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This study is part-1 of the project:
"Numerical Modelling of Regional-Scale Wind Farm Flow Dynamics"

Response to Reviewer 2

Exec: M.A.Khan - February 18, 2025
Revised: Simon Watson - February 18, 2025
Revised: Matthew Churchfield - February 18, 2025

The authors thank the reviewer for the time dedicated to revising the article and suggesting valuable feedback to improve this research and the article. We proceed with answering and clarifying their comments, where possible.

For ease of tracking, our response to each comment is denoted in black, while the reviewer's comments are denoted in blue. The detailed changes made to the manuscript are included in the "track changes" section at the end of this document.

What is role of grid resolution in the conclusions? Certainly, the domain size conclusions will be unchanged, but is the damping strength dependent on grid resolution?

For the investigated parameter space (i.e., Fr between 0.1 to 0.5), the grid resolution used in the study does not a ect the simulation setup. This is because the AGW wavelengths, typically 3 to 45 km are many, many times bigger than the typical grid sizes, 10 to 20 m, used in wind farm LES (the ultimate objective of this research). Since we are fully resolving the AGWs using a resolution typical of LES, the solution is grid-independent. Rayleigh damping contains no length scale, so it is not dependent on grid resolution. This reasoning is mentioned in the text in lines 304-305 and is now highlighted in the conclusions as follows:
"In terms of time-step size and grid resolution, the typical values used in wind farm LES (i.e., $0.5 < t < 1.0$ and $5 < x, y, z < 20$) are more than su cient to resolve the AGWs. Thus, the simulation setups used for the investigated Fr range are independent of the time-step size and grid resolution because the time periods and wavelengths of AGWs are several times bigger."

Why are the simulations three-dimensional? There are only five grid points in the third dimension without any inherent three-dimensionality in the geometry or inflow, so the flow is almost certainly two-dimensional.

It is true that the flow is two-dimensional and including the third dimension was not necessary. However, the wind farm canopy case triggers turbulence. Thus, we considered three-dimensional simulations to resolve at least some turbulence induced by the canopy. For consistency, we used the setup in the hill scenario. Also, we were unsure if flow separation would occur downstream of the hill, which didn't because the slope parameter was constant.

In various places in the manuscript, the authors mix discussion of dimensional terms and non-dimensional terms. Everything seems to be able to be expressed in non-dimensional terms, and I encourage the authors to rewrite the commentary and replot the figures as such where possible. Doing so could also reduce the total number of figures and lines on figures needed. (The need to demonstrate dynamic similarity also seems unnecessary.)

It is correct that every plot and discussion can be expressed in non-dimensional terms; in fact, the plots are all in terms of non-dimensional numbers. To provide additional insight, the plots related to domain length include hill-half widths or canopy lengths, this is indeed redundant as Fr already includes L. Thus, we have revised the plots in Fig. 10a and Fig. 13a by keeping only Fr.
However, we would like to keep the vertical velocity contours in Fig. 12 and the commentary on the same as its visual evidence of AGW-dependency on L for low values of Fr and on U/N for higher values of Fr. Likewise, the plots in Fig. 6 about dynamic similarity and the related discussion are important in introducing critical length and time scales in studying AGWs. Further, such representation helps in interpreting the physical meaning of the non-dimensional number, i.e. Fr. Such as, "Thus, it is deduced that by the definition $Fr = U/NL$, L/U acts like N or the free atmosphere's stability. This means that the stability of the free atmosphere is relative to the size of the disturbance source and the geostrophic wind."

A key conclusion of the manuscript is the need to scale the domain size and damping layer dimensions by

the wavelengths. However, the actual discussion of the wavelengths from the semi-analytical solution for the hill is somewhat limited. The vertical wavelengths are not actually quantified or plotted, in fact. To make the manuscript more useful, the authors should include expressions for the (non-dimensionalized) wavelengths or similar plots as a function of the relevant non-dimensional parameters. That will allow other researchers to consult this work in order to scale their geometries.

The reviewer has rightly indicated the need to include further discussion on the vertical wavelength. As suggested by them, we have included a plot in Fig. 3(b) of the revised manuscript that shows non-dimensional vertical wavelength as a function of Fr. Please check Fig. 3(b) and commentary on it in the "tracked changes" section.

While the authors allude to other e ects of interest (turbulence, ABL, etc.), for future work, this leaves the reader wondering how relevant this work really is. For example, no wind farm is strictly in the free atmosphere, and this seems to be intended application of the authors. How then would the ABL influence the conclusions of this manuscript? While the authors may not have the quantitative answer yet, some scaling argument to demonstrate that the present results would, for example, be a conservative constraint on more complex flow situations would be beneficial.

There are two points to note in the reviewer's comment: first, how relevant is this study in modeling wind farms, and second, how the conclusions of this study may vary when an ABL is included? Firstly, this study is focused on the most important aspect of modeling and understanding wind farm-induced atmospheric gravity waves, which is that the internal and interfacial waves are the most critical ones in the accurate modeling of AGWs, and they occur in the free atmosphere. Moreover, these are the waves that reflect from the domain boundaries, and damping layers are always implemented only in the free atmosphere region of the simulation domain. Thus, focusing on the free atmosphere is the initial and most important piece of the puzzle and by only clearly understanding it, one can piece together the remaining. Altogether, we are confident in proposing this study as the stepping stone to accurately modeling and investigating AGWs. These aspects are already mentioned in the manuscript, such as, "It is critical to note that RDLs are always implemented in the free atmosphere because it is the gravity waves in the free atmosphere that reflect from the boundaries".
For the second question, the findings of this study are already validated for CNBL conditions by the authors in a follow-up study Khan et al. (2024). We have mentioned this in the conclusion besides citing the follow-up study for the readers to consult. Thus, the last paragraph of the conclusions section is modified as follows.
"In further work (Khan et al., 2024), the impact of the inversion layer (inversion Froude number and height) on the setup is explored as well as the impact of turbulence and the Coriolis force. In extending the current study, we validated the findings mentioned above for conventionally neutral boundary layer conditions. The additional observation of the follow-up study is that the Rayleigh damping layer, along with the advection damping layer (details on which are given in (Lanzilao and Meyers, 2023)), is better than the Rayleigh damping layer alone in containing the energy accumulation at the inlet. More importantly, it is intended to develop an inflow boundary condition that can avoid the energy accumulation at the inlet based on an inflow which accounts for the presence of AGWs."

References

Khan, M. A., Watson, S. J., Allaerts, D. J. N., and Churchfield, M.: Recommendations on setup in simulating atmospheric gravity waves under conventionally neutral boundary layer conditions, Journal of Physics: Conference Series, 2767, 092 042, https://doi.org/10.1088/1742-6596/2767/9/092042, 2024.

Lanzilao, L. and Meyers, J.: An improved fringe-region technique for the representation of gravity waves in large-eddy simulation with application to wind farms, Boundary-Layer Meteorology, 186, 567–593, https://doi.org/10.1007/s10546-022-00772-z, 2023.

1 Track Changes

In the following we report the changes made to the paper using latexdi .

Investigating the Relationship between Simulation Parameters and Flow Variables in Simulating Atmospheric Gravity Waves for Wind Energy Applications

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Abstract. Wind farms, particularly offshore clusters, are becoming larger than ever before. Besides influencing the surface wind flow and the inflow for downstream wind farms, large wind farms can trigger atmospheric gravity waves in the inversion layer and the free atmosphere aloft. Wind farm-induced gravity waves can cause adverse pressure gradients upstream of the wind farm, that contribute to the global blockage effect, and can induce favorable pressure gradients above and downstream of the wind farm that enhance wake recovery. Numerical modeling is a powerful means of studying these wind farm-induced atmospheric gravity waves, but it comes with the challenge of handling spurious reflections of these waves from domain boundaries. Typically, approaches which employ radiation boundary conditions and forcing zones are used to avoid these reflections. However, the simulation setup of these approaches heavily relies on ad-hoc processes. For instance, the widely used Rayleigh damping method requires ad-hoc tuning to produce a setup which may be only produce satisfactory results for a particular case. To provide more systematic guidance on setting up realistic simulations of atmospheric gravity waves, we conduct an LES study of flow over a 2D hill and through a wind farm canopy that explores the optimum domain size and damping layer setup depending on the fundamental parameters which determine the flow characteristics.

In this work, we only consider linearly stratified conditions (i.e., no inversion layer), thereby focusing on internal gravity waves in the free atmosphere and their reflections from the domain boundaries. This type of flow is governed by a single Froude number, which dictates most of the internal wave properties, such as wavelength, amplitude, and direction. This in turn will dictate the optimum domain size and Rayleigh damping layer setup. We find the effective horizontal and vertical wavelengths, (the representative wavelengths of the entire wave spectrum), to be the appropriate length scales to size the domain and damping layer thickness, and the optimal Rayleigh damping coefficient scales with the Brunt-Väisälä frequency.

Considering Froude numbers seen in wind farm applications, we propose recommendations to limit the reflections to less than 10% of the total upwards propagating wave energy. Typically, damping is done at the top boundary, but given the non-periodic lateral boundary conditions of practical wind farm simulation domains, we find that damping the inflow-outflow boundaries is of equal importance to the top boundary. The Brunt-Väisälä frequency-normalized damping coefficient should be between 1 and 10. The damping layer thickness should be at least one effective vertical wavelength; damping layers exceeding 1.5 times the vertical wavelength are found to be unnecessary. The domain length and height should accommodate at least one

effective horizontal and vertical wavelength, respectively. Moreover, Rayleigh damping does not damp the waves completely, and the non-damped energy might accumulate over the simulation time.

1 Introduction

The size of a modern wind farm, especially offshore, can extend several tens of kilometers horizontally, involving flow interactions on a regional scale with impacts well into the free atmosphere. The energy and momentum extraction caused by a large wind farm is significant enough to decelerate the flow in the atmospheric boundary layer (ABL) (Frandsen, 1992; Calaf et al., 2010; Smith, 2010), which slowly recovers because of turbulent momentum transfer and the interplay of the large-scale driving pressure gradient and the Coriolis force. This is evident from the consolidated wake of a large wind farm, which extends far beyond the wind farm in the streamwise direction. Researchers frequently study the behavior of a wind farm within the ABL only and do not consider the free atmosphere above. However, a full understanding of large wind farm behavior requires an understanding of the farm's effect on the free atmosphere and vice versa.

The temperature stratification of both the ABL and the free atmosphere is strongly connected to the flow dynamics of large wind farms. For instance, the thickness of the ABL can significantly decrease during stable atmospheric conditions, which will affect the flow at turbine level in a variety of ways. In offshore environments in particular, the height of the capping inversion (the relatively thin strongly stable layer that often forms between the top of the ABL and the free atmosphere) can drop to a few hundred meters at night. In such conditions, wind farms can induce atmospheric gravity waves (AGWs), which include trapped gravity waves (TGWs) in the capping inversion and internal gravity waves (IGWs) in the free atmosphere aloft. Wind farm-induced AGWs were first hypothesized by Smith (2010), who treated wind farms as semi-permeable obstacles that can deflect the flow upwards to displace the capping inversion, resulting in these buoyancy-driven waves.

Wind farm-induced AGWs can affect wind farm performance by creating streamwise pressure gradients that accelerate or decelerate the flow into and through the farm, contributing to the phenomenon of "wind farm blockage", and affecting the way the consolidated wind farm wake recovers. Smith (2010), Allaerts et al. (2018), and Lanzilao and Meyers (2021) believe that AGWs are the main driver behind the wind farm blockage effect at the entrance of a wind farm. Wu and Porté-Agel (2017) theorize that the wind farm blockage effect is the combined effect of AGWs and cumulative turbine induction. The term 'regional efficiency' of wind farms was introduced by Allaerts and Meyers (2018) to characterize the impact of atmospheric gravity waves on wind farm performance. They claim that regional efficiency is of the same order as wake-induced wind farm efficiency losses. Allaerts et al. (2018) estimated a 4 to 6% decrease in the annual energy production of an offshore wind farm caused by the blockage effect created by AGWs. On the other hand, (Lanzilao and Meyers, 2023) and (Stipa et al., 2023) argue that AGWs can also assist wind farm wake recovery because they can create a locally favorable pressure gradient at the downstream end and behind the wind farm. However, the extent of the favorable pressure and whether or not it always improves wind farm wake recovery is still unknown. Because of the sheer scale of AGWs and their interaction with wind farms, little if any experimental data exist to quantify their effects on wind farms; most investigations about wind farm-induced AGWs are numerical.

Numerical flow simulation, particularly large-eddy simulation (LES), is a useful tool to study AGWs. Gravity waves induced by large wind farms were first investigated with LES by Allaerts and Meyers (2017). Since then, wind farm-induced gravity waves have been studied using LES by Allaerts and Meyers (2017, 2018), Allaerts et al. (2018), and Lanzilao and Meyers (2021, 2023), all of which used pseudospectral codes and a forcing fringe region to circumvent the horizontal periodicity that is inherent in the pseudospectral approach. Stipa et al. (2023) and Maas (2022) have also studied wind farm-induced gravity wave effects using simulations with inflow and outflow boundary conditions.

The simulation of wind farm-induced atmospheric gravity waves in a finite domain requires special treatment at all domain boundaries other than the ground to stop the gravity waves from spuriously reflecting off these boundaries that do not exist in reality. AGWs propagate in all directions, so they will spuriously interact with the inflow, outflow, and top boundaries (all of which do not exist in reality). Unfortunately, current approaches to avoid wave reflection at domain boundaries are all ad-hoc and require extensive fine-tuning (Allaerts, 2016; Lanzilao and Meyers, 2023). Thus, setting up reflection-free simulations is currently tedious, computationally expensive, and time-consuming, and there is a lack of clear guidance.

The goal of this study is to make the process more systematic by investigating possible relations between the simulation setup and the physical parameters driving the flow through wind farms under stable atmospheric conditions. We anticipate that appropriate simulation parameters like domain length and height, etc, are related to internal wave properties such as wavelength and direction. We therefore investigate the proper scaling of simulation parameters and their effectiveness in avoiding wave reflections for a range of physical parameters. Understanding the relation between the physical parameters, the internal wave properties, and the simulation setup, enables recommendations to be made on how to set up simulations involving internal atmospheric gravity waves.

As this is, to our knowledge, the first systematic LES study to investigate the relation between simulation setup and physical parameters, we restrict ourselves to the study of internal gravity waves in the free atmosphere for reasons of simplicity. Investigating the free atmosphere separately is vital to handling the reflections, as IGWs propagate both horizontally and vertically and hence can reflect off all domain boundaries. To this end, we consider two linearly stratified flow scenarios: the flow over a bell-shaped 2D hill, and the flow through a wind farm canopy as a simpler surrogate for a wind farm consisting of discrete turbines. The former flow scenario can be solved analytically and is used mainly to validate the simulations and to understand the dependence of AGW properties on the governing flow parameters.

The paper is structured as follows: Section 2 gives an overview of current approaches to avoid AGW reflection at domain boundaries and highlights why it is so challenging to obtain a good simulation setup for inflow/outflow type of simulations. Next, Section 3 describes the flow scenarios studied in this study. In Section 4, the simulation methodology and the methods to analyze the LES data are explained. The numerical models and simulation setups for both flow scenarios are also explained in this section. Section 5.1 presents the results for the hill case, while Section 5.2 contains the results for the wind farm canopy. In Section 6, the conclusions of this research are presented as recommendations on how to effectively and efficiently set up simulations involving atmospheric gravity waves.

2 Current practices to avoid spurious AGW reflection

Two main methods exist to mitigate spurious AGW-domain boundary interaction: radiative boundary conditions and damping layers. Radiative boundary conditions often take the form

$$\frac{\partial \varphi}{\partial t} + c_j \frac{\partial \varphi}{\partial n_j} = 0 \tag{1}$$

where φ is the quantity being transported through a boundary over time t, n_j is the displacement in the direction j normal to the boundary, and c_j is the wave transport velocity in the normal direction. One way to imagine this boundary condition is that the transported quantity φ is set to be exactly the same value as that imparted by the wave as it propagates to the boundary, thus absorbing the wave. One major difficulty is accurately determining c_j. This is often only possible in idealized situations. The radiative boundary condition is capable of allowing wave energy out of the domain only if the waves are moving perfectly normal to the boundary (Durran, 1999). However, internal gravity waves are typically moving at an angle relative to horizontal.

A popular alternative to the radiative boundary condition is a damping layer. Damping layers, sometimes referred to as "sponge zones," attenuate waves as the wave moves through the zone. These zones are placed adjacent to domain boundaries and have a prescribed thickness. A downside to using such zones is that, because they have thickness, the domain size needs to be increased accordingly, which adds some extra computational cost. Typically, damping layers are either of the viscous type or of the relaxation type, with the latter also known as Rayleigh damping layers (RDLs). RDLs are more commonly used than viscous damping and are considered more effective than the latter. For example, a recent study by Lanzilao and Meyers (2023) found that RDLs outperform radiative boundary conditions in minimizing AGW reflections in wind farm simulations. For this reason, we focus on the application of RDLs in this study.

In principle, the quantity of interest passing through an RDL, usually velocity but also sometimes temperature, is relaxed towards a prescribed reference value with a specified time scale as the wave travels through the RDL, reflects off the boundary, and travels back through the RDL once again. Rayleigh damping is introduced into the transport equations as a forcing term which, for the case that the quantity to be damped is a three-dimensional field $\varphi(\mathbf{x})$, takes the form

$$f^{RDL}(\mathbf{x}) = -\frac{1}{\tau} f(\mathbf{x}) \cdot (\varphi(\mathbf{x}) - \varphi_{\mathrm{ref}}(\mathbf{x})) \tag{2}$$

where $\varphi_{\mathrm{ref}}(\mathbf{x})$ is the reference value towards which the quantity of interest in a parcel of fluid is driven along a streamline. For example, where the velocity field is required to be damped then $\varphi(\mathbf{x}) = u(\mathbf{x})$, and $\varphi_{\mathrm{ref}}(\mathbf{x}) = u_{\mathrm{ref}}(\mathbf{x})$. In this case, $u_{\mathrm{ref}}(\mathbf{x})$ could be defined as $[G_x, G_y, 0]$ where G is the geostrophic wind vector. Sometimes the situation is ageostrophic, and a reference horizontal velocity is not appropriate, in which case only the vertical (z) component of velocity is damped towards zero. The Rayleigh damping function, $f(\mathbf{x})$, is a critical part of an RDL that ensures that the wave gradually dissipates energy as it travels through the layer and that the attenuation does not cause waves to reflect from the interface between the non-damped and damped regions. The function $f(\mathbf{x})$ can be a linear, exponential, polynomial, or cosine function. A cosine function is commonly used in the vertical, which, for the upper boundary, typically has the form (Lanzilao and Meyers, 2023)

$$f(z) = \left[1 - \cos\left(\frac{\pi}{s_{\mathrm{ra}}} \frac{z - (L_z - L_{\mathrm{d}})}{L_{\mathrm{d}}}\right)\right] \tag{3}$$

where z is height above the ground, L_z is the height of the computational domain, L_d is the thickness of the damping layer, and s_{ra} is a constant which controls the gradient of the damping function in the vertical. This was tuned by Lanzilao and Meyers (2023) and the best results were obtained for a value of s_{ra}=2. Although this suggests some dependency of wave damping on the shape of the damping function, Perić (2019) found that the form of the damping function had little impact while investigating different approaches to damping internal waves.

The time scale, τ, or damping coefficient, controls how fast the quantity is relaxed toward a reference value. In Lanzilao and Meyers (2023), τ is scaled with the Brunt-Väisälä frequency (N) in pursuit of the optimal value for wind farm simulations, suggesting an optimum value of $\xi = 1/(\tau N)$ of 3, with s_{ra}=2, as mentioned above. By contrast, Allaerts (2016), tuned the RDL damping coefficient and found an optimum value of $1/\tau = 0.0001$ s^{-1}, translating into a typical value of $\xi = 0.017$, though it should be noted that this was for a value of s_{ra}=1; the same value of ξ was used in a number of later studies including: Allaerts and Meyers (2017); Allaerts et al. (2018); Allaerts and Meyers (2018).

The damping layer thickness is an important parameter as it determines the space available for the forcing to dissipate the incoming wave energy. Klemp and Lilly (1978) suggested the thickness to be more than one vertical wavelength based on hydrostatic flow solutions of a single Fourier mode. More recent studies also follow this convention but without any further investigation: Allaerts and Meyers (2017, 2018), and Lanzilao and Meyers (2021, 2023) use a 15 km thick RDL only at the top boundary as the vertical wavelengths in their simulations were always less than 15 km.

As mentioned above, many of the existing LES studies on wind farm interaction with the ABL are performed with codes that are horizontally pseudospectral. Pseudospectral codes have the advantage of nearly exponential error convergence with increasing spatial resolution, but they come with the limitation that all lateral domain boundaries must be periodic. This poses a challenge for wind farm simulations because we wish to advect turbulent ABL flow into the domain and allow that flow along with that produced by the wind turbine wakes to exit the domain. Periodic boundaries normally would cause the wake flow exiting the domain to re-enter at the inflow. The solution to this problem is the introduction of a forcing fringe region adjacent to the downstream boundary which forces the 'contaminated' inflow back toward the pre-computed or concurrently computed pure turbulent ABL inflow solution (Inoue et al., 2014). However, it is important to remember that pseudospectral codes with their periodic lateral boundary conditions effectively have no lateral boundaries. RDLs only have to be applied adjacent to the top boundary. Furthermore, the forcing fringe region used to force the downstream flow back toward the desired inflow state is a form of Rayleigh damping. This means that AGWs will not reflect off the lateral boundaries in pseudospectral LES, rather they will simply exit the domain on one side and re-enter at the inflow. The forcing fringe region will have some effect on damping the lateral progression of the AGWs or may trigger spurious gravity waves, requiring an additional treatment (Lanzilao and Meyers, 2021).

As opposed to pseudospectral simulations, simulations with inflow/outflow boundary conditions, which are more commonly used in engineering applications, do not have lateral periodicity and hence do not need a forcing fringe region. Inflow is simply injected into the domain on the inflow boundary with a Dirichlet condition and the outflow boundary is often a Neumann or advection boundary condition that lets the waked flow exit the domain. There is no need for a fringe zone that forces the waked outflow back to the desired clean inflow. On the downside, this approach requires real lateral boundary conditions that

gravity waves will either reflect off or spuriously interact with. Therefore, RDLs must be placed adjacent to these boundaries as well as adjacent to the top boundary. Very little guidance is given in the literature on how to effectively simulate gravity waves using inflow/outflow boundaries. Stipa et al. (2023) and Maas (2022) used inflow/outflow boundary conditions for their large wind farm LES, but the approach is complicated when modelling AGWs. Any inconsistency between the inflow and the internal flow field, triggers non-physical waves at the inlet which propagate into the domain over time. Besides the generation of spurious waves due to these mismatches, avoiding gravity wave reflections from the domain boundaries is a significant challenge. Reflections are the spurious waves that are triggered by having a boundary that is not there in real but required in these simulations. As shown in Figure 1, the gravity waves triggered by a small hill under linearly stratified conditions should move up and out of the numerical domain. But the reality of a finite domain with boundary conditions that do not exactly depict the actual physical conditions at the boundaries causes these waves to reflect. The non-dissipated wave energy accumulates at the boundaries and propagates back into the domain eventually making the solution non-physical.

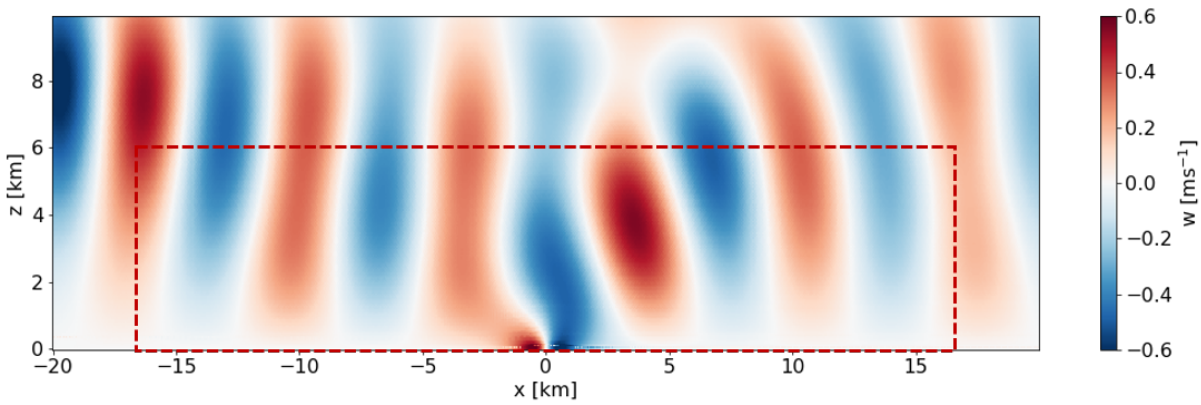

Figure 1. Vertical velocity contours of the flow over a small hill located at x = 0 km, produced by simulating a bell-shaped hill under linearly stratified laminar conditions in SOWFA. The region outside the red dotted box consists of Rayleigh damping layers at the inlet (left), top, and outlet (right).

The current approaches to setting up RDLs in wind farm simulations are rather ad-hoc. Extensive fine-tuning of the damping parameters is required to set up a working simulation which may be applicable only to a specific case (Allaerts, 2016; Lanzilao and Meyers, 2023). As a result, setting up reflection-free simulations is tedious, computationally expensive, and time-consuming. The aim of this paper is to investigate how to make the process less ad-hoc by investigating relationships between the simulation setup and the fundamental physical parameters driving wind farm flow under stable atmospheric conditions.

3 Flow Scenarios

In this study, we consider two simple, two-dimensional flow scenarios that generate internal gravity waves: the flow over a bell-shaped hill, and the flow through a wind farm canopy. Our initial focus lies on the hill case, which is used in various

meteorology and earth sciences studies, such as Vosper and Ross (2020) and Snyder et al. (1985). The reasons to start with the hill case include the simplicity of the flow scenario, computational affordability, and the availability of a semi-analytical solution, which is useful for validation purposes. Moreover, the primary aim of this study is how best to handle atmospheric gravity waves in numerical simulations, and the wave source is thereby of less importance. Nevertheless, the simulated hill heights are similar to typical wind turbine rotor tip heights, and the half-width of the hill is of the same order as typical wind farm lengths. In the second flow case, we study the flow through wind farm canopies to extend the findings from the hill case to wind farm applications. Unlike the rigid hill, a wind farm canopy is a porous region in which the drag force of the wind turbines is applied homogeneously. The details of both flow scenarios are discussed in more detail in the following subsections.

3.1 Scenario 1: Bell-shaped hill

The first flow scenario considers flow over a 2D bell-shaped hill. The height profile h of a bell-shaped hill – sometimes referred to as the Witch of Agnesi (WOA) profile – is defined in the horizontal direction x as

$$h(x) = \frac{H}{1 + (x/L)^2}. \tag{4}$$

This profile is governed by two parameters, i.e., the maximum hill height H and the half-width at half-height L.

To allow a direct comparison between an LES and a linear gravity wave potential flow solution, surface friction and the Coriolis force were excluded, and only a linearly stratified free atmosphere was considered with uniform inflow. Although the inflow was laminar, the LES model could still generate turbulence due to flow separation behind the hill.

A computationally inexpensive semi-analytical flow solution over the hill was used to understand the properties of the IGWs and validate the numerical solutions. We use the analytical solution along with the R-RMSE metric (discussed in Section 4.3) to quantify reflections in the simulations. The following subsection gives an overview of this semi-analytical solution and how it is used in this study.

3.1.1 Semi-Analytical Solution

Linear wave theory, particularly the Taylor-Goldstein equations, is commonly used to study atmospheric gravity waves. Allaerts (2022) used these equations to develop a Python module called Linear Buoyancy Wave Package (LBoW) to solve linear buoyancy wave problems. In this research, we use a part of this code that computes a semi-analytical steady-state solution of the uniform, stratified flow over the WOA hill. The code solves the equations on a grid in frequency space using a Fast Fourier Transform (FFT). The solution is independent of the grid size in the vertical direction but not in the horizontal. A solution at any vertical level can be acquired without requiring a prior solution at lower and higher levels. The grid resolution in the horizontal direction dictates the solution accuracy. The FFT solution deviates from a theoretical Fourier transform of the bell-shaped hill for high horizontal wave numbers due to rounding errors. Mesh size in the range of $20\ m$ to $100\ m$ is recommended to compute the semi-analytical solution.

The steady-state, semi-analytical solution of uniform flow over the hill in a stably stratified free atmosphere with a uniformly increasing potential temperature with height is shown in Fig. 2(a) , where the upward flow deflection by the hill triggers only