Draft for non-linear wind vane correction model

Andreas Rott

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The deflection of the wind direction by the rotor largely results from two factors. Firstly, by the rotation of the rotor and a corresponding counter-rotation of the flow, and secondly by the thrust of the rotor if it is not directly aligned with the flow. For the model shown here, we assume that both causes are independent of each other and additive. This model is limited to the deflection caused by the thrust of the rotor.

The true wind direction is denoted by $\nu \in [-180°, 180°)$ and the wind direction measured by the wind vane by $\mu \in [-180°, 180°)$. Starting from the rotor centre as the coordinate origin, the wind vector in front of the rotor can be represented by the components u_{ref} (along the rotor axis, x-axis) and the lateral component v_{ref} (along the y-axis) (the w-component along the vertical upwards z-axis is neglected). It holds that

$$\tan(\nu) = \frac{v_{\mathrm{ref}}}{u_{\mathrm{ref}}}.$$

Behind the rotor, the wind direction consists of the u_{vane} and v_{vane} components. The v-component remains unchanged, $v_{\mathrm{vane}} = v_{\mathrm{ref}}$. The u-component is decelerated by the axial induction a of the rotor: $u_{\mathrm{vane}} = u_{\mathrm{ref}} \cdot (1-a)$. Since the wind vane is not directly in the rotor plane we added another parameter s, which has to be optimized based on the data: $u_{\mathrm{vane}} = u_{\mathrm{ref}} \cdot (1 - sa)$ This results in the following for the measured wind direction deviation:

$$\tan(\mu) = \frac{v_{\mathrm{vane}}}{u_{\mathrm{vane}}} = \frac{v_{\mathrm{ref}}}{u_{\mathrm{ref}} \cdot (1 - sa)} = \frac{v_{\mathrm{ref}}}{u_{\mathrm{ref}}} \cdot \frac{1}{1 - sa} \Rightarrow \mu = \arctan\left(\tan(\nu) \cdot \frac{1}{1 - sa}\right)$$

we substitute a with the thrust coefficient c_{T} with the relation $a = \frac{1}{2} - \frac{1}{2}\sqrt{1 - c_{\mathrm{T}}}$.

And the thrust coefficient gets a dependence on the misalignment $c_{\mathrm{T}} = c_{\mathrm{T0}} \cdot \cos(\nu)^p$ where p is a parameter that theoretically should be close to 2, but can be optimized based on the given data and c_{T0} is the thrust coefficient for perfect alignment of the rotor into the wind. Thus, overall, the model is given by:

$$\mu_{s,p}(\nu) = \arctan\left(\tan(\nu) \cdot \frac{1}{1 + \frac{s}{2} \cdot \left(\sqrt{1 - c_{\mathrm{T0}} \cdot \cos(\nu)^p} - 1\right)}\right).$$

Figure 1 shows a comparison of the non-linear model (red) and the linear model (black). The parameters of the non-linear model are set to $s = 1, p = 2$, and the linear model has a slope of $c = 1.26$. It can be seen that for smaller misalignment the amplification of the wind direction deviation is stronger than the linear model and for larger misalignment the amplification gets weaker converging to the bisector. To answer the question whether the difference between the non-linear and the linear model within the meaningful range of $\nu \in [-30°, 30°]$ is large enough to justify the increased complexity of the model, further research is needed.

Unfortunately, this model can only be inverted numerically in the domain of definition.

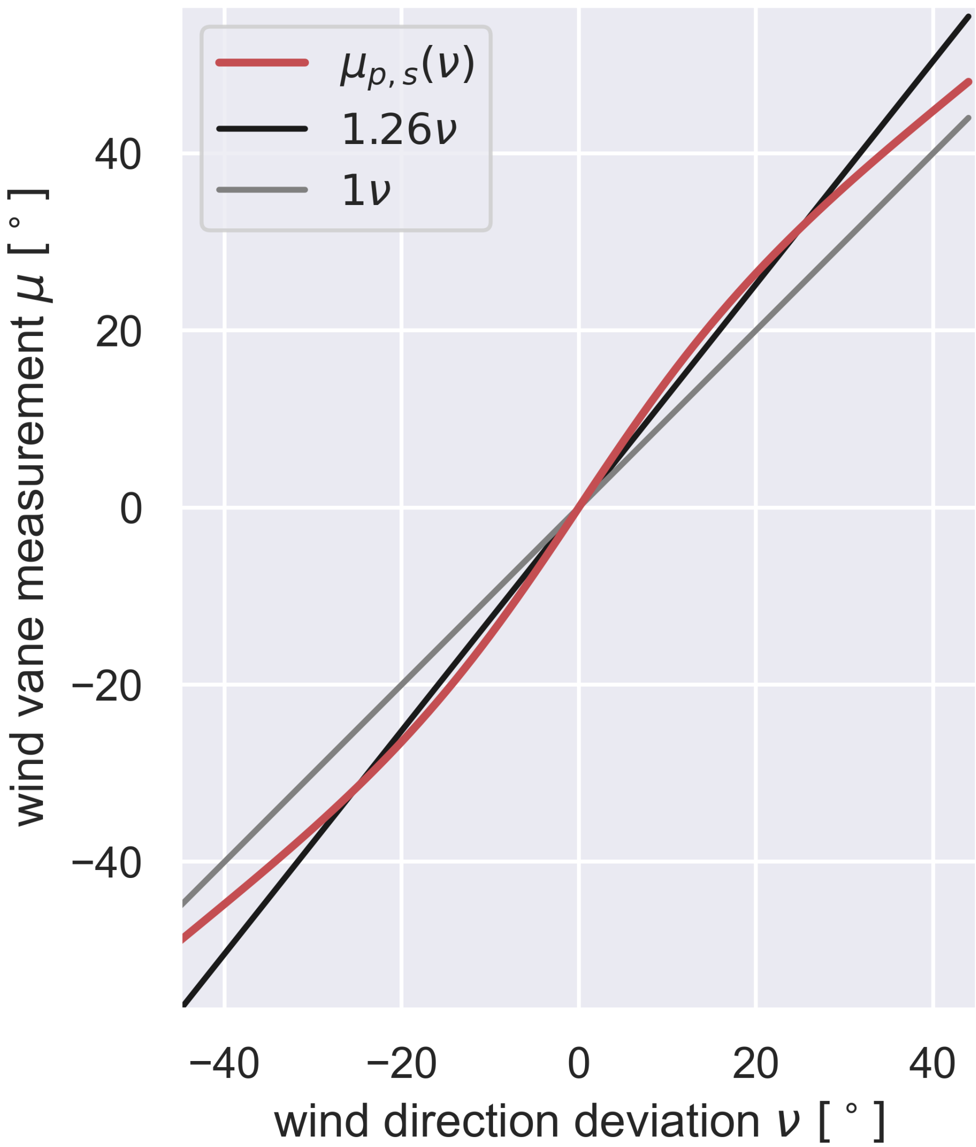

Figure 1: Comparison of non linear model (red), linear model with slope of 1.26 (black) and the bisector (grey)