[wes-2024-111]

Dear Anonymous Referee #2,

Thank you for your detailed review of the paper and the provided comments for improvement. The authors acknowledge the need for including a brief relevant literature review discussing rotor disk theory. Below are the author responses in *italics* and Anonymous Referee #2 comments in blue.

Generally, the paper is well written with a logical development of the equations. It seems that the use of the method of calculus of variations yields an elegant approach for derivation of C_{Pmax}, C_T and C_{Be} for the tip speed ratio going towards infinity and zero, respectively.

However, the paper lacks completely an introduction section giving the work a wider perspective and describing the importance of the work. In particular, the advancement by previous work mentioned in the paper, "Wilson et al., 1974; Hansen, 2008; Manwell et al., 2009; Burton et al., 2011; Wood, 2011; Schaffarczyk, 2014; Sørensen, 2016; Schmitz, 2019) should be elaborated and the relation to the present work should be clear.

As mentioned above an introduction section should be inserted with a relevant literature review in relation to the subject. What advancements have been achieved in the past and how the present work contributes to new insight?

Author Response: Thank you for the positive comments about the writing and mathematics derived in the paper. The authors agree that an introduction section is helpful to highlight developments in optimum rotor disk models over the years. This also provides background for the new insights and coefficients derived in the present work within the constraints of Glauert's theory. Please see Section 1 titled Introduction in the revised manuscript. A total of 10 additional references were added.

Discussing shortcomings in the theory where they appear

- E.g. the jump from equation 29 with the thrust coefficient CT for the whole disc to eq. 30 with the thrust coefficient on an incremental form
    - Is eq. 30 valid in any case?

Author Response: Very good comment, thank you. Additional steps have been depicted showing the derivation of the incremental thrust coefficient from the original definition of C_T. This math can be found in the beginning of Section 4 titled Exact Integral of the Thrust Coefficient C_T Based on Glauert's Optimum Solution, specifically in Eqs. 29-31. Eq. 31 (previously Eq. 30) is valid for defining the increment in thrust coefficient between blade sections for all $\lambda > 0$.

And the shortcomings and approximations behind the Glauert theory e.g. neglecting the pressure variation in the wake due to swirl

Autor Response: The authors agree that some discussion is useful to the present work; Section 6.1 titled Validity of Classical Glauert Theory has been added to the revised manuscript. Also, Glauert's inherent assumptions are now mentioned in the introduction in the context of other works.

This was discussed at an early stage in the development of the aerodynamic theory for wind turbines by de Vries1 followed by e.g. Sharp2 and more recently with a numerical design study that demonstrated a high Cp can be obtained even at low tip speed ratios towards the rotor centre.

As the present study in particular focuses on the power, thrust and bending moment for the tip speed ratio going to zero its proposed to include a discussion of these aspects.

Author Response: The authors have addressed other works related to rotor disk theory in the introduction section, and have added some relevant references. In particular, the results at low tip speed ratio have been brought into context with recent work by Wood & Hammam (2022). Note that the objective of this work is simply an addendum to Glauert and not a proposal for improved performance coefficients at very low tip speed ratio. It is noteworthy, however, that some quantitative comparisons to Wood & Hammam (2022) show that results obtained using Glauert's theory are quite good at low tip speed ratio given the inherent assumptions, see Section 6.1.

Final conclusion of review

The reviewer can recommend publication of the paper but recommends to integrate response to the above review comments.

Author Response: Thanks again for the very good comments. The authors feel that we addressed the helpful suggestions, also in conjunction with comments made by the other reviewer. In the end, this work is an interesting closed-form analytical addition to a fundamental work in wind turbine aerodynamics.