Dear Editor, dear Reviewers,
Thank you very much for your comments and for taking the time to review our work.
In the following we go through your comments and provide, for each one, our responses and the actions we took to accommodate your feedback in the revised manuscript.
Best regards,
The Authors

Reply to Emmanuel Branlard (RC1)

https://doi.org/10.5194/wes-2023-25-RC1

Review Comment

In this paper the authors present a vortex model of the wake behind a lifting-line wing on a circular path. The model is compared with free-votex wake simulations and results from the literature.
This is a nicely written paper, with interesting methods and conclusions. I'd like to congratulate the authors for their work, which is rigorous, well presented, and includes useful aproximations and physical intepretations.
Please find below, and in the PDF enclosed, some comments that I hope can improve the revision of the paper.

My general comments are the following:

Authors Answer

The authors would like to thank the reviewer very much for the congratulations, the time and the interest dedicated to our paper. We feel that the manuscript has increased considerably in clarity thanks to your feedback.

Authors Actions

Please find in the attached PDF the direct answers to all your comments. In most cases, we modified the manuscript accordingly.

Review Comment

1) Higlighting the approximation and domain of validity

-------------- 1.1) One of the main contribution of the work consists of approximating the velocity field induced by a semi-infinite helical vortex, by a half vortex ring and a semi infinite series of vortex rings, and by using the assumption that the control point is close to the wing itself. The notations can take a bit of time to get accustomed to, and the change of variable (eta) (equal to -inf when r=0), and (phi_j,theta_h) can be a bit tricky to visualize. I think it can be valuable to highlight your contribution a bit more, and discuss the implication of the approximation used (removed the theta dependency in the integral, theta_j, phi_j small, and sometimes eta small) in a physical sense, and the domain of validity of your result. Adding a couple of extra sentences would be sufficient.

Authors Answer

Authors Actions

We commented on this at the end of Sect 2 and in the first paragraph of Sect 3.

Review Comment

------------- 1.2) If you have the time, here are additional suggestions along the lines of what I have in mind in my previous point. It could be valuable to somehow show the difference in velocity fields between the full semi-infinite filament and the approximate solution to get an idea of where it fails (most likely away from the filament). For instance, could people use your approximate formulae to get the velocity field near R_j=0, what would the error be? Similarly, what would the error be sligthly upstream or downstream? (This might be difficult to visualize as it's a 3D problem.). My understanding is that the approximations (and choice of variables) were done to be valid in a neighboorhood of the helical filament. You can also study the conditions where the theta-dependency matters, and the difference between Fig2a and Fig 2b becomes significant (e.g. large pitch angles). This could give additional context on the range of validity of your model. -

Authors Answer

This is an interesting point. We try to accommodate this request without adding any additional mathematical derivation, which, we feel, would be slightly out of scope. The axial induced velocities due to the near wake w^n_i can be evaluated at any radial (via eta) and tangential position (via theta_j), but not upwind and downwind (the dependence on phi_j is lost with the ring assumption). The mathematical derivation for the far wake is developed to exclusively find the induced velocities at the wing centre produced by the two vortex ring cascade. However, the induction produced by a vortex ring in a generic evaluating point can be easily computed by manipulating eq. 35 and 36. Therefore, the induction produced from the near and the far wake can then be studied as a function of eta (we modified Fig. 3 to show this study). Note that the first time eta is assumed small is in sect 3.1 and this assumption is only used in Sect. 3 to get to an analytical understanding of the near wake.
Concerning the induction along the tangential direction, we feel that Sect. 5.5 already partially addresses the topic. We prefer not to expand further on this topic to limit the paper in length, but we can further investigate this topic in future works.

Authors Actions

We modified Fig. 3, to show the difference in induced velocities between the helicoidal and the ring vortex systems as a function of eta and we elaborated on it.

Review Comment

------------- 1.3) On the topic of approximation of helices by vortex rings, there are a couple of numerical methods in wind energy that have used this, one of the earliest one I know is the following: https://doi.org/10.1115/OMAE2014-24227. But maybe you can find an older one . It could be worth citing at least one of them.

Authors Answer

Authors Actions

We added a citation in Sect. 4.3 to a paper from the same authors and the same year: https://doi.org/10.1088/1742-6596/555/1/012025 . We feel that the formulation in the chosen paper is more comparable with our formulation.

Review Comment

2) In your comment of figure 5, you mention that straight lines are a good approximation of the near wake. Similar to my comment above, I'm guessing that there are cases where this approximation breaks (e.g. small radius, fast rotation, large helical pitch, leading to large curvature and out-of-plane angles), and it would be interesting to study the domain of validity. But I agree that the first order effect is captured by a straight line.

Authors Answer

To get to this result, we had to assume eta to be small. Therefore, for small turning radii, we expect this approximation to start breaking. For wing types with more trailed vorticity close to the tips, we also expect this approximation to start breaking because strong vortices would be trailed at larger |eta| positions.

Note that here we compare the approximation of the half circle with the straight lines, so the helical pitch does not play any role. Comparing the straight lines with the half helical rotation would not be fully meaningful. Indeed, the two vortex systems of Fig. 2a and 2b are the most in agreement when the full system is studied. You could think that the approximation of the second half of the helix (from theta = pi to 2pi) to the correspondent semi-ring is -somehow-balancing the approximation of the first half of the helix (the near wake) to the correspondent semi-ring. In other words, the first half of the helix is flattened in the rotor plane, while the second half is flattened further from the rotor plane (at one pitch). This mechanism helps to get to the right induction in the rotor plane when considering the full system.

Authors Actions

We added a discussion about it.

Review Comment

3) In the text you mention "As vortex filaments move along iso-velocity lines in the fluid, their velocity in the far wake should be equal in modulus to their velocity when they are trailed". I'm not sure this is correct, I would need to think it through or be convinced. Please make sure you double check this.

Authors Answer

This is clearly not correct, thanks for pointing it out.

Authors Actions

We corrected the sentence accordingly.

Review Comment

You'll find other comments in the pdf enclosed. Please note that my comments are mostly suggestions, but I hope that addressing them with small modification of the text can be beneficial to other readers. Again, congratulation for your work, I'll be looking forward to review a revised version of this paper.
Emmanuel

Authors Answer
Thank you very much for your review.

Authors Actions
We answered the comments directly on the attached pdf.

Reply to Anonymous Referee#2 (RC2)

https://doi.org/10.5194/wes-2023-25-RC2

Review Comment
The paper develops a vortex-based model to obtain induced velocities (and thus the true relative flow velocity) at an airborne wind energy (AWE) system, more precisely, a crosswind kite power system. The motivation is to develop a model for reliable estimation of aerodynamic performance of such systems. This is an interesting paper and would be useful for both academic researchers and AWE practitioners. Thanks to the authors for their contribution to the field of AWE. Overall, the paper reads well, the mathematical formulations have been explained sufficiently, and the figures are clear and clean. However, I have a few comments/questions, and I invite the authors to consider them to improve the paper.

Authors Answer

The authors would like to thank the reviewer very much for the appreciation, the time and the interest dedicated to our paper. We feel that the manuscript has increased considerably in clarity thanks to your feedback.

Authors Actions

Review Comment
Technical comments:

1. In Section 5.3, a comparison is made between axial inductions obtained from the present model, and those obtained from QBlade, modified wake model from Gaunaa et al. (2020), and modified momentum model from Kheiri et al. (2018). In the absence of experimental data, I strongly suggest comparing your numerical results also with some CFD results, such as those from Reynolds-averaged Navier-Stokes (RANS) flow simulations or Large Eddy Simulations (LES). For example, you may refer to

Haas et al. (2017) for LES results (particularly, Figures 2 and 5) or Akberali et al. (2021) for RANS results (particularly, Figure 15).

Authors Answer

At the time of deciding which high-fidelity code to use for the validation we went for a free wake because it has similar assumptions to our model.

RANS and LES could also be used, but would require new computations as the suggested references cannot be directly used for comparison. Indeed, Haas et al. (2017) study the induced velocities from a AWES system composed of 3 wings moving in the same disc and our model assumes a single wing moving in the disc. An extension of our model to 3 wings AWE systems would need a dedicated (and interesting) work. Akberali et al. (2021) instead computed the axial induction by averaging the velocities over the rotor plane with the method developed for conventional turbines in https://doi.org/10.1002/we.127. In our work, the induction is a local induction at the lifting line. As our formulation is developed to be valid in the neighbourhood of the AWES itself, we cannot find the induction at a large angular distance of the AWES and therefore we cannot compute the average induction over the disc. We feel that our extensive validation (including Appendix B) is enough for this work and that comparisons with CFD, not necessarily carried out by us, would deserve a dedicated work.

Authors Actions

We added the reference to Akberali et al. (2021) in the introduction, which was missing.

Review Comment

2. On page 2, the authors mention that “The root and tip corrections for AWESs would however differ largely from conventional wind turbines.” Can you please explain why? What is fundamentally different between a conventional wind turbine and a crosswind kite in the straight downwind configuration (i.e., flying a circular path on a plane normal to the wind)?

Authors Answer

The difference is in the required corrections. The tip corrections for WT blades are developed to typically extend to approx 80 % of the blade from the tip. The root correction for WT blades is developed for blades extending to almost the rotation axis. For AWES, we would need two corrections for the two (left and right) wings. The example of our paper considers a kappa_0 of 0.15, corresponding to a wing span b equal to 30 % of the rotational radius R_0. We would need then to derive dedicated tip corrections for AWES to use momentum theory to find the induced velocities at the AWES. Our work, in a sense, can be seen as the derivation of these tip corrections. We refer to Gaunaa (2020) for more details.

Authors Actions

We briefly expanded on this in the introduction.

Review Comment

3. What is the physical implication/importance of this finding that “the induced drag of an elliptic wing due to near wake to be similar to the induced drag of the same wing in forward flight”?

Authors Answer

The importance of this is that the models assuming straight trailed vorticity are a good approximation only of the near wake. A couple of examples:

1) Let's say that we want to perform an aerodynamic analysis of an AWES with CFD methods. One could resolve the full helicoidal aerodynamic wake in an expensive, but correct, CFD computation. In this case, only one turning radius can be evaluated. If one assumes the same AWES in a forward motion in the CFD, then only the near wake is studied in a simplified way. The far wake can be included as a correction in the post process with our formulation, letting the analysis generic for any turning radius.
2) Let's say that we want to develop an engineering aerodynamic model for AWESs. We can use state-of-the-art engineering aerodynamic models, developed for wings in forward flight, for the near wake.

Authors Actions

We added a sentence summarising this point in the conclusions.

Review Comment

4. Concerning equation (23), are you neglecting the wind speed? If yes, please mention it in the text and provide some justifications.

Authors Answer

Yes, it is neglected. This follows the typical assumption used in AWE engineering model of G^2>>1.

Authors Actions

We added 2 equations to detail the assumptions and the derivation.

Review Comment

Editorial comments:

1. L. 69, "The goal of this study is [to] analytically …"
2. L. 76, "hereby" is extra
3. In equation (7), "dθ" should be removed
4. L. 145, "… be re-written as [an] infinite summation …"
5. "vortices" is more common than "vortexes"
6. In equation (30), the first CL on the r.h.s. should also be squared
7. L. 360, "… with [a] higher fidelity code…"
8. L. 429, remove the extra dot after the parenthesis
9. L. 449, "… is expressed in term[s] of …"
10. L. 457, remove the extra "induced"
11. Use "aero-servo-elastic" instead of "aero-servo-dynamic-elastic". Commonly, the term "aeroelasticity" also includes "dynamics"

References

Akberali, A.F.K., Kheiri, M. and Bourgault, F., 2021. Generalized aerodynamic models for crosswind kite power systems. Journal of Wind Engineering and Industrial Aerodynamics 215.

Haas, T. and Meyers, J., 2017, May. Comparison study between wind turbine and power kite wakes. In Journal of Physics: Conference Series (Vol. 854, No. 1). IOP Publishing.

Authors Answer

Authors Actions
Thank you again for these corrections. We have implemented all the suggested editorial changes.