Author Response to Reviews of

Spatial development of planar and axisymmetric wakes of porous objects under a pressure gradient: a wind tunnel study

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RC: Referee Comment,   AC: *Author Comment*

Referee 1

1. General considerations

RC: The article presents an experimental study of porous disk and cylinder wakes under zero and adverse pressure gradients. Moreover, the experimental data is compared with predictions of analytical wake models under pressure gradient for planar and axisymmetric wakes. The findings of the study are not necessarily new; however, they are consistent with and support the findings of several previous experimental and numerical studies on the topic. As argued by the authors, it is the first study that investigates both planar and axisymmetric wakes under comparable flow conditions in the same test facility. The study can be useful for the community and I will recommend its publication once the concerns highlighted in the following sections are addressed.

AC: *We thank the reviewer for their positive comments and the time invested in the present review. We are glad to see that they appreciated our manuscript. We are also glad that they recommend its publication, once we address the comments highlighted below. Major changes done to the main manuscript are highlighted in blue.*

2. Specific comments:

RC: 1. The authors claim that the cylinder wake matches the wake of a vertical axis turbine. However, the cylinder used in the study is a two-dimensional cylinder resulting in a planar wake (as suggested by the title of the article). The wake of a vertical axis wind turbine, on

the other hand, is not a planar wake. Maybe for very large aspect ratios, it could be similar to a planar wake, but it cannot be true in general. The authors need to clarify this and specify under which conditions the wake of a vertical axis turbine matches with a cylinder (planar) wake.

AC: *The referee raises a fair point, as we only studied the planar wake, which is not, in general, the same as the wake of a VAWT. Moreover, whether the wake of a cylinder matches that of the wake of a VAWT, strongly depends on the solidity of the VAWT. The higher the solidity, the higher the resemblance of a bluff body wake (Araya, Colonius, and Dabiri, 2017). Furthermore, even if the aspect ratio and the solidity match, the VAWT wake is only similar to that of a cylinder in the far wake. Finally, the aspect ratio is important in the wake recovery process for a VAWT. The vertical convection (which would be out-of-plane for this planar wake) and $x - z$ Reynolds stresses play a dominant role (Araya, Colonius, and Dabiri, 2017, Boudreau and Dumas, 2017), likely more than the in-plane recovery processes (Deijl et al., 2022). Therefore, ultimately the planar wake studied in this work and its model would be a poor match for some streamwise ranges of a VAWT. Nevertheless, because of this dependency on aspect ratio, the edge case of an infinite aspect ratio planar wake is still relevant. Consequently, it is still important to model the planar wake generated by a porous cylinder.*

AC: *In the new manuscript, a sentence has been added in the introduction to include the fact that the cylinder wake is not the same as an actual VAWT wake.*

RC: 2. How is the theoretical baseline velocity estimated? The authors must provide the equation used for it.

AC: *We thank the reviewer for remarking this omission in our manuscript. The tunnel area at x is given by:*

$$A(x) = (H + 2x \tan(\gamma))(W + 2x \tan(\gamma)), \tag{1}$$

AC: *with x being the distance from the start of the divergent section and γ the angle of the walls. The height of the test section of the tunnel H is 2.6 m and the width of the test section of the tunnel W equals 2.4 m. The expected baseline velocity U_b at x is then related to the measured velocity in the test section U_∞ by:*

$$\frac{U_b(x)}{U_\infty} = \frac{A(0)}{A(x)}. \tag{2}$$

AC: *This equation has now been added to section 3.*

RC: 3. Page 10 lines 221-223, the authors state that they have verified that the ratio of velocity deficit and wake width is same under zero and adverse pressure gradients, however, they have not included a supporting figure. Is it excluded for brevity? If so, please specify.

AC: *It was indeed excluded for the sake of brevity. It is now added in the manuscript and shown in figure 1 in this rebuttal as it is indeed a key point in the model. From this figure it follows that, for a given wake generator, the ratio of $U_b(x)C(x)/\delta(x)$ has a very similar slope, regardless of the pressure gradient. Figure 1 corresponds therefore well to the results of Liu (Liu, Thomas, and Nelson, 2002) and Thomas and Liu (Thomas and Liu, 2004), whose work form the basis of this assumption. However, small variations ($<20\%$) between the APG and the ZPG cases remain, which is ultimately important for calculating the expected wake width. In the work of Liu (Liu, Thomas, and Nelson, 2002) and Thomas and Liu (Thomas and Liu, 2004), these absolute differences were also present and they were attributed to changes in Re-number. Due to the different set-ups for the ZPG and APG cases (such as a slightly different blockage ratio), small absolute differences between the wakes are therefore unavoidable. For the calculation of the differential equations of equation 2 and equation 9 of the main manuscript, these absolute variations are negligible and most parameters can be non-dimensionalised. Nevertheless, an absolute difference of λ of around 20% may result in a significant change in the calculated wake width, as it can be deduced from equation 4 of the main manuscript. To further show this, the error due to λ has been added to the model results.*

AC: *Figure 1 frim this rebuttal has been added to the main manuscript as figure 7. A discussion on λ has been added to the manuscript (section 4.2). A recommendation for future work has been added to the conclusions regarding the estimation of λ.*

RC: 4. The authors show self-similarity of wake deficit in figure 5c and 5d. According to the data, for porous disks, the wake is self-similar even at x=1D. Why is that? I believe this could be related to the geometry of the disk. In principle, for a turbine wake the self-similarity is valid in the far wake, which scales with turbulence intensity (the lower the turbulence intensity, the further from the turbine is the onset of the far wake). Given the very low turbulence intensity (0.25%), it is surprising to see self-similarity at x=1D.
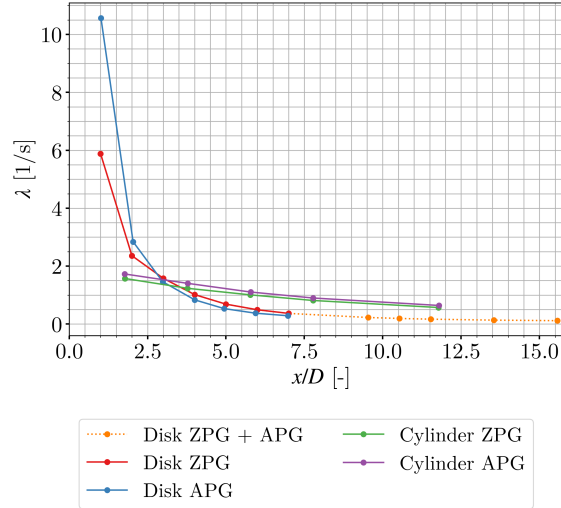
Figure 1: $\lambda(x) = U_b(x)C(x)/\delta(x)$, for all generators at $Re_D = 3.9 \times 10^5$.

Perhaps, the authors can explain this in more detail.

AC: *We thank again the reviewer, as this comment made us find a problem with the presentation of our results. Indeed, in the legend of figures 5c and d from the previous version of the manuscript, the entry for $x = 1D$ is included, but those lines are not present in figure 5c. As can be seen in figure 5a, the velocities in the wake at $x = 1D$ (and also $x = 2D$ for the APG case) appear to become negative and could not be registered by the pressure sensors. Therefore, the self-similarity could not be tested because of the unknown maximum velocity deficit. These profiles were not included in figure 5c, but erroneously appeared in the legend. The figure has been corrected, to figure 2 below. Still, for the ZPG case, the profile at $x = 2D$ is already quite similar to the other profiles. This is indeed likely due to the geometry of the porous disk, which is axisymmetric and the porosity is evenly spread, in contrast with a turbine with a fixed number of blades. In the new manuscript, figure 5 and the corresponding discussion have been modified accordingly to this discussion.*

RC: 5. In section 5, the authors use power law fit (equations 5 and 6) to model the ZPG wake. These equations have 5 parameters (A, α, B, β,). It appears to me that for each case (either porous disk or cylinder), the authors have directly fitted the equations to the experimental data resulting in 5 fitted parameters. My first question is how general or universal are these parameters and are they valid only for the cases presented in this study? Secondly, if you
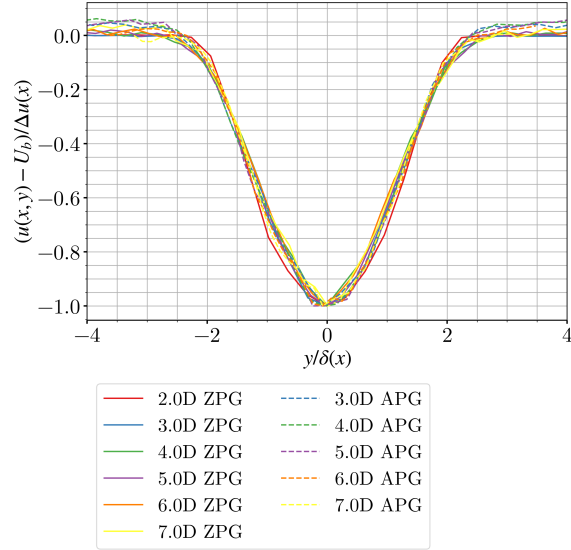
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Figure 2: Velocity profiles normalized using the centreline velocity deficit $\Delta u(x)$ and the wake width δ for the disk.

need the experimental wake under ZPG to fit these parameters (i.e. the ZPG wake needs to be known a-priori), then why not directly used the experimental wake under ZPG for the input of the pressure gradient models? Why go through an extra step to fit power law equations to the ZPG wake and then use it for the pressure gradient model?

AC: *The referee is correct that this step is, in principle, not necessary. One should only need the parameters of the ZPG wake and the pressure gradient as input to the model. However, because of the relatively low spatial resolution of the experimental wake, a smooth fit has to be made to feed into the differential equation solver. While this could have been achieved by a simple cubic spline or any fitting function, the power laws that were chosen as fitting function are based on the theory by Townsend (Townsend, 1976) and George (George, 1989) from which the streamwise evolution of the velocity deficit and wake width in the wake of a bluff-body are derived. The Townsend-George theory also provides the actual values of the α and β parameters (the other three values are scaling factors and a change of the power law origin). This theory assumes an axisymmetric self-similar wake and the classic Richardson-Kolmogorov energy cascade. Whether these power laws apply in general to wind turbine wakes is a topic of itself (Neunaber, Peinke, and Obligado, 2022), but they have previously been applied to wind turbine wake models (Porté-Agel, Bastankhah, and*

Shamsoddin, 2020). The universality of the fitting parameters is still under debate, as they have been found to depend not only on the generator bu also on the background turbulence level and the tip-speed ratio of the rotor (see the recent work of Bourhis and Buxton, 2024 for a detailed discussion on this point).

AC: *Therefore, by assuming such power laws for the streamwise evolution of averaged quantities, a smooth function was created as input to the ODE, that still satisfies the expected theoretical streamwise evolution. By tuning the values of the power laws to the data, we ensure a better fit than the theoretical values such that the focus is on the pressure gradient model performance, instead of the fit of these power laws. Nevertheless, the exact values that have been used in this case are not applicable in a general sense.*

AC: *In conclusion: for a general application of the model, it depends on the quality and resolution of the input whether this step is required. Ideally, if the data is smooth enough, it could directly be fed into an ODE solver. If it is not, we think that a power law is a suitable fit for the streamwise evolution of the velocity deficit and the wake width.*

AC: *A clarification on the application of these power-law-based scalings to the data has been added to section 2.1.*

RC: 6. Page 16 lines 276-277, how do the authors estimate that the far wake starts from $x/D \leq 3$? In addition, how practical is it to pick the velocity deficit at the start of the far wake from the experimental data?

AC: *The reason for the estimation of the onset of the far wake at $x/D \geq 3$ is two-fold. First of all, from a practical stand-point, the measured velocities in the wake of the disk showed negative velocities at $x = 1D$ and $x = 2D$ for the APG case. The exact velocity deficit and wake width were therefore difficult to estimate and to use in the model. For $x/D \geq 3$, this problem vanishes. Second of all, from the self-similarity figures 5c-d, it appears that the profiles for $x/D \geq 3$ are all self-similar, which is a requirement for the model. At $x/D = 2$, for both the disk but especially the cylinder, the profile deviates slightly from the other profiles. In consequence, we consider that $x/D = 2$ could still be considered within the near-wake range, at least from an averaged velocity perspective.*

AC: *Regarding the second point about the practicality of picking the velocity deficit from the experimental data, we agree that this may not be indeed ideal, as it means that one would need information a priori at a certain point about the APG wake before modelling it from*

that starting point. Dar and Porté-Agel (Dar and Porté-Agel, 2022) propose the equation shown in section 2.2 to estimate the velocity deficit in the near wake, which can be used instead. However, this does add an extra assumption to the model, that sums up to the unknown velocity deficit in the near wake in the case of the disk. In consequence, we chose to use the velocity deficit that was known. This was done to focus on the predicted evolution of the deficit by the model.

AC: *A brief sentence in section 2.2. has been added to clarify that this starting point is where the wake is deemed self-similar, which is approximately for $x/D \geq 3$.*

RC: 7. It would be useful if the authors add a comparison of the full velocity deficit profiles between the experiments and the model.

AC: *This is indeed a good suggestion. This comment actually highlighted the model's sensitivity to λ of equation 4 mentioned above. As discussed in comment 3, the ratio of velocity deficit and wake width is not exact in an absolute sense between the ZPG wake and the APG wake. While the model has a robust prediction of the velocity deficit, the wake width is relatively sensitive to this ratio λ. This is now highlighted in the new manuscript by adding the model output of the wake width as figure 11. As can be seen from these figures, the wake width has a larger error due to λ. We therefore recommend that in future work the relation between the velocity deficit and the wake width is calculated in a more robust sense, for instance by using the full profiles and not only the centreline evolution.*

AC: *We have added figure 11 showing the wake width output of the model, visualizing the sensitivity of the model output of velocity deficit and wake width to λ. We have also added a discussion on λ in section 4 and added a recommendation to the conclusions. We have not included the full velocity profiles, as figures 9, 10 and 11 show the velocity deficit and the wake width output and combined they constitute the same velocity profiles.*

3. Technical comments

RC: Page 3 line 65, the averaged streamwise velocity 'deficit' is self-similar in the far-wake, please correct.

AC: *'Deficit' has been added to this sentence.*

RC: Why is the velocity deficit plotted as $(1 - C(x))$? Is there a specific reason for this?

 AC: *Showing $1 - C(x)$ or $C(x)$ is ultimately the same. We think that it might be slightly easier to see the asymptote to 1 than the asymptote to 0 in these graphs.*

 AC: *We thank again the reviewer for this review and the valuable comments and suggestions they made. We hope they will find the new version of the manuscript suitable for publication in Wind Energy Science.*

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