Message to editor on: Convergence and efficiency of global bases using proper orthogonal decomposition for capturing wind turbine wake aerodynamics

Dear Cristina,
Please find attached our two responses to the second review of our article. As you can tell, we have decided to push back on some of the comments by the second reviewer, as we think they are misplaced and would make the article unfocused in our opinion. We have provided elaborate justifications on these points and why we do not include them as requested in the article.
Best regards,
Juan Felipe, Juan Pablo and Søren

Second response to Referee #1: Convergence and efficiency of global bases using proper orthogonal decomposition for capturing wind turbine wake aerodynamics

Dear Authors,
After a second review of your article, I believe it has improved since the initial revision. The Abstract and Introduction now outline the article's relevance to the wind energy community. The text is clearer and somewhat easier to read.

Thank you for the comments to the paper. We have clarified aspects and provided responses to your comments below in blue.

The article can be still improved in some points (lines numbers refer to the ATC document):

- Abstract: briefly explain what is a global POD base.

We have clarified the definition of a global POD basis, i.e. that it is based on multiple flow cases.

- Line 137: "and it is approximately constant (trough the wind turbines in the farm) at 0.8 and 0.3".

Fixed. "through the wind turbines in the farm" was added.

- 144: "The fully-developed or "infinite" wind farm is typically reached after the first 5-6 wind turbines [Andersen et al. (2020)]." This concept is applied to discuss the findings in section 3.4. I suggest clarifying the idea of fully-developed wind farm flow and its relation to assessing the performance of the POD reconstruction technique.

Fully-developed conditions are beneficial if there is no discernible difference between inflow to for example turbine number 8 and turbine 9. Hence, even a local POD basis would be similar for both turbines, and therefore obviously a global basis will quickly converge to cover these conditions.

- 181: "The expected sub-optimality of a global POD basis raises a number of central questions on the effectiveness relative to a local POD basis and on the required number and which datasets to include in the construction of a global POD basis". This sentence is not clear (the required number of what?). Please clarify it.

Re-phrased to: "The expected sub-optimality of a global POD basis raises several questions on how effective a global basis is compared to a local basis. For example, how many datasets should be used and which datasets should be included to create a global basis with high-quality performance across the parameter space compared to a local basis"

- 187: "The new global basis is again evaluated". Isn't it "the POD resulting from the new global basins is again evaluated"?

No, the POD is not resulting from the new global basis. Instead, POD is used to create the new global basis (global because it includes more than one dataset). Then this new basis is evaluated again across the parameter space. This has been clarified:

'[...]. Secondly, a new flow case is added, and POD is applied to find the corresponding new basis, which is "global" because it was formed with more than one dataset. The new global basis is again evaluated across all flow cases before a new dataset can be added.'

- 189-190: "the dataset is added to the decomposition is the one with the worst performance to maximize the reduction of the overall error". Please clarify this sentence.

The text has been clarified: "the next dataset added to the decomposition corresponds to the flow case with the maximum error across the parameter space. The reduction of the overall error is maximized with each iteration by including data from the specific flow case with the largest error."

- Figure 6: make the horizontal and vertical axes equal so the rotor becomes a circle.

Implemented.

- 234-239: "is difficult to qualitatively assess that P1 is significantly better than P9." Recall the objective of your evaluation and why it matters that P1 does not surpass P9.

The key point is not that P1 can not surpass P9. In mathematical terms, P1 will be slightly better than P9, as P1 is optimal at capturing the variance. The point is that the two bases are equally good for all practical purposes. We have rephrased to: "The error fields of the two reconstructions are basically indistinguishable with only minor differences. The similarity in both reconstructed velocities and errors clearly shows that the two different bases are equally efficient at reconstructing the flow for all practical purposes."

- 378: "how different global modes are actuated". I think the verb "actuated" is wrong here.

We have change the verb to "active", but the intended meaning is the same. The point is that different flow scenarios are captured by different modes, where some modes are very important for capturing certain flow conditions. This "activation" of different modes is shown in Figure 11, and we refer to similar analysis in Andersen and Murcia, 2023.

- 402: "The iterative procedure particularly identifies adding more datasets with low CT". Clarify this sentence.

Changed to: "The iterative procedure particularly identifies that more datasets from flows corresponding to low C_T should be included"

- 411: "Selection of datasets apriori would typically require domain knowledge to identify key scenarios with different physics", thus it can be impractical (?).

Requiring domain knowledge is in our opinion not impractical, but simply implies that selecting datasets apriori would require physical understanding of the flows, e.g. knowing that freestream, single wake and multiple wake corresponds to different physics.

- 423: "indicating when fully-developed wind farm flow conditions are achieved dynamically and how this is linked to the first few modes". Clarify.

We have elaborated previously how the fully-developed wind farm flow conditions relate to the convergence of the global POD basis. This convergence is clearly shown in the convergence of the distributions of selected modes (Figures 11). Once, converged it is a clear indication that the dynamics of the flows are comparable.

- 425: different (flow) realizations

Implemented.

- *433: "However, the efficiency and convergence of the linear global POD bases also gives promises that it is possible to utilize nonlinear dimensional reduction techniques, such as autoencoders, to increase efficiency further, i.e. reduce the number of modes required". Explain better what you mean.*

We have included a couple of references on how non-linear methods such as autoencoders can provide more efficient dimensional reduction compared to POD.

- *439: "parameter space". Recalls which are the parameters.*

Changed to: "[...] for a parameter space covering all wake-affected turbines in the wind farm during different operating conditions (thrust coefficient)."

- *443: "The performance of the global basis has a basis error with respect to the local POD basis". Rewrite this sentence.*

Changed to: "The global basis gives an error compared to the optimal local basis. However, [...]"

- *Conclusions need to be revised. Begin by summarizing the problem you aim to address, with straightforward explanations about wind farm physics. Next, highlight the tools employed in the study and summarize their key performance findings. Finally, discuss anticipated future advancements in research on this topic.*

The conclusion was re-structured keeping this comment in mind.

Second response to Referee #2: Convergence and efficiency of global bases using proper orthogonal decomposition for capturing wind turbine wake aerodynamics

The manuscript's clarity has been improved. However, I have still a few comments about the modified parts, that should be taken into account.

Thank you for the comments to the paper. We have clarified aspects and provided responses to your comments below in blue.

- abstract: " Wind turbine wake aerodynamics ,,, but are highly turbulent". I think this sentence should be rephrased (The aerodynamics are turbulent?)

Rephrased for clarity.

- page 3: "nacelle or tower, but this only has a minor influence.." I do not agree with this statement, since De Cillis et al. Wind Energy (2021) have proven that the presence of the tower have a strong influence on the POD modes in the wind turbine's wakes. They used actuator line modeling instead of actuator disk, but since the former model is more accurate then the latter, their conclusion is still to be taken into account and mentioned. Moreover, the nacelle has been found to have a strong effect in the generation of low-frequency oscillations of the wake, such as wake meandering, by many other studies (see for instance, the recent experimental work of Biswas & Buxton, J. of Fluid Mechanics, 2024, among many other previous studies). I thus ask for a deeper discussion on this important point.

In our opinion, the reviewer overestimates the importance of towers on the flow in large wind farms, particularly in terms of importance for this article. First, after the first review, we included a reference to Zahle and Sørensen, 2008, who performed blade-resolved simulations of a single wind turbine, including a tower, in uniform inflow and sheared inflow. Blade resolved simulations are higher fidelity than both actuator discs and lines as used by De Cillis et al., and despite the lack of inflow turbulence Zahle and Sørensen conclude that the tower has limited influence ($1-2\%$), which we deem to be "minor influence". With increasing fidelity also comes increasing computational costs, and it is not feasible to perform LES large wind farm simulation with blade resolved turbines nor is it academic standard to include towers in LES of large wind farms. Second, the results in De Cillis et al., albeit interesting, are also based on simulations without inflow turbulence and a relatively low tip-speed ratio of 3. The tip-speed ratio and inflow turbulence have a significantly higher impact on the wake development, see, e.g., Equation (4.12) in [1]. The main impact of the tower is to introduce asymmetry (as also noted by Biswas and Buxton), which initializes the tip vortex instability and breakdown. Once the tip vortices have broken down, the wake transitions to small-scale turbulence and eventually far wake[1], where distinct tip vortices and tower wake will have disappeared. This is also clearly seen in the experimental results by Biswas and Buxton, see for instance Figure 4, where the Gaussian fit is very good for most distances, at least from $x/D > 1.5$, a clear indication that the wake has transitioned. In comparison, the present simulations are for turbine spacing of $6D$. Biswas and Buxton comment on how practical limitations result in experimental towers and nacelles with a relative size of 3 times larger than real-life turbines. Clearly, experimental campaigns can still give important insights to the physics, but the impact of oversized tower and nacelle will be overestimated (Biswas and Buxton, page 980). Despite this overestimation, Biswas and Buxton also clearly state that tip vortices dominate in the near-wake, not the tower wake. In highly turbulent conditions such as wakes, the breakdown outlined above will happen even faster and reduce the impact of tip vortices and towers. Third, the method, analysis and metrics are generic and does not depend on these numerical details, but could as well be applied on a dataset including towers and nacelle. To summarize, if the reviewer had provided specific references on the importance of modeling tower and nacelle in the highly turbulent conditions inside wind farms, then we would happily include it. But the suggested references do not sufficiently substantiate the importance to warrant a deeper discussion in the context of our article, which focus on the development and efficiency of reduced order models and how global POD can be used to analyze and compare various flow conditions, where we already refer to the interesting paper by De Cillis et al. in the context of using POD to provide physical interpretability.

- Figure 6: "all velocity components as the details of the LES are not reconstructed with only eight modes". I agree with that, but what is the point of showing a bad reconstruction that does not structurally reflects the real flow? The figure clearly shows that the error is of the same order of the velocity (unless I miss some normalization in the last two columns, it seems that the error reaches around 100% of the LES velocity value), This is indeed a proof that a global measure is not a sufficiently good way to ensure the level of convergence of the reconstructed flow. The authors should demonstrate that their POD basis is able to provide a fair reconstruction, built up using more modes, able to reproduce also the finer vortical structures of LES, otherwise the whole comparison among the used POD basis, is pointless.

First, let us remind the reviewer of their comment in their first round:
Finally, I think that the performance of the POD basis cannot be measured only using an integral quantity such as the velocity error, since even if the integral error is rather low, the flow field might have some important structural differences with the simulated one. The POD-reconstructed flow fields need to be shown and compared with the LES snapshots at given times, by showing a velocity error field for each velocity component.
We acknowledged this good idea and have therefore included Figure 6, which adds significant value to the manuscript. However, we fail to understand why the reviewer now thinks this is 'pointless' compared to the first review. To us, the point is that the figure clearly shows that the two reconstructions as well as the two errors are basically indistinguishable, i.e. the global POD basis provides a reconstruction, which is as good as the local POD basis. Therefore, the magnitude of the error is not important as we can always include more modes to reduce the error (see Figure 9). We have intentionally chosen to only include a few modes to emphasize how well the global basis performs, which in our opinion is a direct and fair comparison based on the initial suggestion by the reviewer. Furthermore, we note that the turbine itself acts as a spatial filter [2], so the finer vortical structures are less important for a reduced order model aimed at providing wind turbine load calculations. However, the higher modes will tend to capture more isotrophic scales, which both the local and global basis will capture. So even though, smaller vortices are less important for our application, we include the same figure for 8, 20, 50, and 100 modes in appendix and below for the reviewer. As seen, the performance of 1P and 9P is the same for all reconstructions, while the error decreases as more modes are included.

- Figure 7 and related discussions: as stated in the last comment, a global measure is not a sufficiently good way to ensure the level of convergence of the reconstructed flow. Thus, the authors should evaluate the convergence of the POD basis also using local measures, such as for instance the percentage of the maximum or rms error dU with respect to U_{mean}.

We believe we have substantiated this sufficiently in our previous answer, that the global POD basis provide spatial reconstructions, which are indistinguishable from the local POD basis, and therefore a global measure is appropriate. The article already discusses that different metrics could be used to evaluate and identify which dataset to add. As described in the article, our motivation for the chosen metric is that 'it is a direct measure of the variance in the original flow', where local POD is optimal in terms of capturing the variance. Again, the overall aim is to show that global POD bases are approximately as efficient as local POD bases, but come with additional advantages in terms of constructing reduced order models and providing physical insights to different flows.

References

[1] Jens Nørkær Sørensen, Robert Mikkelsen, Dan S. Henningson, Stefan Ivanell, Sasan Sarmast, and Søren Juhl Andersen. Simulation of wind turbine wakes using the actuator line technique. *Royal Society of London. Philosophical Transactions A. Mathematical, Physical and Engineering Sciences*, 373(2035):20140071–20140071, 2015.

[2] S. J. Andersen and J. P. Murcia Leon. Predictive and stochastic reduced-order modeling of wind turbine wake dynamics. *Wind Energy Science*, 7(5):2117–2133, 2022.

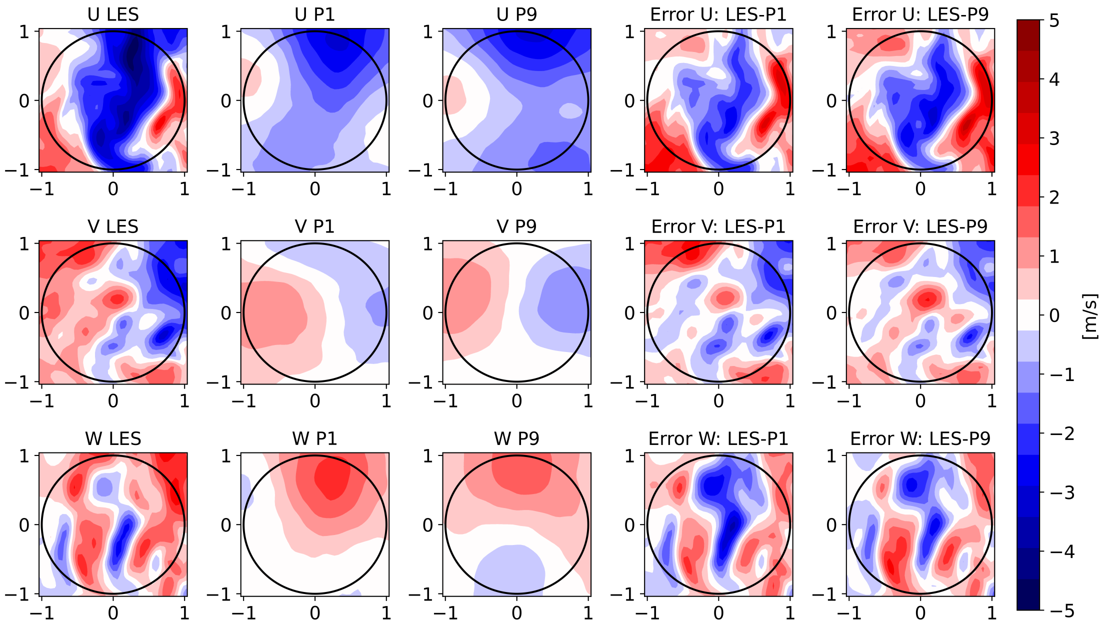

Figure 1: Figure 6 from the article, flow fields of LES and reconstruction using P1 and P9, using 8 modes.

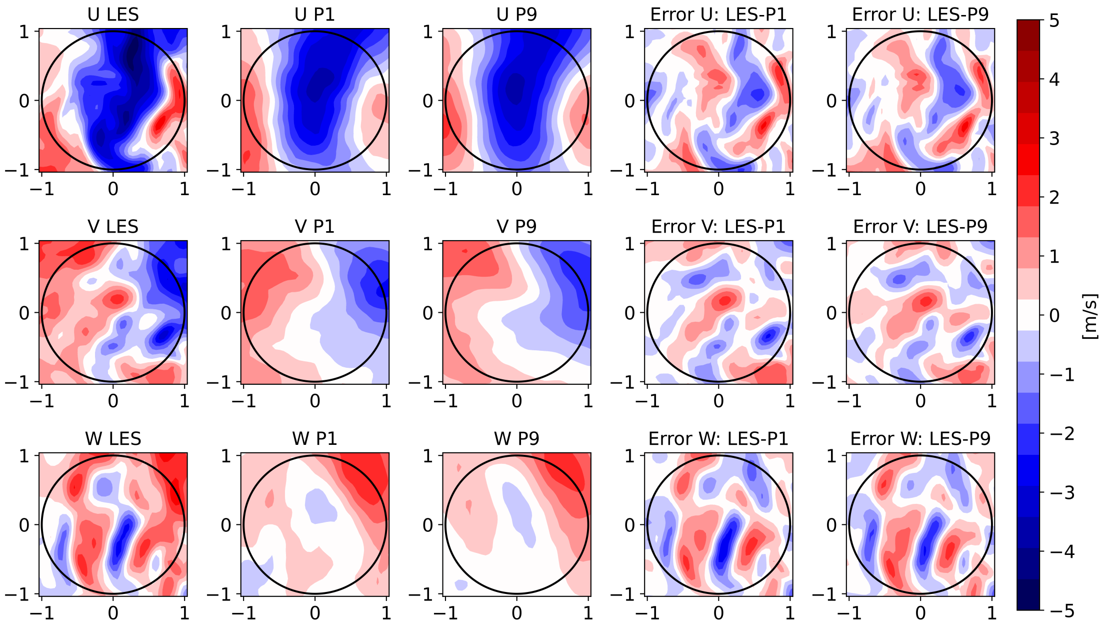

Figure 2: Figure 6 from the article, flow fields of LES and reconstruction using P1 and P9, using 20 modes.

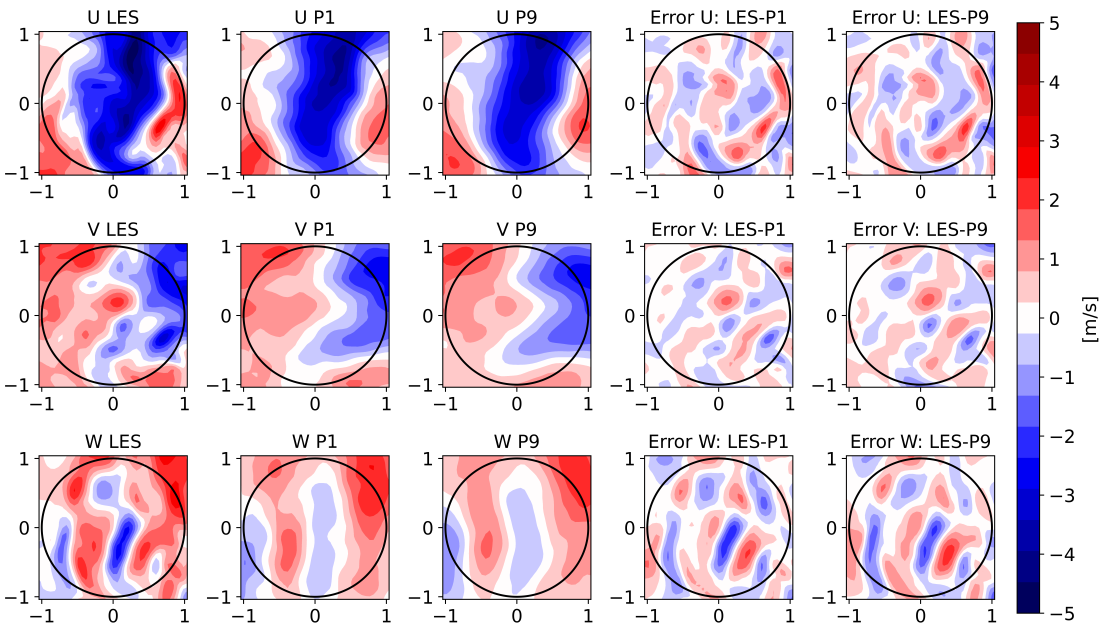

Figure 3: Figure 6 from the article, flow fields of LES and reconstruction using P1 and P9, using 50 modes.

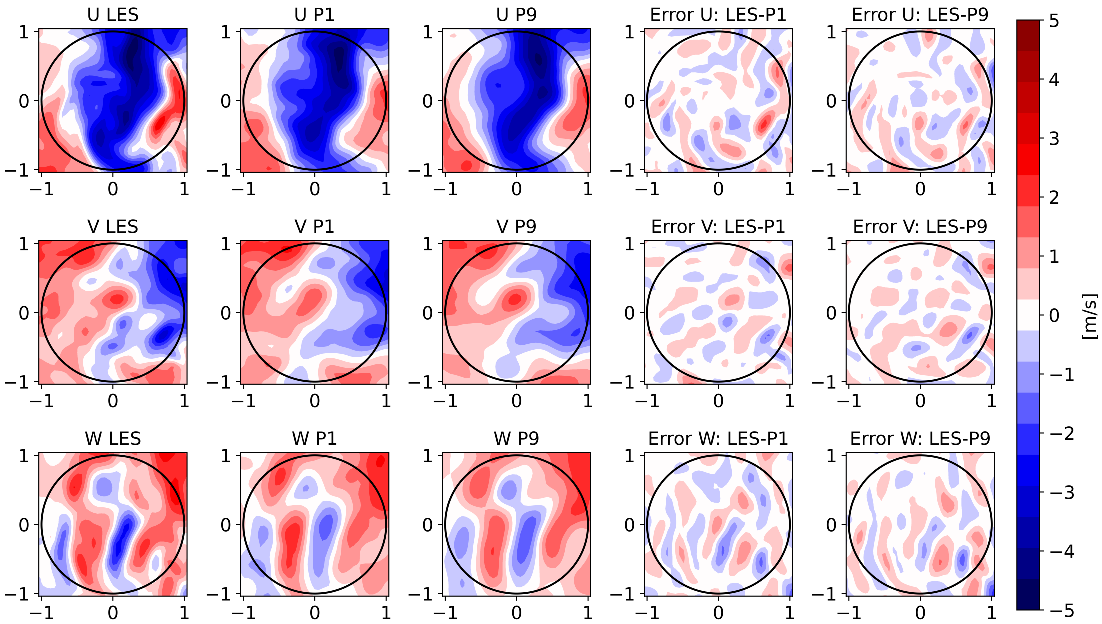

Figure 4: Figure 6 from the article, flow fields of LES and reconstruction using P1 and P9, using 100 modes.