https://doi.org/10.5194/wes-2024-42
Preprint. Discussion started: 12 April 2024

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Effect of Blade Inclination Angle for Straight Bladed Vertical Axis Wind Turbines

Laurence Morgan[1], Abbas Kazemi Amiri[1], William Leithead[1], and James Carroll[1]

[1]EEE, University of Strathclyde, Glasgow, G1 1XW, Scotland

Correspondence: Laurence Morgan (laurence.morgan@strath.ac.uk)

Abstract. Vertical Axis Wind Turbines (VAWTs) have received renewed research interest in the offshore environment due to a number of design synergies that have the potential to decrease the cost of energy for offshore wind. Many studies have been completed on the rotor design for straight bladed (H) rotors however there is sparse information on the effect of blade inclination angle on VAWT aerodynamic performance, and the optimal blade design of VAWTs with inclined blades (V-rotors) for maximum power capture.

This paper presents a systematic study into the effect of blade inclination angle, chord distribution, and blade length on VAWT performance. In the case of fixed chord length blades, it is found that significant power gains are available through blade inclination, between 10% and 68%, dependent on blade length. This is driven by the increase in rotor swept area. Further investigation indicates that despite this, under maximum blade stress limitations the most economical solution for fixed chord length blades are H-rotors.

Optimal chord distributions to maximise the rotor power coefficient are then obtained, and a natural blade taper is observed. Significant power gains, between 10% and 69% dependent on blade length, are observed through blade inclination. However, consideration must be taken to limit blade mass. For a given power rating, whilst satisfying limitations on maximum blade root bending stress, it is found that blade volume can be reduced between 9% and 42% dependent on blade length, and rotor torque can be reduced between 3% and 9%. This indicates the potential of V-rotors to reduce the cost of energy compared to H-rotors in traditional VAWT designs. Additionally, inclined blades are shown to increase the operational tip speed ratio, demonstrating their applicability to turbines using secondary rotors, such as the X-Rotor. (1) please explain how?

1 Introduction

1.1 background

Renewable energies are key to combating the ongoing climate crisis, and offshore wind energy is a integral pillar of this response (IRENA, 2022). It has be estimated that a three fold increase in the rate of deployment of wind energy is required by 2030 to meet our climate goals (Lee and Zhao, 2021) and decreasing the cost of energy for offshore wind can significantly increase the rate of offshore wind deployment. In this context Vertical axis wind turbines (VAWTs) have been identified as a technology that has the potential to significantly reduce the cost of offshore wind energy due to a number of design synergies

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(Borg et al., 2012) (Sutherland et al., 2012). Further to this, recent work has demonstrated that a number of mechanisms allow for VAWT wakes to re-energies significantly faster than those of traditional horizontal axis wind turbines (HAWTs), facilitating increased wind farm density (Huang, 2023) and further reductions in the cost of energy. This has lead to significant academic and commercial interest in the design concept (Hand and Cashman, 2020).

VAWTs have many potential configurations, with any 2D shape with vertical symmetry representing a possible rotor configuration. For large offshore structures, practical design considerations have led to a convergence on 2 key designs: Darrieus VAWTs, with curved blades attached at both the blade root and tip, and straight bladed VAWTs, typically configured with blades orientated parallel to the axis of rotation in an H configuration. Straight bladed designs with inclined blades have also been proposed (Ljungström et al., 1974; Musgrove, 1977; Sharpe and Taylor, 1983; Shires, 2013a; Leithead et al., 2019), these rotors will be henceforth referred to as V-rotors.

The use of V-rotors was first introduced by Ljungström (Ljungström et al., 1974), initially referred to as 'Y' rotors. It was noted in this work that straight bladed VAWTs facilitated cyclic pitching more easily than Darrieus type. This rotor concept was also briefly described by Park (Park, 1976). A variable geometry H-Rotor with inclinable blades was later proposed by Musgrove (Musgrove, 1977), this used a reefing system to incline the blades as a means of reducing power capture at high wind speeds. Wind tunnel tests on a scale model conducted by Willmer (Willmer, 1980) demonstrated that the power coefficient decreased significantly as the blade inclination angle was increased from 0° to 60°. Figure 1a) shows the 'Musgrove 250' turbine with blades in the 'reefed' position.

David Sharpe further developed Ljungström's rotor concept from 1983 onwards (Sharpe and Taylor, 1983). These designs did not utilise a cross-arm, with the blades attached directly to the rotor hub. Listed benefits of the design included: The reduction in capital costs due to the short rotor tower and the lack of cross-arm, the ease of manufacturing of straight (untwisted) blades, the potential for control with either variable geometry or variable pitch, and the increased portion of the rotor swept area sampling higher wind speeds at high altitudes. An initial study determined the power coefficient for inclinations angles of 35°, 45° and 55° for an blade aspect ratio of 11.9, finding the maximum power coefficient occurring with a inclination angle of 55°. Additionally, increasing blade aspect ratio was found to increase the peak power coefficient for an inclination angle of 45° (Sharpe and Taylor, 1983). The aerodynamic simulation work was completed using a BEM based double multiple streamtube theory (DMS) approach described in (Sharpe, 1984). Further work was completed, validating the aerodynamic simulation work and demonstrating how the rotor could be controlled using a tip-pitch system in (Robotham et al., 1985), and a 5kW system was field tested in 1987 (Sharpe et al., 1987), this system is shown in figure 1b). Following these tests, a single bladed design utilizing a counterbalancing weight was proposed (Shires, 2013a), this design is shown in figure 1c). Following this period, the development of V-Rotor concepts was predominantly undertaken by private engineering companies. Altechnica continued to work on the concept through the 'Taylor V-Turbine' as shown in figure 1d).

The next significant academic work on V-rotors was undertaken in the NOVA project with in the design of the Aerogenerator-X as shown in figure 1e) (Shires, 2013a). This work was conducted as partnership of a number of British academic institutions in conjunction with Wind Power ltd. Here, a 10MW V-rotor was designed, the rotor blades were augmented with sails, aimed and counteracting the aerodynamic overturning moments inherent in V-VAWT designs, the rotor design work is presented in

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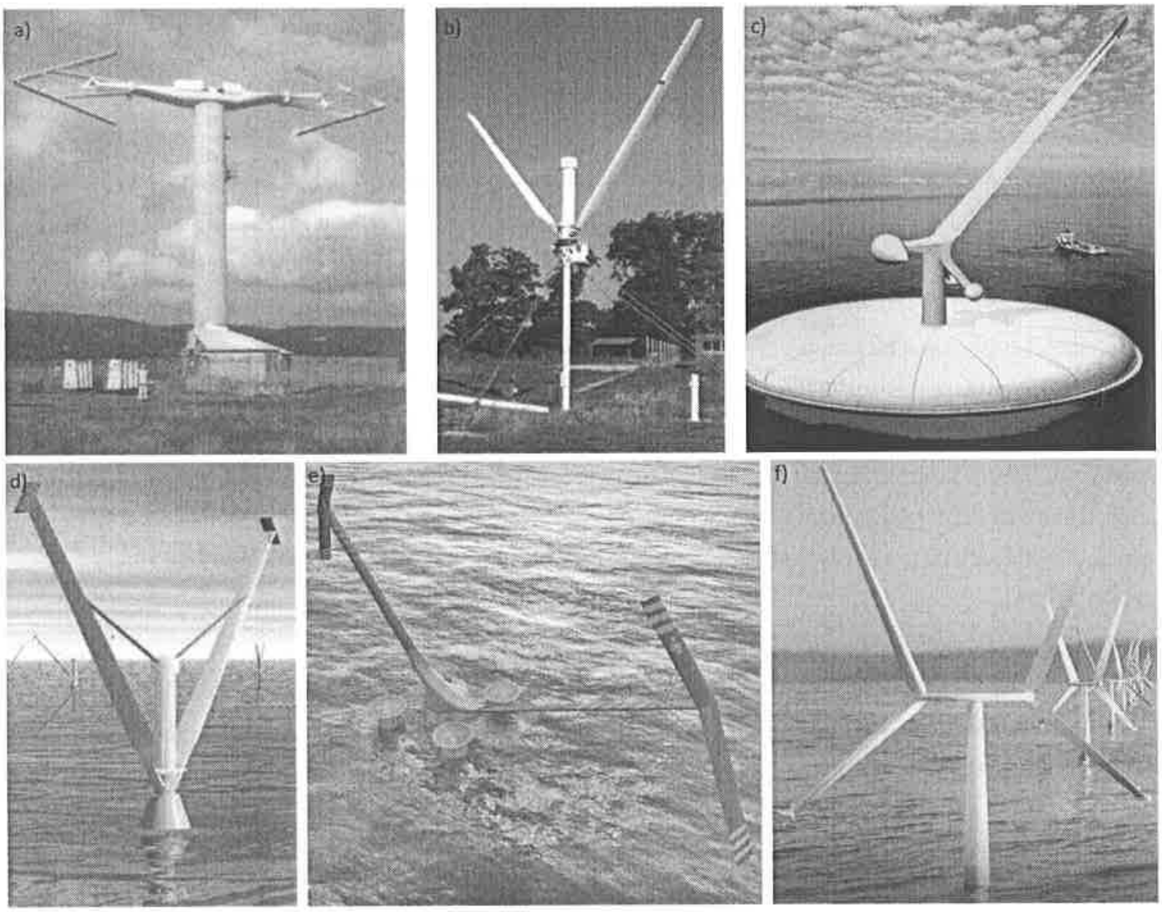

Figure 1. a) Musgrove 250 turbine (Price, 2006), b) 5kW V-Rotor tested by Sharpe and Taylor (The Open University, 2015), c) Single bladed V-VAWT concept (Shires, 2013a), d) Taylor V-Turbine (The Open University, 2015), e) The aerogenerator concept (Shires, 2013a), and f) the X-Rotor wind turbine concept.

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(Shires, 2013a). A gradient based optimisation approach was coupled to a DMS based aerodynamic solver (Shires, 2013b) and was used to find a rotor geometry that minimised the rotor volume (used as a proxy for the rotor cost) under a number of design constraints including ensuring a rated power of 10MW at 13m/s with rated rotor speed of 4rpm, the limitation of the maximum overturning moment and a constraint on the maximum length of unsupported sails and struts. The completed optimisation was found to decrease the rotor volume by 3% whilst preserving the aerodynamic power and decreasing the overturning moment by 11%. For some of the optimisations presented, the inclination of the rotor blades was a free variable, however the optimisation procedure did not significantly alter the initial inclination angle of 59°.

Following the completion of the NOVA project, the X-Rotor wind turbine concept began development at the university of Strathclyde (Leithead et al., 2019) and further research is underway in the form on an EU H2020 project (Cordis, 2023). The turbine consists of an X-shaped primary rotor, made up of an upper and lower V, which utilises secondary rotors, attached to the tips of the lower V, for power take off. The use of inclined rotor blades is critical in the X-Rotor concept as a means of increasing the primary rotor tip speed ratio, which is key to ensuring efficient power conversion between the primary and secondary rotors Morgan et al. (2024). A rendering of the turbine concept is shown in figure 1e). The primary rotor blades are attached to a central cross-arm with a radius of 25m and extend to a tip radius of 75m, with inclination angles of 30° and 130° respectively. The lower blades reduce the overturning moment in a similar manner to the sails of the aerogenerator concept, and provide an attachment location for the secondary rotors. The aerodynamic and structural design of the rotor is presented in (Leithead et al., 2019), however the aerodynamic design process of the primary rotor was not presented. A DMS model was developed and validated against higher fidelity lifting line free vortex wake simulations to simulate the X-Rotor turbine in (Morgan and Leithead, 2022) and comparison of multiple aerodynamic codes simulating the primary rotor is presented in Giri Ajay et al. (2023).

Absent from the current work published on V-rotors, as represented in this literature review, is a systematic study on the effect of key rotor design parameters, particularly the blade inclination angle, on aerodynamic performance. This hinders the design of V-rotors at the conceptual stage. There is some sparse data on the effect on rotor inclination angle for specific blade/rotor designs at the lab scale (Willmer, 1980; Sharpe and Taylor, 1983), however these show conflicting trends. Additionally, whilst the effect of solidity and aspect ratio have been well studied for H-rotors, thus far these parameters have not been significantly investigated for V-rotors. In the case of rotor solidity, this may be critically important as V-rotor blade sections operate at variable tip speed ratios, indicating that blades with a variable chord distribution are likely to increase aerodynamic efficiency.

(4) Please explain how this is accomplished.

Additionally, the current literature on V-rotors does not explicitly describe the effect of rotor geometry on power capture. Results are typically presented with respect to the rotor power coefficient C_P, however it is unclear whether the normalisation area is defined with the initial turbine area, or the rotor with inclined blades. A key benefit of the V-rotor concept is the increase in swept area achievable by inclining a blade of fixed length, this must be explicitly accounted for when presenting results.

1.2 Research contribution

(5) ?

This paper presents the results from a systematic numerical study of straight bladed VAWT configurations, completed to understand the effect of blade inclination angle, rotor aspect ratio, and blade chord length on rotor aerodynamic performance

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and investigates optimal blade design for V-rotors. Firstly, fixed chord length blades are considered and the effect of chord length, inclination angle, and rotor aspect ratio are investigated in terms of power production, thrust loading, and optimal tip speed ratio. Following this, the data from these simulations are used to define plan-form blade designs that are optimised for maximum power capture. The effect of blade inclination angle and design tip speed ratio on the optimal blade plan-form are then discussed. Finally, the aerodynamic performance of these optimal blades is presented, and the opportunity for cost of energy reduction is discussed.

A systematic approach based on a grid search, rather than an optimisation tool, is used as it allows the wider design space to be studied and understood, allowing for broad trends to be identified, and engineering judgements to be applied without the formulation of specific cost functions. The investigation will use a single aerofoil section (NACA0018) and will not consider twisted or pitched rotor blades, as the introduction of these design variables would considerably increase the design space, these limitations are discussed in section 2.3.

In conclusion this paper provides:

1. A systematic study into the effect of blade inclination angle and rotor aspect ratio on the optimal design and aerodynamic performance of straight bladed VAWTs.

2. A study into the optimal chord distribution of inclined VAWT blades.

3. An evaluation of the potential for inclined blades to lower blade volume and rotor torque.

1.3 Paper structure

Section 2 describes the methodology used for numerical simulation, rotor geometry generation and blade geometry generation. Section 3 presents the key results from the numerical study, including the rotor performance parameters and optimum blade design, and section 4 discusses these results. Finally section 5 provides an overview of the key research outcomes and discusses avenues where work could be continued.

2 Methodology

2.1 Numerical Simulation

The numerical simulation of VAWTs is considerably more complex than that of HAWTs, both at the rotor scale and at the blade element scale. At the rotor scale, the flow interacts with the rotor surface twice, initially in the upwind, then in the downwind sweep. At the blade element scale, large cyclic variations in the angle of attack experienced along the blade which further complicate numerical simulation. This work utilises a 2 Dimensional Actuator Cylinder (2DAC) approach for the numerical simulation of the rotor aerodynamics.

The 2DAC model, first developed by Madsen (Madsen, 1988), discretises the rotor along it's vertical axis into 2D rotor segments. These circular rotor segments are treated as 2D actuator surfaces, and the induced velocity field is obtained through

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solving the Euler equations in two dimensions. The loading of the actuator surface is coupled to the integrated loads on the blade element, which are calculated using tabulated polars. These are coupled back to the solution of the flow field as the blade loads are dependent on the both the relative velocity and angle of attack at the blade element. The 2DAC model used in this study was developed by TUDelft as part of the X-Rotor project (Ferreira, 2021), a linearised solution to the Euler equations is employed, as presented in (Madsen et al., 2013), and a Prandtl tip loss function is applied (Prandtl et al., 1927). Due to the large number of simulations required, simulations were completed with a course discretisation of 20 blade elements per rotor, with an angular step of 5°.

The 2DAC model was selected as it is momentum based, and can therefore complete simulations in significantly less computational time than higher fidelity approaches such as free vortex wake models and viscous computational fluid dynamics. This was important due to the large number of simulations required for this parametric study. Whilst DMS methods are also computationally light, it has been shown that 2DAC models more accurately reproduce the results of higher fidelity models over a range of rotor design parameters and operating conditions (Ferreira et al., 2014). Specifically 2DAC modelling has recently been shown to accurately reproduce rotor averaged performance parameters for inclined rotors in the case where pitch offsets remain small (Giri Ajay et al., 2023). There are, however, many ways in which this model does not directly reflect the behaviour of an actual rotor installed in realistic environmental conditions. These limitations in the modelling methodology are listen below:

1. Dynamic stall modelling: The delay of flow separation around a blade section as the angle of attack exceeds the static stall angle, and the corresponding increase in the lift coefficient beyond the maximum static value is referred to as dynamic stall. This effect is present during VAWT operation (Laneville and Vittecoq, 1986), and has been shown to considerably alter VAWT rotor performance (Marten et al., 2016; Bianchini et al., 2018). As the blade root of a V-VAWT is operating at a low local tip-speed ratio and often with a large chord to radius ratio, the flow conditions are often out-with the range of validity that engineering based dynamic stalls models have been validated, i.e. at a local tip speed ratio bellow 1. Dynamic stall corrections have therefore not been included in order to limit the uncertainty in model prediction, and to provide transparency in the results.

2. Flow curvature modelling: Flow curvature effects have been shown to be significant for VAWT aerodynamic behaviour (Rainbird et al., 2015), however have been ignored in this investigation. This study is attempting to provide a general view of the effect of rotor geometry, rather than provide a highly accurate solution to the aerodynamic behaviour of a rotor with a specific geometry. A single aerofoil section (NACA0018) represented by a single set of polars, uncorrected for flow curvature, are used throughout this study. It could be considered that the geometrical blade sections have undergone an inverse conformal mapping procedure as in (Rainbird et al., 2015), or that a twist distribution has been included that accounts for the effective incidence angle as demonstrated in (Bianchini et al., 2016).

3. Modelling of 3D effects: Due to inclined rotor blades, induced velocities in the vertical direction that impact the flow conditions at each blade section will be present. In addition to these, radial flows will also be present and are expected to modify the blade element characteristics. Both of these effects are inherently ignored in the 2DAC model. However,

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quasi-2D models have successfully been used to study rotors with inclined blades (Shires, 2013b), and have been shown to successfully reproduce the aerodynamic characteristics of 3D models for rotor with inclined blades (Morgan and Leithead, 2022).

4. Inclusion of Wind shear: This study has ignored the effects of wind shear both on power capture and on rotor design in order to preserve the generality of the results. Appendix A discusses this issue further. Any detailed rotor design process should include the effects of wind shear.

5. Strut modelling: The aerodynamic drag arising from strutting has been shown to reduce rotor power performance for VAWTs (Worstell, 1981), however in this study the rotor support structure has not been considered. There is a wide range of strut configurations proposed for different V-rotor concepts, as shown in figure 1, and for this reason only the aerodynamically active parts of the rotor have been considered in this analysis.

In order to to provide a realistic optimal aerodynamic design for an inclined rotor, each of these limitations must be addressed in the modelling. However, this paper is not attempting to find a single optimum rotor design, but is providing a survey of the design space in order to inform readers on the trends associated with increasing rotor inclination angle, and to demonstrate the potential benefits in power production available to rotors with inclined blades.

2.2 Rotor geometry generation

In this paper, the rotor geometry refers to the shape of the rotor swept area, independent of the blade geometry. As the rotors considered in this study are straight bladed, the rotor geometry can be defined by the inner radius r_0, the blade length l, and the inclination angle ψ. For generality, the blade length can be non-dimensionalised by the inner radius, $l_r = l/r_0$; this is equal to double the rotor aspect ratio for an H-rotor. For this study rotor geometries are generated with $1 \leq l_r \leq 4$, and $0° \leq \psi \leq 50°$. As mentioned in section 2.1, only the aerodynamically active sections of the rotor are considered here, and neither the tower nor rotor struts are modelled. The variables l, r_0 and ψ are labelled in figure 2 for clarity. For a given rotor configuration, the frontal area can be calculated with

$$A = r_0^2 \left[2l_r \cos(\psi) + l_r^2 \cos(\psi)\sin(\psi)\right]. \tag{1}$$

2.3 Blade plan-form generation

The blade designs considered within this study are split into two categories, the first are fixed chord length blades, and the second are blades with an optimised chord distribution. The chord length is presented as non-dimensionalised by the inner radius of the rotor, and the chord lengths considered in this study ensure that the maximum solidity at the blade root does not exceed 1, i.e.

$$\sigma = \frac{Nc}{r_0} cos(\psi) \leq 1. \tag{2}$$

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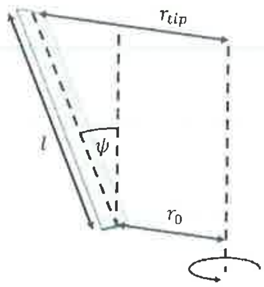

Figure 2. Geometry of a V-rotor.

where N and c represent the number of blades and chord length, respectively. For a two bladed rotor, the maximum non-dimensional chord length ranges from 0.5 for $\psi = 0°$ up to 0.775 for $\psi = 50°$. The full range of non-dimensionalised chord lengths considered for this study is given in table 1.

In order to find the optimum chord distribution, the the 2D rotor segments are assumed independent, an assumption inherent within the 2DAC approach, and the optimum blade plan-form shape is generated through finding the chord length that locally maximises the power coefficient for each rotor segment, using data from the fixed chord length simulations.

This approach can be repeated over a range of tip speed ratios, $1.5 \leq \lambda' \leq 6$, defining a set of blades optimised over a range of tip speed ratios. The optimum blade design for power capture can then be readily identified as the iteration that maximises C_P. The full range of solutions can also be explored to find potentially more practicable solutions and to identify tends within the design space. This approach can be repeated over the full range of rotor geometries, giving a set of optimal blade shapes designed for maximum C_P over a 3 dimensional design space, covering λ', ψ, l_r. The full details of the completed simulations are given in table 1, in total 7332 rotor simulations were completed.

In this study the NACA0018 aerofoil section was used over the full blade length, with blade polars were generated at a Reynolds number (15 million) using X-Foil (Drela, 1989) and extrapolated using the Viterna method (Viterna and Janetzke, 1982). Whilst the optimal blade plan-form is dependent on the blade section used over the blade, this study is attempting to understand the general trends in rotor behaviour based on the rotor geometry, rather than provide a detailed optimal design case. It is expected that trends in rotor design will be consistent over a range of symmetric aerofoils, therefore the use of a single aerofoil section is not considered a significant limitation. The NACA 0018 section was selected as it is commonly used for VAWT rotors, and the Reynolds number was selected to be consistent with multi-MW offshore VAWT turbine concepts such as the X-Rotor.

The effect of blade number is also absent from this investigation. As this study focuses on the rotor averaged performance parameters and utilises an aerodynamic model based on revolution averaged loads, the blade number has only 2 effects: firstly to modify the rotor solidity, and secondly to reduce the tip losses; marginally increasing both power production and rotor loads. The dominant effect is expected to be the change in rotor solidity, which can be readily understood through an equivalent change in the blade chord through equation 2. Whilst the effect of blade number is not directly studied, and the effect of blade

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ψ [°]	l_r	c/r_0	λ'
0	[1,2,3,4]	[0.025,0.05,0.075...0.5]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]
10	[1,2,3,4]	[0.025,0.05,0.075...0.5]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]
20	[1,2,3,4]	[0.025,0.05,0.075...0.525]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]
30	[1,2,3,4]	[0.025,0.05,0.075...0.575]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]
40	[1,2,3,4]	[0.025,0.05,0.075...0.65]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]
50	[1,2,3,4]	[0.025,0.05,0.075...0.775]	[1.5,2,2.5,2.75,3,3.25,3.5,3.75,4,4.5,5,5.5,6]

Table 1. List of rotor design and operational variables permuted over to generate data for this study.

number cannot be directly obtained from the existing data due to the changes in tip losses, the trends in rotor behaviour will be reproduced in terms of solidity, and this is not considered a significant limitation to the study.

3 Results

Section 3.1 presents the results of studying the effect of rotor inclination angle on fixed chord length blades, examining the relationship between blade inclination angle and rotor averaged variables, including the thrust, power, and optimal tip speed ratio. Variables pertinent to the cost of energy are introduced, and limitations on the maximum bending stress in the blade are investigated. Next, (12)

~~Following this~~, the blade optimisation procedure described in section 2 is demonstrated, and the resultant optimum blade plan-form shapes are discussed in section 3.2.

Section 3.3 evaluates the aerodynamic behaviour of rotor designs utilising the optimised blades presented in section 3.2. Next, ~~Following this~~, variables influential on the cost of energy are discussed, and limitations on maximum bending stresses are once again imposed to evaluate the practicality of rotor designs.

In order to understand how the rotor geometry effects rotor design, two variables are introduced here, the power product, and the thrust product. They are given by the relevant rotor performance coefficients scaled by the rotor area, to demonstrate aerodynamic and geometrical effects of rotor design:

$$\Phi_P = C_P A \tag{3}$$

(13) This scaling does not preserve the whole concept of similitude as grounds for comparison. Can you please explain how this scaling is better than the traditional non-dimensional groups. for comparison of performance? ??

$$\Phi_T = C_T A \tag{4}$$

where A represents the rotor area. When normalised with respect to the values obtained for an equivalent H-Rotor, these variables allow the direct comparison between the power capture and rotor loads for rotors of different swept area.

(14) This needs more clarification. What is an "equivalent H-rotor"? How Φ_P & Φ_T then allow comparison of loads if the choice of A is different? Traditionally Cp & CT would be normalised for same reference swept area when comparing two different turbines with different swept areas.

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(15) You should be able to get the same results for Cp & CT by normalising using same swept area as that of a H-rotor!

3.1 Survey of design space fixed chord

Figure 3 displays the maximum power product, the corresponding thrust product, and the optimal tip speed ratio at which the maximum power coefficient is reached as a function of the rotor inclination angle and the chord length of the blade. For ease of comparison, the power and thrust products are normalised by the values obtained for the optimal H-Rotor. The blue square marker represents the location of the optimal H-Rotor design, whilst the red diamond marker indicates the largest achievable power product. The grey shaded area represents the region which has not been simulated, due to the restrictions on chord length discussed in section 2. This is repeated over the range of aspect ratios introduced in section 2.

The rotors were simulated up to a maximum tip speed ratio of 6. It is clear that with the large inclination angles and high blade lengths, the optimum power coefficient is not reached for the low chord length cases. This implies that the decrease in maximum power product for these cases is artificially smaller due to the limited search range, and that a high power product may be achievable. (16) No, this is not correct unless proven. Both higher solidities and TSR have been observed to produce less power or reduce Cp & CT.

Figure 4 (a) presents the maximum power product for the optimal straight H-Rotor blade ($c = 0.2r_0$) as a function of rotor inclination angle for a range of rotor aspect ratios alongside the rotor area. Both variables are normalised by the respective values for $\psi = 0$. Figure 4 (b) presents the corresponding tip speed ratio at which these maximum variables are achieved. From this figure it is again clear that, for combinations of high aspect ratios and high inclination angles, the maximum tip speed ratio range considered in this study is not sufficiently large to capture the maximum achievable power coefficient. Cases where the optimal tip speed ratio is limited by the design range used within the study are highlighted in red.

(19) only in terms of rotor size.

Whilst the potential for power gains are clear from figures 3 and 4, it is important to understand the effects of any rotor design on the cost of energy. At an early design stage, blade cost can be assumed to scale with blade mass, and whilst a detailed design structural design is required to obtain blade mass estimates, the blade volume can be used as a proxy variable to understand the relative change in blade mass between different designs. This is given by

$$V = l\int_{0}^{1} k_1 c(x)^2 dx, \tag{5}$$

where k_1 represents a shape parameter, and x represents the non-dimensional spanwise coordinate. In this study, the shape parameter is constant along the blade as a single aerofoil section is used. In addition to blade mass changes, variations in the rotor geometry, specifically the tip radius r_{tip}, and design tip speed ratio effect speed/torque input to the drivetrain. Assuming similarity scaling, the drivetrain torque can be used as a proxy for the generator mass and cost. The rotor torque is given by:

$$Q = \frac{P}{\Omega} \tag{6}$$

with the rotor speed, Ω, given by

$$\Omega = \frac{\lambda' U_0}{r_{tip}}, \tag{7}$$

and the tip radius given by

$$r_{tip} = r_0^2\left[1 + l_r \sin(\psi)\right]. \tag{8}$$

Better to show the effect of ψ for same swept areas rotor designs!

(20) The authors need to address the comments so that the later sections that build on these become more clear.

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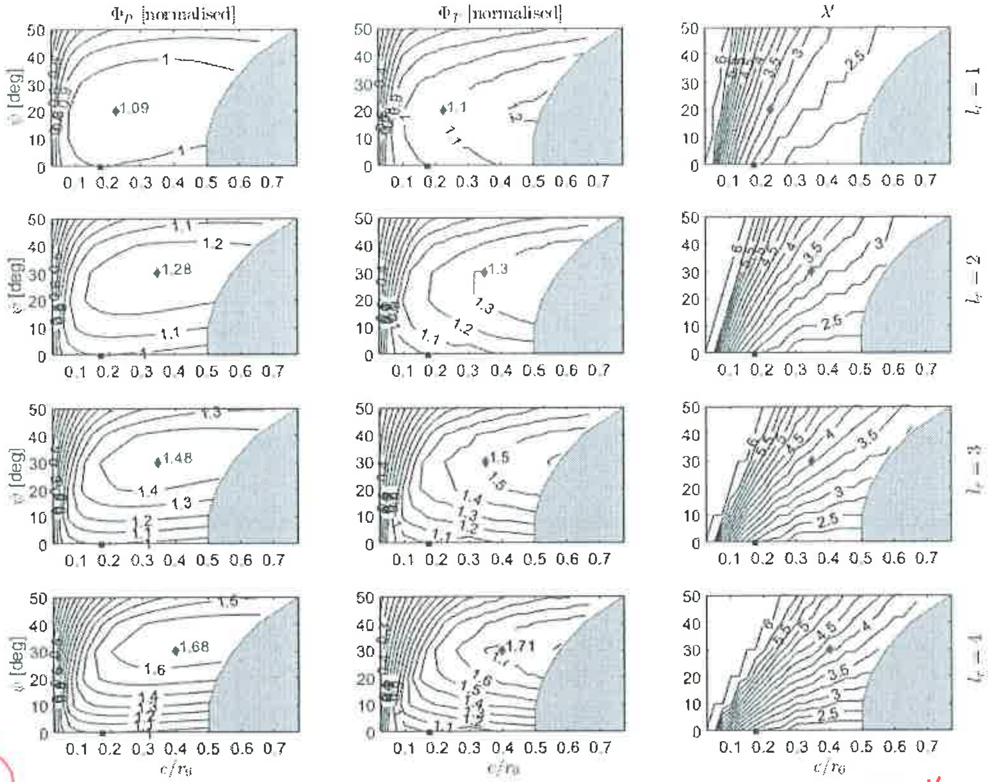

(17) Please explain the effect of each variable in the discussion.

Figure 3. Power product, thrust product and optimal tip speed ratio, for untapered VAWT designs, presented as a function of c/r_0 and ψ, with thrust and power normalised by the optimal H-Rotor design. Results shown for blade lengths of $l_r = [1, 2, 3, 4]$. The optimal H-rotor design is indicated by a blue square, and the optimal V-rotor design is indicated by a red diamond.

In order to compare values over different rotor designs, it is instructive to compare rotors with equal power production, this can be achieved through introducing the linear scaling parameter s, with

$$\Phi_P^{[1]} = s^2 \Phi_P^{[2]}. \tag{9}$$

The scaling parameter defines how the length scale of rotor design [2] must be modified in order to equal the power rating of design [1]. The inner radius, blade length, blade chord scale linearly with s, the change in rotor swept area, blade volume, and rotor torque then scale with equations 1, 5, 6 respectively.

A final consideration is the bending stresses within the blade. A blade with a specific material design will be capable on withstanding a certain maximum bending stress. In comparing blade designs, it is important to compare designs that have a similar maximum bending stress, otherwise material innovations are implicitly included within the comparison. Treating the

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(18) Again these Fig. 4 results are not clear! and perhaps misleading. e.g. comparing the 2 circled points. $l_r = 3$ is a larger rotor than $l_r = 2$. Normalizing with $\psi = 0$, will of course lead to larger ϕ_p or ϕ_T by intuition. But it does not convey the advantage of one ψ over the other unless areas are same! So you need to show results for same scaled swept areas to see advantage.

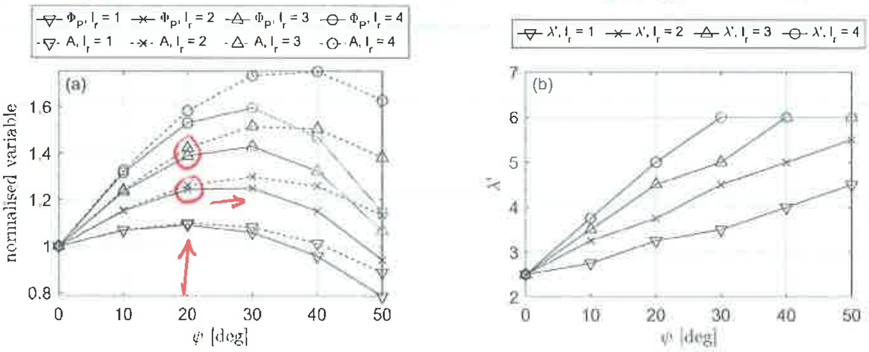

Figure 4. (a) Rotor power and rotor area for a blade of $c_r = 0.2$ normalised by the H-rotor values presented as a function of inclination angle. (b) Corresponding optimal tip speed ratio. Values where trends indicate the optimal tip speed ratio extends beyond $\lambda' = 6$ are marked in red.

blade a simple beam, the out of plane bending stresses at span-wise location x, and a distance δ from the structural center of the blade (normalised by the chord length c) are given by

$$\sigma(x) = \frac{\delta c(x)}{k' t(x)^3 c(x)} L \int_x^1 F_N(\tilde{x}) \tilde{x} d\tilde{x} = \frac{\delta}{k'' c(x)^3} L \int_x^1 F_N(\tilde{x}) \tilde{x} d\tilde{x}, \tag{10}$$

where $k'' \equiv k' \left(\frac{t}{c}\right)^3$ represents a shape function depended on the thickness of the aerofoil section, as a constant aerofoil section is used, k'' remains constant. As the blade has a fixed chord length, it is clear that the maximum bending stresses occur at the blade root. For the comparison presented here, the spanwise force distribution that maximises the bending stress at the design tip speed ratio is selected.

Figure 5 displays the blade volume and rotor torque for rotor designs of equal rated power as function of inclination angle and blade chord length. The values are normalised by the design values for the optimal H-Rotor. Alongside these contours, the maximum operational bending stresses are presented, normalised by the maximum bending stress for the optimised H-rotor. The constant blade volume contours have a positive gradient up to 20° for the $l_r = 1$ and up to 30° for $1 \leq l_r \leq 4$, indicating that rotors utilising inclined blades can reduce the cost of blades for rotors of the same power output. However, the maximum bending stress contours clearly indicate that inclined rotor blades are subject to larger aerodynamic bending stresses. In all cases, the gradient of the maximum bending stress contours are larger than the gradient of the blade volume contours, this implies that by moving along each constant stress contour the minimum blade volume and rotor torque is obtained at an inclination angle of 0°. This results implies that any material innovation that facilitates a blade design which can withstand larger aerodynamic bending stresses will be best applied to lower the chord length of the H-Rotor design if the objective is to minimise blade costs.

Similarly, the contours in of constant maximum bending stress have a larger gradient than the rotor torque contours, again indicating the most optimal solution to reduce rotor torque, for a given blade material layup, is the H-rotor design. These