[wes-2024-111]

Dear Anonymous Referee #1,

Thank you for your detailed review of the paper and provided comments. The authors have acknowledged the necessity of including a small section discussing the validity of Glauert's rotor disk theory, particularly at low tip speed ratios. Below are the author responses in *italics* and Anonymous Referee #1 comments in blue.

The calculus of variations (CoV) is a powerful mathematical tool for the analytic or semi-analytic determination of optimal solutions for a wide range of problems. Its first application to rotors that I am aware of is by Breslin & Andersen (1994, Additional References below) for lightly-loaded propellers. Using the Kutta-Joukowsky form of the thrust equation for an actuator disk representing the rotor, they maximised thrust for a given power and showed the critical importance of the pitch of the trailing vorticity; it is the ratio of torque to thrust. It follows that an optimal rotor has constant pitch throughout its wake. Wood & Hammam (2022) used the converse CoV analysis for wind turbines in which power is maximised for a given thrust. They also used the Kutta-Joukowsky thrust equation which avoids the need to consider pressure in the wake. They removed the restriction to light-loading and showed the same importance of the vortex pitch as the ratio of torque to thrust. No assumptions were made about the uniformity of the velocity through the rotor as is made in the present analysis based on the work of Glauert. The three other important results from Wood & Hammam (2022) were:

1. Their figure 4 shows that the axial induction factor is quadratic in radius at low tip speed ratio, λ, but becomes approximately constant at higher λ,
2. The disk loading – the angular velocity behind the disk, which is proportional to the bound circulation - behaves in the same way. As $\lambda \to 0$, the loading is quadratic in radius whereas it is constant with radius at high λ,
3. Optimal performance at low λ is constrained by the need to avoid recirculation in the wake.

The first two results certainly, and the third possibly, cannot be obtained from the Glauert assumption of constant induction so the present results are valid only for λ that is sufficiently high for the axial induction factor to be approximately constant.

<u>*Author Response:*</u> *The authors have added Section 6.1 titled Validity of Classical Glauert Theory in the revised manuscript. Glauert's assumptions in solving the 1-D problem are listed both in the introduction and section 6.1. The authors also acknowledge the approach taken by Wood & Hammam.*

To quantify the differences in the analyses: Wood & Hammam (2022) found the thrust coefficient CT $\to$ 0.357 as $\lambda \to 0$ which is less than half the value in figure 1 of the present submission. Interestingly, the differences in power coefficient are small at λ = 1: Wood & Hammam (2022) obtained 0.4381 compared to 0.4155 here whereas their CT = 0.738 compares to 0.846 here. Thus, allowing for radial variation in the parameters leads to a more optimal solution at low λ. To my knowledge, there are, unfortunately, no experimental studies that would help decide the question of optimal performance at low λ.

<u>Author Response:</u> The authors have added more details in Section 6.1 titled Validity of Classical Glauert Theory of the revised manuscript. It is evident that Wood & Hammam's approach yields more correct solutions at low λ, however it is restated that the purpose of this work was to simply build upon Glauert's classical theory, including all the associated assumptions.

<u>Additional References</u>

Breslin, J. P., and Andersen, P. (1994). Hydrodynamics of ship propellers. Cambridge University Press.

CT. Optimal performance of actuator disc models for horizontal-axis turbines. Frontiers in Energy Research, 10, 971177.

<u>Author Response:</u> The authors have added the suggested and other references (10 total) to the manuscript.