Review of paper: wes-2024-111:

Glauert's Optimum Rotor Disk Revisited – A Calculus of Variations Solution and Exact Integrals for Thrust and Bending Moment Coefficients

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Brief summary

The authors present an amendment to Glauert's optimum rotor disk solution for the maximum power coefficient, CPmax, as a function of tip speed ratio, using a simple alternative solution based on calculus of variations. Next deriving corresponding exact integrals for the thrust coefficient CT and bending moment CBe. Then using L'Hôpital's rule up to the third derivative of the equations, the thrust coefficient and power coefficient for the tip speed ratio going to zero are determined.

Overall comments

Generally, the paper is well written with a logical development of the equations. It seems that the use of the method of calculus of variations yields an elegant approach for derivation of CPmax, CT and CBe for the tip speed ratio going towards infinity and zero, respectively.

However, the paper lacks completely an introduction section giving the work a wider perspective and describing the importance of the work. In particular, the advancement by previous work mentioned in the paper, "Wilson et al., 1974; Hansen, 2008; Manwell et al., 2009; Burton et al., 2011; Wood, 2011; Schaffarczyk, 2014; Sørensen, 2016; Schmitz, 2019) should be elaborated and the relation to the present work should be clear.

Specific comments and proposal for improvement

- As mentioned above an introduction section should be inserted with a relevant literature review in relation to the subject. What advancements have been achieved in the past and how the present work contributes to new insight ?
- Discussing shortcomings in the theory where they appear
    - E.g. the jump from equation 29 with the thrust coefficient CT for the whole disc to eq. 30 with the thrust coefficient on an incremental form
        - Is eq. 30 valid in any case ?
- And the shortcomings and approximations behind the Glauert theory e.g. neglecting the pressure variation in the wake due to swirl

This was discussed at an early stage in the development of the aerodynamic theory for wind turbines by de Vries[1] followed by e.g. Sharp[2] and more recently with a numerical design study[3] that demonstrated a high Cp can be obtained even at low tip speed ratios towards the rotor centre.

As the present study in particular focuses on the power, thrust and bending moment for the tip speed ratio going to zero its proposed to include a discussion of these aspects.

Final conclusion of review

The reviewer can recommend publication of the paper but recommends to integrate response to the above review comments.

[1] Vries, O. de., North Atlantic Treaty Organization, AGARD, NATO, and OTAN. 1979. Fluid Dynamic Aspects of Wind Energy Conversion :

[2] Sharpe, D. J. 2004. “A General Momentum Theory Applied to an Energy-Extracting Actuator Disc.” Wind Energy 7 (3): 177–88. https://doi.org/10.1002/we.118.

[3] Johansen, Jeppe, et al. “Design of a Wind Turbine Rotor for Maximum Aerodynamic Efficiency.” Wind Energy, vol. 12, no. 3, 2009, pp. 261–73, https://doi.org/10.1002/we.292.