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Brief communication: A note on the variance of wind speed and turbulence intensity

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Abstract. This note addresses the issue that several papers in the peer-reviewed literature of wind energy applications have used an incorrect equation that equals the variance of wind speed (σ_U^2) to the sum of the variances of the wind components. This incorrect equation is often used to calculate turbulent intensity (TI), which, as a consequence, is often incorrectly estimated too. While exact analytical equations do not exist, here two approximate analytical equations are derived for σ_U^2 and TI, both functions of the variances of the wind components. Both formulations are validated with samples from a prior field campaign and perform satisfactorily.

1 Introduction

The standard deviation of wind speed, which is the square root if the variance, is an important parameter in wind energy applications because it is part of the definition of turbulence intensity used in the International Electrotechnical Commission (IEC) standard (International Electrotechnical Commission, 2019), which wind turbines must comply with. Since wind turbines always face the wind, especially in the first experiments that were conducted in wind tunnels and in idealized simulations, the convention has always been to align the x-axis along the wind direction. As such, the standard deviation of wind speed is simply the standard deviation of the u-component of the wind, i.e., the component along x. By contrast, in geophysical applications, such as meteorological field campaigns or simulations of weather events, the convention is to align the x-axis along the east-west direction (and the y-axis along the north-south). Since the wind direction changes during the diurnal and seasonal cycles, the wind-aligned system of coordinates used in the wind energy community has become unpractical when studying real wind farms. The geophysical system of coordinates, therefore, has been adopted for field measurements and simulations of wind farms. As a consequence, the standard deviation of wind speed is no longer a simple function of the standard deviations of the wind components. While an exact analytical equation is impossible to obtain, an incorrect expression is often found in the literature, namely, the sum of the variances of the wind components, and often treated, incorrectly, as an exact definition. Since turbulence intensity is a function of the variance of wind speed, errors are introduced in the calculation of turbulence intensity too. This note addresses this issue by proposing an analytical approximation for the wind speed variance and one for turbulence intensity.

The issue of the relationship between the standard deviation of wind speed and those of the wind components is relevant because turbulent kinetic energy, which is an important variable that is predicted by weather prediction models and that is

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affected by wind farms, is a function of the standard deviations of the wind components, while turbulence intensity is a function of the standard deviation of wind speed. Converting between the two is therefore important in validating model results against observations, especially with reference to the IEC standard that uses turbulence intensity.

2 Definitions

Let us use the geophysical system of coordinates and, for the sake of simplicity, assume that wind is a two-dimensional vector. The two wind components along x and y are u and v, respectively, and the magnitude U is a non-linear function of both:

$$U = f(u,v) = \sqrt{u^2+v^2}. \tag{1}$$

The means $\bar{u}$, $\bar{v}$, and $\overline{U}$, calculated over a set of N measurements u_t, v_t, and U_t, each taken at time t, are:

$$\bar{u} = \frac{1}{N}\sum_t u_t, \tag{2}$$

$$\bar{v} = \frac{1}{N}\sum_t v_t, \tag{3}$$

$$\bar{U} = \frac{1}{N}\sum_t U_t. \tag{4}$$

The variances σ_u^2, σ_v^2, and σ_U^2 are:

$$\sigma_u^2 = \frac{1}{N}\sum_t (u_t-\bar{u})^2 = \overline{(u-\bar{u})^2} = \overline{u^2 - 2u\bar{u} + \bar{u}^2} = \overline{u^2} - 2\bar{u}^2 + \bar{u}^2 = \overline{u^2} - \bar{u}^2, \tag{5}$$

$$\sigma_v^2 = \frac{1}{N}\sum_t (v_t-\bar{v})^2 = \overline{v^2} - \bar{v}^2, \tag{6}$$

$$\sigma_U^2 = \frac{1}{N}\sum_t \left(U_t-\bar{U}\right)^2 = \overline{U^2} - \bar{U}^2 = \overline{u^2} + \overline{v^2} - \bar{U}^2 = \sigma_u^2 + \sigma_v^2 + \bar{u}^2 + \bar{v}^2 - \bar{U}^2. \tag{7}$$

Eq. 7 may not be simplified analytically any further because:

$$\bar{u}^2 + \bar{v}^2 \neq \bar{U}^2 = \left(\overline{\sqrt{u^2+v^2}}\right)^2. \tag{8}$$

As a consequence:

$$\sigma_U^2 \neq \sigma_u^2 + \sigma_v^2 \tag{9}$$

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and

$$TI^2 = \frac{\sigma_U^2}{\overline{U}^2} \neq \frac{\sigma_u^2 + \sigma_v^2}{\overline{U}^2} = \frac{2\,TKE}{\overline{U}^2}, \tag{10}$$

where:

$$TKE = \frac{\sigma_u^2 + \sigma_v^2}{2}. \tag{11}$$

In order to obtain an expression for the variance of wind speed, we first need to recognize that the wind is intrinsically turbulent and therefore we can use the Reynolds averaging approach. The turbulent fluctuations, usually denoted with a prime ($'$), in this case coincide exactly with the differences from the means (δ) as follows:

$$u_t = \bar{u} + u_t' = \bar{u} + \delta u_t, \tag{12}$$

and similarly for v_t and U_t. Therefore the variances can be rewritten exactly as:

$$\sigma_u^2 = \frac{1}{N}\sum u_t'^2 = \frac{1}{N}\sum (\delta u_t)^2 = \overline{u'^2} = \overline{(\delta u)^2}, \tag{13}$$

and similarly for σ_v^2 and σ_U^2.

3 Proposed formulation

Following the approach of Ackermann (1983) and Baird (1962), we introduce the only approximation of this manuscript: that the δ's coincide with the differentials. This is equivalent to assuming that the fluctuations (and the δ's) are smaller in magnitude than their respective means, which is realistic, but may or may not be true in all atmospheric conditions. The goal is to derive formulations for σ_U^2 and TI that depend only on statistics of the wind components.

First, we use the assumption that the δ's can be approximated as differentials as follows:

$$(\delta U)^2 \approx \left(\frac{\partial U}{\partial u}\right)^2 (\delta u)^2 + \left(\frac{\partial U}{\partial v}\right)^2 (\delta v)^2 + 2\left(\frac{\partial U}{\partial u}\right)\left(\frac{\partial U}{\partial v}\right)\delta u \delta v. \tag{14}$$

Note that, in Eq. 14, the partial derivatives are to be evaluated at the "point" of the function $U = f(u,v)$ around which there are the fluctuations, thus for the mean values of $\bar{u}$ and $\bar{v}$. Thus, the two partial derivatives are actually:

$$\left(\frac{\partial U}{\partial u}\right) = \left.\frac{\partial U}{\partial u}\right|_{\bar{u},\bar{v}} = \frac{1}{2}\left(u^2+v^2\right)^{-\frac{1}{2}}(2u)\Big|_{\bar{u},\bar{v}} = \frac{\bar{u}}{\sqrt{\bar{u}^2+\bar{v}^2}}, \tag{15}$$

$$\left(\frac{\partial U}{\partial v}\right) = \left.\frac{\partial U}{\partial v}\right|_{\bar{u},\bar{v}} = \frac{1}{2}\left(u^2+v^2\right)^{-\frac{1}{2}}(2v)\Big|_{\bar{u},\bar{v}} = \frac{\bar{v}}{\sqrt{\bar{u}^2+\bar{v}^2}}, \tag{16}$$

which are not a function of time t. Replacing Eqs. 15 and 16 into Eq. 14 leads to the following expression for σ_U^2:

$$\sigma_U^2 = \frac{1}{N}\sum (\delta U)^2 = \overline{(\delta U)^2} \approx \tag{17}$$

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$$\overline{\left.\frac{\partial U}{\partial u}\right|^2_{\bar{u},\bar{v}}(\delta u)^2+\left.\frac{\partial U}{\partial v}\right|^2_{\bar{u},\bar{v}}(\delta v)^2+2\left.\frac{\partial U}{\partial u}\right|_{\bar{u},\bar{v}}\left.\frac{\partial U}{\partial v}\right|_{\bar{u},\bar{v}}}\delta u \delta v=$$

$$\frac{\bar{u}^2}{\bar{u}^2+\bar{v}^2}\sigma_u^2+\frac{\bar{v}^2}{\bar{u}^2+\bar{v}^2}\sigma_v^2+2\frac{\bar{u}\bar{v}}{\bar{u}^2+\bar{v}^2}\sigma_{uv},$$

where σ_{uv} is the covariance of u and v, which can be positive or negative. Therefore:

$$\sigma_U^2 \approx \frac{\bar{u}^2\sigma_u^2+\bar{v}^2\sigma_v^2+2\bar{u}\bar{v}\sigma_{uv}}{\bar{u}^2+\bar{v}^2}<\sigma_u^2+\sigma_v^2. \tag{18}$$

Using $\sigma_u^2+\sigma_v^2$ as an approximation for σ_U^2 generally causes an over-estimation of the variance of U, especially when $\bar{u}$ and $\bar{v}$ are of opposite sign (e.g., in the second and fourth quadrants) and the covariance is positive, or vice versa when $\bar{u}$ and $\bar{v}$ are of the same sign and σ_{uv} is negative.

If the two variables u,v were independent (but they are not), their covariance σ_{uv} would be zero; since σ_{uv} is often unknown, it can be set to zero as an approximation:

$$\sigma_U^2 \approx \frac{\bar{u}^2\sigma_u^2+\bar{v}^2\sigma_v^2}{\bar{u}^2+\bar{v}^2}<\sigma_u^2+\sigma_v^2. \tag{19}$$

Note that, when the wind turbine is yawed along the prevailing wind, $\bar{v}$=0 and therefore $\sigma_U^2 \approx \sigma_u^2$ from Eq. 18. This result is technically incorrect because lateral fluctuations (i.e., δv) do contribute to σ_U^2, but it is consistent with the derivation in Appendix B by Larsén (2022).

To obtain an expression for TI, we derive an approximation for $\overline{U}$ as follows:

$$\bar{U}=\sqrt{(\bar{u}+u')^2+(\bar{v}+v')^2} \tag{20}$$

$$=\sqrt{\bar{u}^2+\bar{v}^2}\sqrt{\frac{(\bar{u}+u')^2+(\bar{v}+v')^2}{\bar{u}^2+\bar{v}^2}} \tag{21}$$

$$=\sqrt{\bar{u}^2+\bar{v}^2}\sqrt{1+\frac{2u'\bar{u}+u'^2+2v'\bar{v}+v'^2}{\bar{u}^2+\bar{v}^2}}. \tag{22}$$

The term under the square root can be simplified via the binomial approximation for $\alpha=1/2$:

$$(1+x)^{\alpha}\approx(1+\alpha x), \tag{23}$$

valid for $|x|<1$ and $|\alpha x|<<1$, which are generally true in Eq. 22 due to the assumption that the fluctuations are small with respect to the means, as follows:

$$\bar{U}\approx\sqrt{\bar{u}^2+\bar{v}^2}\,\overline{1+\frac{u'\bar{u}+v'\bar{v}}{\bar{u}^2+\bar{v}^2}+\frac{1}{2}\frac{u'^2+v'^2}{\bar{u}^2+\bar{v}^2}}. \tag{24}$$

Using the Reynolds averaging properties, the final expressions for $\overline{U}$ and $\overline{U}^2$ are:

$$\bar{U}\approx\sqrt{\bar{u}^2+\bar{v}^2}\left(1+\frac{1}{2}\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}\right), \tag{25}$$

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$$\bar{U}^2 \approx \left(\bar{u}^2 + \bar{v}^2\right)\left(1 + \frac{1}{2}\frac{\sigma_u^2 + \sigma_v^2}{\bar{u}^2 + \bar{v}^2}\right)^2. \tag{26}$$

Since the term in parenthesis in Eq. 26 is greater than 1, not only is the inequality in Eq. 8 confirmed, but it can be further expanded to:

$$\bar{U}^2 > \bar{u}^2 + \bar{v}^2. \tag{27}$$

One could be tempted to replace the expression for $\overline{U}^2$ from Eq. 26 in Eq. 7, but doing so would cause the expression for the variance of wind speed to become negative because the error introduced by the binomial approximation, although small when used for $\bar{U}$, is amplified in $\bar{U}^2$, especially when it is used in a difference of terms of similar magnitudes as in Eq. 7. When used in the denominator and alone, however, as is the case for TI from Eq. 10, Eq. 26 is acceptable and we obtain:

$$TI^2 = \frac{\sigma_U^2}{\overline{U}^2} \approx \frac{\bar{u}^2\sigma_u^2 + \bar{v}^2\sigma_v^2 + 2\bar{u}\bar{v}\sigma_{uv}}{\left(\bar{u}^2 + \bar{v}^2\right)^2\left(1 + \frac{1}{2}\frac{\sigma_u^2 + \sigma_v^2}{\bar{u}^2 + \bar{v}^2}\right)^2}. \tag{28}$$

4 Application

Five-minute average measurements from the 10-m sonic anemometer mounted on the 49-m meteorological tower used during the VERTEX (VERTical Enhanced fluX) field campaign (Archer et al., 2019; Wu and Archer, 2021), conducted in coastal Delaware between September and November 2016, are used to demonstrate the validity of the proposed formulation for small and large turbulent perturbations and compare its performance against that of the inexact equations discussed above. Two different time series of six hours on September 11, 2016 are considered. The first (Fig. 1a) is characterized by small perturbations of wind speed, u, and v components (i.e., standard deviation much lower than the mean). The second time period (Fig. 1b) exhibits a decrease in the magnitude of the u-component with time, which causes a decrease in wind speed and introduces large perturbations around the means (i.e., standard deviation larger than or about the same as the mean).

Several conclusions can be drawn after looking at Table 1. First, the proposed formulations for σ_U^2 (Eq. 18) and for TI (Eq. 28) perform well in absolute terms, with percent errors lower than 11% in general and close to zero for σ_U^2 for the case with small perturbations. Using $\sigma_u^2 + \sigma_v^2$ as an approximation for σ_U^2 always causes an overestimation (i.e., positive error), as mentioned earlier, with en error that is larger in magnitude than that of Eq. 18 (9.50% versus -0.28% for small perturbations and 14.07% versus 3.30% for large). The error for TI is larger than that for σ_U^2, due to the additional approximation introduced by the division of Eq. 18 by Eq. 26. In all cases, the proposed formulations exhibit a lower percent error for small than for large perturbations, as expected due to the assumption of small perturbations. TI is always grossly overestimated by using the approximation $\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}$ because the numerator overestimates, while the denominator underestimates.

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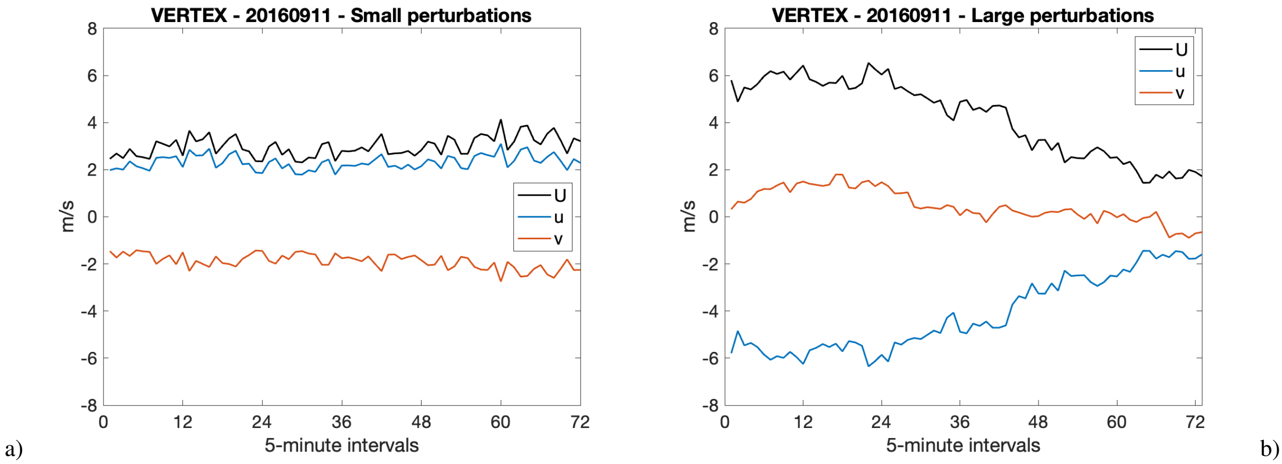

Figure 1. Time series of 5-minute average wind speed, u, and v components during the VERTEX field campaign on 11 September 2016 in the VERTEX: a) a 6-hour period with small perturbations and b) another 6-hour period with large perturbations.

	Small perturbations		Large perturbations	
	Value	Error (%)	Value	Error (%)
$\bar{u}$	2.32 m/s	-	-4.13 m/s	-
$\bar{v}$	-1.90 m/s	-	0.49 m/s	-
$\bar{U}$	3.00 m/s	-	4.20 m/s	-
$\bar{U}^2$	9.00 m^2/s^2	-	17.63 m^2/s^2	-
σ_u^2	0.10 m^2/s^2	-	2.50 m^2/s^2	-
σ_v^2	0.10 m^2/s^2	-	0.47 m^2/s^2	-
σ_U^2	0.18 m^2/s^2	-	2.60 m^2/s^2	-
σ_{uv}	-0.08 m^2/s^2	-	-0.92 m^2/s^2	-
TI^2	0.0195	-	0.1476	-
Eq. 25: $\bar{U} \approx \sqrt{\bar{u}^2+\bar{v}^2}\left(1+\frac{1}{2}\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}\right)$	3.03 m/s	0.98	4.51 m/s	7.48
Eq. 26: $\bar{U}^2 \approx \left(\bar{u}^2+\bar{v}^2\right)\left(1+\frac{1}{2}\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}\right)^2$	9.17 m^2/s^2	1.97	20.36 m^2/s^2	15.52
Eq. 18: $\sigma_U^2 \approx \frac{\bar{u}^2\sigma_u^2+\bar{v}^2\sigma_v^2+2\bar{u}\bar{v}\sigma_{uv}}{\bar{u}^2+\bar{v}^2}$	0.18 m^2/s^2	-0.28	2.69 m^2/s^2	3.30
Eq. 28: $TI^2 \approx \frac{\bar{u}^2\sigma_u^2+\bar{v}^2\sigma_v^2+2\bar{u}\bar{v}\sigma_{uv}}{\left(\bar{u}^2+\bar{v}^2\right)^2\left(1+\frac{1}{2}\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}\right)^2}$	0.0191	-2.05	0.1320	-10.57
$\sigma_u^2+\sigma_v^2$	0.19 m^2/s^2	9.50	2.97 m^2/s^2	14.07
$\frac{\sigma_u^2+\sigma_v^2}{\bar{u}^2+\bar{v}^2}$	0.0214	9.74	0.1719	16.46

Table 1. Statistics of the two observed cases of small and large perturbations during the VERTEX field campaign and performance of the formulations discussed in Section 3.

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5 Conclusions

An analytical equation that approximates the variance of wind speed as a function of the variances of the wind components is derived under the assumption that the turbulent fluctuations of the wind components are small with respect to their means. The approximation for the variance of wind speed is then used, after a few steps, to derive another approximation for turbulence intensity. Although a through validation is beyond the scope of this note, both formulations appear to perform well against a few samples of observations obtained during the VERTEX field campaign of 2016 and to outperform the two incorrect equations that have been used at times in the literature.

Competing interests. Archer is a member of the editorial board of Wind Energy Science.

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