5. Specifically, the rigid-lid case under-estimates power by 1.5% if compared to the subcritical case, while power is overestimated by 3% with respect to the supercritical case. Finally, in light of what stated above, the error is expected to reduce if higher values of the free atmosphere lapse rate are considered while
695 those adopting a fully neutral ABL may over- or underestimate wind farm power, depending on the structure of the potential temperature profile.

Finally, the present analysis did not investigate the sensitivity of our results to different values of the inversion height. Nevertheless, based on previous evidence (Lanzilao and Meyers, 2023), AGW effects are expected to fade away with increasing values of H , with both the rigid lid and the full AGW solutions likely approaching the fully neutral case.

700 ~~P-MWerr %subcritical 139.5 -1.5supercritical 133.3 3.0 rigid-lid 137.4 Overall wind farm power obtained from LES simulations in subcritical and supercritical conditions, as well as employing the rigid-lid approximation. The error of the latter with respect to the first two cases is also reported.~~

5 Conclusions

In this study, we introduced an approach that allows the effects of wind farm self-induced atmospheric gravity waves to be modeled without actually resolving these waves in the simulation. The proposed method couples the LES solution below the inversion layer with the MSC model developed by Stipa et al., 2023b. Specifically, the vertical perturbation to the inversion layer produced by the wind turbines, evaluated with the MSC model, is used to vertically deform the top boundary in the LES domain. Since prescribing the inversion displacement automatically establishes the pressure field below, the resulting LES velocity field contains the influence of gravity waves. If ~~conventionally neutral boundary layers (CNBLs)~~ CNBLs are
710 simulated, temperature transport becomes irrelevant as the flow is neutrally stratified inside the domain. ~~Atmospheric gravity wave~~ AGW feedback with the wind farm is provided by running the MSC model with multiple coupling iterations. The proposed method implies a computational domain that only requires ~~15~~ $\approx 12.7\%$ of the cells used in the conventional AGW-resolved method. Moreover, ~~referring to finite volume codes, the~~ simultaneous solution of a concurrent-precursor and the use of a fringe region are not required, as there are no gravity waves in the domain. ~~Thus, incoming ABL turbulence can be~~ prescribed using simple inflow-outflow boundary conditions. More generally, tedious and complex measures to avoid spurious gravity waves reflections, such as the Rayleigh damping layer and the ~~fringe advection damping~~ region, are no longer required.

The proposed approach has been verified against the LES simulations conducted in Stipa et al., 2023b. These are characterized by a ~~set-up~~ setup that allows to resolve ~~gravity waves~~ AGWs, and correspond to subcritical and supercritical regimes of interfacial waves within the inversion layer. ~~Our~~ The results show that the proposed method is able to accurately capture

720 the impact of gravity waves on pressure and velocity ~~, including with good accuracy, correctly estimating~~ blockage effects. Moreover, the row-averaged thrust and power distributions are in good agreement with that of the AGW-resolved approach.

Overall, our analysis shows that the proposed AGW-modeled method allows to model the impact of atmospheric gravity waves on wind farm performance at a reduced computational cost and with ~~great sufficient~~ accuracy. A drawback of the approach is that its performance depends on how accurately the MSC model captures the displacement of the inversion
725 layer. Moreover, the MSC model is currently limited to stationary and conventionally neutral boundary layers. For this reason, future work aims at including internal stability and time-dependency into the MSC model, enabling the ~~AGW-modeling~~ ~~AGW-modeled~~ method to simulate evolving and arbitrary ABL inflow conditions within LES at a low computational cost. ~~We also plan on extending the verification of the AGW-modeled approach on different atmospheric conditions using data from Lanzilao and Meyers (2023).~~

730 Furthermore, ~~we used the AGW-modeling method~~ ~~the AGW-modeled method has been used~~ to study the implications of adopting the ~~rigid-lid rigid lid~~ approximation (Smith, 2023). The latter neglects the inversion layer displacement produced by gravity waves ~~by only considering the effects of flow confinement imposed by the unperturbed inversion layer height, thus only considering approximated flow confinement effects.~~ While details due to wind farm gravity waves are expectedly absent, ~~flow confinement alone~~ ~~the rigid lid~~ still yields global wind farm blockage, and leads to ~~surprisingly small errors (less than an~~ ~~overestimation (3%) difference) and an underestimation (-1.5% difference)~~ in overall wind farm power when compared to
735 the ~~full gravity wave solution. To this end, the~~ ~~supercritical and subcritical cases, respectively, employing the AGW-modeled approach. To further investigate the relation between the full AGW solution, the rigid lid approximation, and fully neutral conditions (i.e. absence of stratification), the~~ MSC model has been used to systematically map the error in global wind farm power produced by the ~~rigid-lid rigid lid~~ approximation with different values of inversion strength and free atmosphere strat-
740 ification. The ~~rigid lid~~ approximation performs worse for supercritical interface wave regimes ($F_r < 1$) and low values of the lapse rate ~~(4—7% difference), while~~ γ . ~~Conversely,~~ the error reduces with increasing free atmosphere stability ~~, with greater sensitivity to $\Delta\theta$ rather than γ . Truly neutral conditions yield the lowest blockage and the greatest wake effects. The overall wind farm efficiency given by considering the full AGW solution can be lower or greater than the fully neutral case, depending on the vertical potential temperature profile structure, whereas the rigid lid approximation seems to yield an upper limit for the~~ ~~wind farm efficiency, which increases with increasing free atmosphere stability. This highlights the importance of considering the potential temperature profile structure when assessing wind farm performance.~~

Appendix A: Effect of Atmospheric Turbulence on Thrust and Power Averages Precursor Simulations

This section presents the results of the off-line precursor simulations used to generate the turbulent inflow for the AGW-resolved and AGW-modeled simulations. While they share the same input parameters, reported in Section 3, these precursor simulations employ a different domain size in the spanwise direction. Specifically, a domain of $6 \times 3 \times 1$ km has been prescribed in the off-line precursors used to initialize the flow in the concurrent precursor method within the AGW-resolved simulations. Conversely, the off-line precursor conducted to generate the inflow data for the AGW-modeled simulations employed a domain size of $6 \times 21 \times 1$ km. Figure A1 reports the horizontally-averaged vertical profiles of wind speed magnitude, wind angle, potential temperature and shear stress for the subcritical and supercritical cases, for the two different domain sizes. The inversion jump $\Delta\theta$, lapse rate γ , reference potential temperature θ_0 , inversion width Δh , inversion height H , friction velocity u^* , geostrophic wind G and geostrophic wind angle ϕ_G are also reported for each case in Table A1. The first five parameters have been obtained by fitting the potential temperature profile averaged between 100,000 s to 120,000 s and 100,000 s to 140,000 s for the AGW-resolved and AGW-modeled cases, respectively, using the Rampanelli and Zardi (2004) model. Since all simulations employ the potential temperature controller described in Stipa et al. (2023a), these quantities exactly match in all cases, which is further confirmed by the potential temperature profiles of Figure A1. The shear stress profile also agrees almost perfectly and all cases are characterized by the same final friction velocity u^* . Conversely, the wind speed magnitude and wind angle shows some minor differences, which we attribute to the use of the geostrophic damping technique.

domain size [km]	$\Delta\theta$ [K]	γ [K/km]	θ_0 [K]	Δh [m]	H [m]	u^* [m/s]	G [m/s]	ϕ_G [deg]
$6 \times 3 \times 1$	7.312	1	300.0	98.1	500.0	0.43	10.5	-24.0
$6 \times 21 \times 1$	7.312	1	300.0	98.1	500.0	0.43	10.3	-23.7
$6 \times 3 \times 1$	4.895	1	300.0	95.1	500.0	0.43	10.5	-23.8
$6 \times 21 \times 1$	4.895	1	300.0	95.2	500.0	0.43	10.3	-23.6

Table A1. Inversion jump $\Delta\theta$, lapse rate γ , reference potential temperature θ_0 , inversion width Δh , inversion height H , friction velocity u^* , geostrophic wind G and geostrophic wind angle ϕ_G evaluated from the off-line precursor simulations used for the AGW-resolved and AGW-modeled cases. The first five parameters have been obtained by fitting the potential temperature profile with the Rampanelli and Zardi (2004) model, after averaging in time from 100,000 s to 120,000 s and from 100,000 s to 140,000 s for the AGW-resolved ($6 \times 3 \times 1$ km domain) and AGW-modeled ($6 \times 21 \times 1$ km domain) cases, respectively.

Geostrophic damping is used to eliminate inertial oscillations produced by a geostrophic momentum imbalance when initializing the flow without any knowledge about the geostrophic wind. As explained in Stipa et al. (2023a), this situation occurs when one tries to control the horizontally-averaged wind velocity somewhere inside the boundary layer. In this case, it is impossible to know *a-priori* what the geostrophic wind will be, and G — required to apply the geostrophic damping action — has to be retrieved by inverting the equations for the geostrophic balance, using the pressure gradient calculated by the velocity controller. In turn, the pressure gradient is calculated by horizontally averaging the wind components at each iteration

770 during the simulation and thus it is expected that the averaging procedure yields slightly different values when using domains of different size. However, the difference in the final values of geostrophic wind between the small and the large domains is only about 0.2 m/s, which is acceptable in the context of the present study.

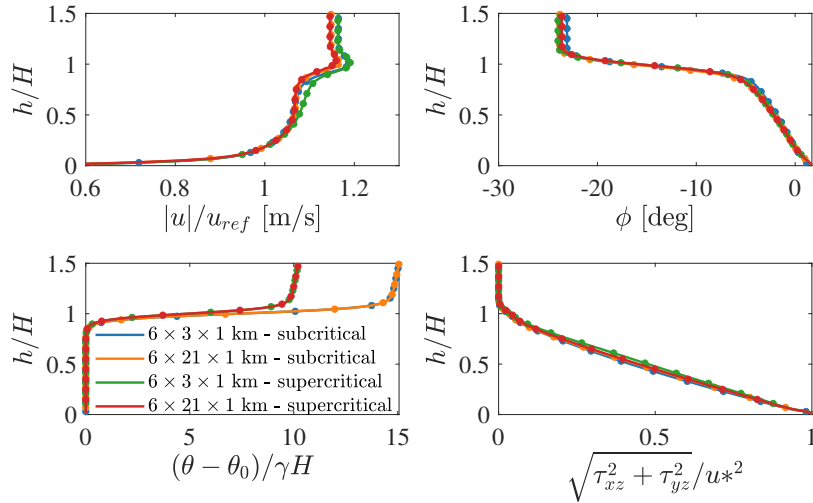


Figure A1. Horizontally and time averaged wind speed magnitude (top-left), wind angle (top-right), shear stress (bottom-right) and potential temperature (bottom-left) evaluated from the off-line precursor simulations used for the AGW-resolved and AGW-modeled cases. Time averaging has been performed from 100,000 s to 120,000 s and from 100,000 s to 140,000 s for the AGW-resolved ($6 \times 3 \times 1$ km domain) and AGW-modeled ($6 \times 21 \times 1$ km domain) cases, respectively.

Appendix B: Effect of Atmospheric Turbulence on Thrust and Power Averages

As mentioned in Section 3, although precursor simulations for the AGW-resolved and AGW-modeled cases shared the same inputs parameters, they run on domains of different size. As a consequence, ~~although while~~ simulations corresponding to the same CNBL conditions ~~produce~~ produced almost identical horizontally-averaged fields, ~~they also result in~~ (see Appendix A), these feature different realizations of ~~their the~~ time-resolved turbulent field. ~~While this is not a problem~~ Although this does not represent an issue for the small eddies, large turbulent structures may change the freestream velocity obtained by averaging over a time window that is comparable with their size. This is evident from Figure B1b, ~~where we report which reports~~ the instantaneous wind, averaged among the first row turbines, for the AGW-resolved and AGW-modeled approaches corresponding to both subcritical and supercritical conditions. When averaging time histories of e.g. turbine power or thrust, this effect ~~could can~~ introduce a consistent bias as these quantities depend on the cube and square of velocity, respectively. Unfortunately, for a time window of the order of the one ~~used in the present study~~ available in the AGW-resolved simulations (15,000 s), such effect introduces a variability in wind farm thrust and power that is comparable with the effect ~~if of~~ blockage. This is shown in Figure B1a, where we report the time history of the velocity sampled at the domain inlet and close to the

785 wind turbine located at the first row center, for both the subcritical and supercritical AGW-modeled simulations. As can be noticed, the two curves are vertically shifted due to blockage effects. However, even though 15,000 s can be considered a large averaging window, the variations in average velocity obtained by hypothetically shifting this average window in time are expected to be comparable with, if not bigger than, global blockage effects larger than, the vertical shift produced by blockage.

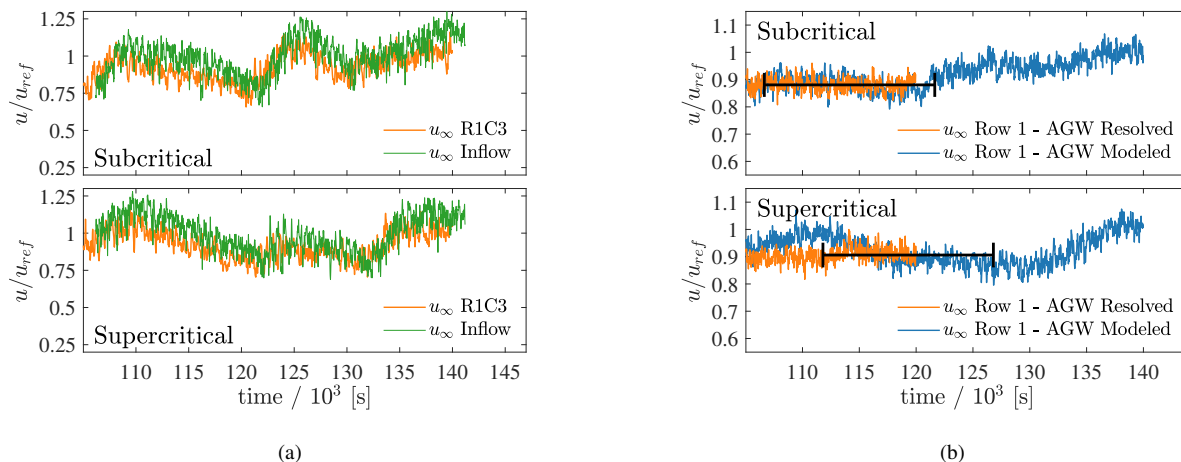


Figure B1. (a) Hub-height wind speed at the domain inlet (green) and as sampled by the wind turbine located in the middle of the first row (orange). Data correspond to the subcritical and supercritical AGW-modeled simulations. (b) Wind speed averaged among the first row wind turbines for the AGW-resolved (orange) and AGW-modeled (blue) cases. Data correspond to the both subcritical and supercritical cases. The black bar indicates the time window used to average turbine data in the AGW-modeled simulations, when these are compared against the AGW-resolved simulations.

For this reason, when comparing turbine power and thrust between AGW-modeled and AGW-resolved methods under the same CNBL conditions, we chose the averaging window for the former case as follows. First, we ensure that the same window is used for both cases, i.e. 15,000 s, corresponding to the entirety of the data available from the AGW-resolved analyses. Then, we shift the averaging window in the AGW-modeled cases until the freestream velocity averaged among the first row turbines matches the same quantity obtained from the AGW-resolved simulation. The averaging window resulting from this approach is reported in black in Figure B1b, for both subcritical and supercritical conditions. Finally, turbine thrust and power from the AGW-modeled cases are averaged over this window, ensuring that the wind farm sees the same inflow velocity in the two cases. Regarding flow variables, these We emphasize that this procedure is only applied when looking at turbine data, while the flow variables are always averaged throughout the entire simulation, i.e. from 105,000 to 120,000 s for the AGW-resolved and from 105,000 s to 140,000 s for the AGW-modeled cases. While-Although it is true that the approach described above likely forces the first row average power to match between AGW-resolved and AGW-modeled cases characterized by the same CNBL conditions, it should be recognized that it also allows thrust and power distributions to vary in the remaining rows. As a

consequence, this does not impair our ability to assess the accuracy of the AGW-modeled technique in capturing AGW effects on the row-by-row power production.

Code availability. TOSCA is available at <https://github.com/sebastipa/TOSCA>

805 *Author contributions.* Conceptualization, S.S, M.A.K., D.A., J.B.; methodology, S.S., M.A.K.; software, S.S; validation, S.S.; formal analysis, S.S.; investigation, S.S., M.A.K; computational resources, J.B.; data curation, S.S.; writing–original draft preparation, S.S., M.A.K.; writing–review and editing, J.B., D.A.; visualization, S.S.; supervision, J.B., D.A.; project administration, J.B.; funding acquisition, J.B.. All authors have read and agreed to the published version of the manuscript.

Competing interests. No competing interests are present.

810 *Acknowledgements.* The present study is supported by UL Renewables and the Natural Science and Engineering Research Council of Canada (NSERC) through Alliance grant no. 556326. Computational resources provided by the Digital Research Alliance of Canada (www.alliancecan.ca) and Advanced Research Computing at the University of British Columbia (www.arc.ubc.ca) are gratefully acknowledged.

References

- Allaerts, D. and Meyers, J.: Boundary-layer development and gravity waves in conventionally neutral wind farms, *Journal of Fluid Mechanics*, 814, 95–130, <https://doi.org/10.1017/jfm.2017.11>, 2017.
- 815 Allaerts, D. and Meyers, J.: Gravity Waves and Wind-Farm Efficiency in Neutral and Stable Conditions, *Boundary-Layer Meteorology*, 166, <https://doi.org/10.1007/s10546-017-0307-5>, 2018.
- Allaerts, D. and Meyers, J.: Sensitivity and feedback of wind-farm-induced gravity waves, *Journal of Fluid Mechanics*, 862, 990–1028, <https://doi.org/10.1017/jfm.2018.969>, 2019.
- Bastankhah, M. and Porté-Agel, F.: A new analytical model for wind-turbine wakes, *Renewable Energy*, 70, 116–123, <https://doi.org/https://doi.org/10.1016/j.renene.2014.01.002>, special issue on aerodynamics of offshore wind energy systems and wakes, 2014.
- 820 Bennett, A. F.: Open Boundary Conditions for Dispersive Waves, *Journal of Atmospheric Sciences*, 33, 176 – 182, [https://doi.org/https://doi.org/10.1175/1520-0469\(1976\)033<0176:OBCFDW>2.0.CO;2](https://doi.org/https://doi.org/10.1175/1520-0469(1976)033<0176:OBCFDW>2.0.CO;2), 1976.
- Béland, M. and Warn, T.: The Radiation Condition for Transient Rossby Waves, *Journal of Atmospheric Sciences*, 32, 1873 – 1880, [https://doi.org/https://doi.org/10.1175/1520-0469\(1975\)032<1873:TRCFTR>2.0.CO;2](https://doi.org/https://doi.org/10.1175/1520-0469(1975)032<1873:TRCFTR>2.0.CO;2), 1975.
- 825 Centurelli, G., Vollmer, L., Schmidt, J., Dörenkämper, M., Schröder, M., Lukassen, L. J., and Peinke, J.: Evaluating Global Blockage engineering parametrizations with LES, *Journal of Physics: Conference Series*, 1934, 012 021, <https://doi.org/10.1088/1742-6596/1934/1/012021>, 2021.
- Devesse, K., Lanzilao, L., and Meyers, J.: A meso-micro atmospheric perturbation model for wind farm blockage, Submitted to *Wind Energy Science Journal*, <https://arxiv.org/abs/2310.18748>, 2023.
- 830 Inoue, M., Matheou, G., and Teixeira, J.: LES of a Spatially Developing Atmospheric Boundary Layer: Application of a Fringe Method for the Stratocumulus to Shallow Cumulus Cloud Transition, *Monthly Weather Review*, 142, 3418 – 3424, <https://doi.org/https://doi.org/10.1175/MWR-D-13-00400.1>, 2014.
- Jonkman, J., Butterfield, S., Musial, W., and Scott, G.: Definition of a 5MW Reference Wind Turbine for Offshore System Development, National Renewable Energy Laboratory (NREL), <https://doi.org/10.2172/947422>, 2009.
- 835 Klemp, J. and Lilly, D.: Numerical Simulation of Hydrostatic Mountain Waves, *J. Atmos. Sci.*, 35, 78–107, [https://doi.org/10.1175/1520-0469\(1978\)035<0078:NSOHMW>2.0.CO;2](https://doi.org/10.1175/1520-0469(1978)035<0078:NSOHMW>2.0.CO;2), 1978.
- Klemp, J. B. and Durran, D. R.: An Upper Boundary Condition Permitting Internal Gravity Wave Radiation in Numerical Mesoscale Models, *Monthly Weather Review*, 111, 430 – 444, [https://doi.org/10.1175/1520-0493\(1983\)111<0430:AUBCPI>2.0.CO;2](https://doi.org/10.1175/1520-0493(1983)111<0430:AUBCPI>2.0.CO;2), 1983.
- 840 Lanzilao, L. and Meyers, J.: An Improved Fringe-Region Technique for the Representation of Gravity Waves in Large Eddy Simulation with Application to Wind Farms, *Boundary-Layer Meteorology*, <https://doi.org/10.1007/s10546-022-00772-z>, 2022a.
- Lanzilao, L. and Meyers, J.: Effects of self-induced gravity waves on finite wind-farm operations using a large-eddy simulation framework, vol. 2265, <https://doi.org/10.1088/1742-6596/2265/2/022043>, 2022b.
- Lanzilao, L. and Meyers, J.: A parametric large-eddy simulation study of wind-farm blockage and gravity waves in conventionally neutral boundary layers, 2023.
- 845 Lin, Y.-L.: *Mesoscale Dynamics*, Cambridge University Press, 2007.
- Maas, O.: Large-eddy simulation of a 15 GW wind farm: Flow effects, energy budgets and comparison with wake models, *Frontiers in Mechanical Engineering*, 9, <https://doi.org/10.3389/fmech.2023.1108180>, 2023.

- Nappo, C. J., ed.: An Introduction to Atmospheric Gravity Waves, vol. 102 of *International Geophysics*, Academic Press, 850 <https://doi.org/https://doi.org/10.1016/B978-0-12-385223-6.00014-8>, 2012.
- Nieuwstadt, F. T. M.: On the solution of the stationary, baroclinic Ekman-layer equations with a finite boundary-layer height, *Boundary-Layer Meteorology*, 26, 377–390, <https://doi.org/10.1007/BF00119534>, 1983.
- Rampanelli, G. and Zardi, D.: A Method to Determine the Capping Inversion of the Convective Boundary Layer, *Journal of Applied Meteorology - J APPL METEOROL*, 43, 925–933, [https://doi.org/10.1175/1520-0450\(2004\)043<0925:AMTDTC>2.0.CO;2](https://doi.org/10.1175/1520-0450(2004)043<0925:AMTDTC>2.0.CO;2), 2004.
- 855 Smith, R.: The wind farm pressure field, *Wind Energy Science Discussions*, 2023, 1–14, <https://doi.org/10.5194/wes-2023-56>, 2023.
- Smith, R. B.: Linear theory of stratified hydrostatic flow past an isolated mountain, *Tellus*, 32, 348–364, <https://doi.org/https://doi.org/10.1111/j.2153-3490.1980.tb00962.x>, 1980.
- Smith, R. B.: Interacting Mountain Waves and Boundary Layers, *Journal of the Atmospheric Sciences*, 64, 594 – 607, <https://doi.org/10.1175/JAS3836.1>, 2007.
- 860 Smith, R. B.: Gravity wave effects on wind farm efficiency, *Wind Energy*, 13, 449–458, <https://doi.org/https://doi.org/10.1002/we.366>, 2010.
- Stipa, S., Ajay, A., Allaerts, D., and Brinkerhoff, J.: TOSCA - An Open-Source Finite-Volume LES Environment for Wind Farm Flows, *Wind Energy Science*, 2023a.
- Stipa, S., Ajay, A., Allaerts, D., and Brinkerhoff, J.: The Multi-Scale Coupled Model: a New Framework Capturing Wind Farm-Atmosphere Interaction and Global Blockage Effects, *Wind Energy Science Discussions*, 2023, 1–44, <https://doi.org/10.5194/wes-2023-75>, 2023b.
- 865 Teixeira, M. A. C.: The physics of orographic gravity wave drag, *Frontiers in Physics*, 2, <https://doi.org/10.3389/fphy.2014.00043>, 2014.