Optimizing the Operation of Energy Islands with Predictive Nonlinear Programming - A case study based on the Princess Elisabeth Energy Island.

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Response to Reviews

This document contains the response to the paper's reviewers *Optimizing the Operation of Energy Islands with Predictive Nonlinear Programming - A case study based on the Princess Elisabeth Energy Island.* The authors thank the reviewers and editor for their valuable suggestions. Each comment, suggestion, or recommendation was analyzed in detail and answered in the context of the proposed paper. All changes are referenced in this response as a text box.

1 First reviewer

Authors' comment

We appreciate the time and effort of the reviewers in revising our contribution and providing us with valuable comments for possible improvement for its possible publication in this excellent journal.

Reviewer Comment 1.1

The article presents a power system modelling approach using non-linear programming applied to a hybrid AC/DC power system, with the Princess Elisabeth Energy Island used as a case study. Based on my review, I believe the manuscript requires major revisions, as outlined below:
Lack of Clear Novelty: The manuscript does not clearly communicate the novelty of the proposed approach. While a non-linear programming method is applied to model a hybrid AC/DC system, it remains unclear what differentiates this work from existing literature. The authors should explicitly highlight the unique contribution and innovation of their methodology.

Authors' response 1.1

We thank the reviewer for their valuable feedback regarding the clarity of the proposed approach's novelty. To address this concern, we have revised subsection 5.2 to explicitly highlight the unique contributions and advantages of our nonlinear programming framework for hybrid AC/DC energy islands. Specifically, we emphasize that the nonlinear power flow formulation for the optimal energy island operation was chosen to capture the full operational capabilities of the energy island, particularly in reactive power control. Unlike DC or linear power flow models, which neglect voltage magnitudes and reactive power flows, our approach enables precise reactive power dispatch from wind turbines and the battery energy storage system (BESS). This facilitates voltage regulation and achieves approximately a 1% reduction in power losses, as demonstrated in our case study. To illustrate this, we have added Figure 10, which shows the correlation between turbine-level reactive power injections and nodal voltages in the autumn scenario, underscoring the role of reactive power in maintaining voltage stability under high wind generation or power export conditions. This aspect is addressed in Subsection 5.2 of the manuscript, as follows:

> *5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island*
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> Figure 10 presents the nodal voltages and turbine-level reactive power injections for the representative autumn scenario, clearly evidencing the correlation between reactive power dispatch and voltage magnitudes. This relationship is fundamental in offshore AC networks, where voltage regulation plays a critical role in ensuring system stability—particularly during periods of high wind generation or significant power export. Capturing this interaction requires a modeling framework capable of representing both voltage magnitudes and reactive power flows, which is not possible in simplified DC power flow approximations where reactive power is neglected and voltages are assumed constant. To address this, the proposed nonlinear AC power flow formulation explicitly models and optimally dispatches reactive power from both the wind turbines and the Battery Energy Storage System (BESS), enabling coordinated voltage support and reduction of active power losses. In our case study, dispatching reactive power from the wind turbines and the BESS reduced total system losses by approximately 1% compared to an equivalent scenario without such reactive power support.

Furthermore, we highlight the computational efficiency of the proposed framework, leveraging the sparsity of the network admittance matrix to complete simulations for a representative autumn daily profile in approximately 27 seconds on a standard laptop (Intel Core i5-1235U, 16 GB RAM, Python 3.11). This demonstrates the framework's tractability for multiperiod and seasonal studies, offering a significant improvement over less accurate DC approximations. This is also included in the manuscript, as follows:

> *5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island*
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> Furthermore, the high sparsity of the network admittance matrix ensures competitive computational performance despite the increased modeling detail: for example, the autumn representative day simulation was completed in approximately 27 seconds on a standard laptop (Intel Core i5-1235U, 16 GB RAM, Python 3.11). This demonstrates that the AC-based nonlinear formulation not only delivers a more accurate and operationally meaningful representation of the energy island but also remains computationally tractable for extended multiperiod and seasonal analyses.

Additionally, to further clarify the novelty and contributions, we have updated the abstract and introduction to include an explicit contribution on uncertainty analysis. Using Monte Carlo simulations, our framework quantifies the economic impact of wind power and electricity price forecast errors, enhancing the robustness of operational planning for hybrid energy islands. These revisions collectively strengthen the presentation of our methodology's innovation and its distinct advantages over existing approaches.

Abstract

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To address these challenges, this work proposes a comprehensive framework for the optimal operation of hybrid AC/DC energy islands, addressing: (i) active and reactive power dispatch, incorporating BESS and hydrogen production; (ii) a detailed wind resource characterization based on one year of hourly data obtained using a realistic wind model with local measurements, including wake losses and turbine-level forecasts, used to define representative seasonal and spatial production patterns that inform typical operating conditions; (iii) operational optimization of a realistic test system based on the Princess Elisabeth energy island, and (iv) uncertainty analysis via Monte Carlo simulations, quantifying the impact of wind power and electricity price forecast errors, set up using commercial wind power planning tools and advanced forecasting software, and validated with Pyomo/Python.

1 Introduction

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Therefore, this paper proposes a comprehensive framework for the optimal operation of AC/DC energy islands using nonlinear programming, with four key contributions:

1. Development of a detailed optimization model integrating active and reactive power dispatch, incorporating battery energy storage system (BESS) management and hydrogen production, while leveraging the reactive power capabilities of wind power plants, BESS, and HVDC systems to minimize power losses and enhance voltage regulation.

2. Detailed wind resource characterization based on one year of hourly data generated using a realistic method with local measurements from the Federal Public Service Economy of Belgium (2024), including wake losses and turbine-level forecasts, to identify representative seasonal and spatial patterns that define typical daily operating conditions.

3. Application of the proposed framework to a realistic test system modeled after the Princess Elisabeth Energy Island, incorporating detailed turbine layout designs for offshore wind power plants.

4. Uncertainty analysis via Monte Carlo simulations to quantify the economic impact of wind power and electricity price forecast errors, enabling robust operational planning under uncertainty.

Reviewer Comment 1.2

Insufficient Model Validation: The model validation process is not adequately addressed. Using the Princess Elisabeth Energy Island as a case study alone does not constitute validation. The authors should compare their modelling results with real-world data or provide a sensitivity analysis to demonstrate the robustness and reliability of the model.

Authors' response 1.2

We thank the reviewer for their valuable feedback, which allows us to clarify the scope and validation process of our study. In response to the reviewer's request for sensitivity analysis, we have added a new subsection (Subsection 5.3, "Optimizing Curtailment Mitigation with BESS and Hydrogen Systems under Constrained Transmission Conditions") that evaluates the model's performance under a contingency scenario where High-Voltage Direct Current (HVDC) link capacities are reduced to 33% of their nominal value. This sensitivity analysis, illustrated in Fig. 16, demonstrates the robustness of the proposed nonlinear optimization framework by showing its ability to adapt dispatch strategies, battery energy storage system (BESS) operation, and hydrogen production scheduling to mitigate curtailments under severe transmission constraints.

> *5.3 Optimizing Curtailment Mitigation with BESS and Hydrogen Systems under Constrained Transmission Conditions*
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> This subsection presents a case study to evaluate the effectiveness of the proposed approach in mitigating curtailments under constrained transmission conditions. To simulate a contingency scenario, the capacity of all HVDC transmission interconnections is reduced to 33% of their nominal value, representing a severe limitation in power export capability, such as might occur during maintenance or unexpected outages. This case scenario analysis tests the robustness of the proposed nonlinear optimization framework, described in Section 3, by assessing its performance under atypical operating conditions. The optimization objective, focused on maximizing revenue from offshore wind generation, indirectly minimizes curtailments by prioritizing efficient resource allocation, including BESS charging/discharging and hydrogen production scheduling. The results demonstrate how the BESS stores excess energy during periods of high wind generation and constrained transmission, while the hydrogen production system absorbs surplus power to meet daily production targets, thereby reducing curtailments and enhancing economic performance.
> Fig. 16 illustrates the dispatched power profiles to the onshore grids of Belgium (BE), Great Britain (GB), and Denmark (DK), alongside the operational profiles of the BESS and hydrogen production systems for the representative day under constrained HVDC capacity. During hours 4–8, the optimization framework prioritizes power dispatch to GB, as shown in Fig. 16, driven by peak electricity prices in GB during this period, as observed in Fig. 13. This strategic allocation maximizes revenue by capitalizing on high market prices, aligning with the optimization objective.

From hour 8 onward, the dispatched power to Belgium reaches the maximum allowable capacity under the constrained HVDC limits, as depicted in Fig. 16. This preference for Belgium is primarily due to its proximity to the Princess Elisabeth Energy Island, which minimizes transmission losses compared to GB and DK. Additionally, electricity prices in Belgium during these periods are generally comparable to or higher than those in GB and DK, as shown in Fig. 13, further incentivizing exports to Belgium. In contrast, the dispatched power to GB exhibits lower peaks around hours 8 and 12, as seen in Figure 16, reflecting the influence of lower electricity prices in GB during these hours, consistent with Fig. 13.

Beyond hour 8, the dispatched power to all three countries frequently reaches the maximum constrained HVDC capacity, as observed in Fig. 16. This behavior is driven by a significant increase in available wind power from the wind power plants (WPPs), particularly WPP PE_I around hour 12 and WPP PE_III from hour 16 onward, as indicated by the wind profiles in Fig. 13. To mitigate curtailments under these conditions, the optimization framework prioritizes hydrogen production from hour 8, as shown in Fig. 16, where the electrolyzer power $P_e(t)$ increases to absorb surplus wind generation, ensuring that daily hydrogen production targets are met while minimizing unutilized renewable energy.

The BESS complements the hydrogen production system by dynamically adjusting its operation to balance the constrained transmission capacity and variable wind generation. As illustrated in Fig. 16, the BESS discharges during the early hours when wind power availability is low, supporting power exports to the onshore grids. During periods of high wind generation, particularly around hour 12 for wind power plant PE_I and from hour 16 for wind power plant PE_III, the BESS charges to store excess energy, as shown in Fig. 16, thereby preventing curtailments.

Regarding the reviewer's concern on the model validation, we emphasize that our methodology has been rigorously validated across multiple operating points and conditions. Additionally, as detailed in Subsection 5.2, we compared our results with the AC/DC power flow analysis software Pyflow, which serves as a benchmark to confirm the precision of our model. Furthermore, we highlight that the test system used is based on the real-world designated locations for the Princess Elisabeth Energy Island project. As stated in the paper, "Given that these wind farms are yet to be developed, this study defines and simulates their layout within the designated areas, employing commercially available tools from Youwind (2025)." Power turbine forecasts were estimated using hourly production data over a full year, incorporating simulations of wake losses in the wind parks and utilizing wind speed and direction time series from the Federal Public Service Economy of Belgium (2024).

Moreover, the technical characteristics of the power system elements, such as the capacities of the wind power plants and their interconnections with onshore grids, are based on project-specific information, as described in Section 4. We believe that the focused scope on the Princess Elisabeth Energy Island, combined with rigorous validation against established tools like Pyflow and project-specific data, provides a robust foundation for our findings. Finally, to address the reviewer's suggestion to enhance validation through additional case studies or sensitivity analyses, we have incorporated into the future work section an item to extend the analysis to other energy island projects, such as Denmark's Energy Islands and the Bornholm Energy Island, to further validate and generalize the methodology.

6 Conclusions

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Potential directions for future research may include:

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iii) Extending the analysis to other energy island projects, such as Denmark's Energy Islands and the Bornholm Energy Island, to verify the proposed methodology across different geographical and operational contexts, leveraging project-specific data and configurations to enhance the generalizability of the mode

Reviewer Comment 1.3

Manuscript Structure: The structure of the article needs refinement. In particular, all modelling outcomes should be clearly presented under a dedicated Results section, separate from other discussions or methodological content.

Authors' response 1.3

We appreciate the reviewer's valuable suggestion to refine the manuscript structure by clearly separating modeling outcomes into a dedicated Results section. To address this comment, we have restructured the paper to include two distinct sections:

- **Section 4: Energy Island Princess Elisabeth: Test System Description and Simulation Scenarios**, which includes the subsections "Test System Definition: Setup Based on the Princess Elisabeth Energy Island", "Hydrogen Production System: Electrolyzer Model Description", and "Measurement-Based Optimization for Estimating Green Hydrogen Production Models." These subsections outline the methodological framework and test system setup.
- **Section 5: Results and Analysis of the Princess Elisabeth Energy Island Operation**, which includes the subsections "Wind Power Profile Analysis of the Princess Elisabeth Energy Island", "Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island" and "Impact of Wind and Price Uncertainty using Monte Carlo Analysis." These subsections present all modeling outcomes and analytical results.

This reorganization ensures a clear distinction between methodological content and results, enhancing the manuscript's clarity and structure in line with the reviewer's recommendation.

Reviewer Comment 1.4

Simplistic Optimisation Objective The optimisation objective function used in the model is overly simplistic, focusing solely on maximising revenue from offshore wind generation. The authors should justify this choice and consider including additional parameters in the objective function such as operational costs, curtailment, and dispatch down to reflect a more realistic and holistic optimisation scenario.

Authors' response 1.4

We thank the reviewer for their comment regarding the need to justify the choice of the optimization objective function. To address this, we have revised subsection "3.5 Objective Function" to explicitly justify the selection of maximizing revenue from offshore wind generation. This objective is chosen to prioritize the economic performance of hybrid AC/DC energy islands while indirectly minimizing power losses through efficient resource utilization.

> *3.5 Objective Function*
> The objective function of the proposed framework is designed to maximize revenue from offshore wind power generation, reflecting the primary economic driver of hybrid AC/DC energy islands. This focus ensures optimal utilization of renewable energy resources while indirectly minimizing system power losses through efficient active and reactive power dispatch. By optimizing resource allocation within the nonlinear AC/DC power flow model, the framework achieves a reduction in power losses, as demonstrated in subsection 5.2. Additionally, the model accounts for curtailments through network constraints, with BESS and hydrogen production mitigating potential curtailments by storing or converting excess energy, as detailed in subsection 5.3. This approach enhances operational efficiency and economic performance while maintaining computational tractability for multiperiod and seasonal analyses. The mathematical expression for the revenue maximization objective is as follows:
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> $$\max\ z = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{C}} C_{i,t} P^{m}_{i,t}, \tag{1}$$
>
> where $C_{i,t}$ represents the power price at country i during period t, and $P^{m}_{i,t}$ denotes the active power delivered to country i in period t within the time window $\mathcal{T}$.

Additionally, by optimizing resource allocation within the nonlinear AC/DC power flow model, the framework achieves a reduction in power losses, as demonstrated in subsection 5.2:

> *5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island*
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> To address this, the proposed nonlinear AC power flow formulation explicitly models and optimally dispatches reactive power from both the wind turbines and the Battery Energy Storage System (BESS), enabling coordinated voltage support and reduction of active power losses. In our case study, dispatching reactive power from the wind turbines and the BESS reduced total system losses by approximately 1% compared to an equivalent scenario without such reactive power support.

Furthermore, curtailments are explicitly addressed within the proposed framework. The AC/DC network model incorporates constraints that account for curtailments, while the integration of BESS and hydrogen production mitigates these by storing or converting excess energy, thus enhancing operational flexibility.

Finally, to further address the reviewer's suggestion for a more holistic optimization scenario, we have added Subsection 5.3, "Optimizing Curtailment Mitigation with BESS and Hydrogen Systems under Constrained Transmission Conditions," as detailed in our response to Reviewer Comment

1.2. This scenario analysis, illustrated in Fig. 16, demonstrates how the proposed approach with the revenue-focused objective function effectively mitigates curtailments by optimizing BESS and hydrogen production operations under severe transmission constraints, thereby enhancing the framework's applicability to operational challenges. Regarding operational costs, based on the authors' experience, these are significantly lower than revenues in the operational phase of energy islands, making their inclusion in the objective function less critical for this context. However, we acknowledge their importance in planning stages, where cost considerations are more prominent.

Reviewer Comment 1.5

Weak Conclusions: The conclusion section lacks depth. It should be substantially revised to better reflect and interpret the key findings of the study, offering a more comprehensive summary and critical insights into the implications of the results.

We appreciate the reviewer's feedback on the need for a more comprehensive conclusions section. To address this, we have revised the conclusions to better summarize key findings, incorporating the impact of wind and price uncertainties on economic profits, the benefits of reactive power control for reducing system losses by approximately 1% and supporting voltage regulation. Additionally, we have added a new future work subsection proposing techno-economic assessments, stochastic optimization, and analysis of other energy island projects to enhance the methodology's applicability. These changes ensure a concise and insightful conclusions section aligned with the reviewer's recommendations, as detailed in the revised conclusions below:

6 Conclusions

This study presents a predictive nonlinear optimization framework for the operation of AC/DC energy islands, validated through a case study of the Princess Elisabeth Energy Island. The nonlinear power flow formulation facilitated reactive power dispatch from wind turbines and the battery energy storage system, reducing system losses by approximately 1% and enhancing voltage regulation. The linear model for green hydrogen production, derived through measurement-based optimization, achieved a mean modeling error below 1.5%, surpassing conventional constant-efficiency models. Seasonal analyses underscored the framework's adaptability, optimizing energy dispatch, battery management, and hydrogen production to maximize economic revenues while ensuring secure operation. Monte Carlo simulations evaluating wind power and electricity price uncertainties revealed that price uncertainty significantly impacts economic profits, increasing mean profits by 3.58% compared to the deterministic case, while wind uncertainty reduces profits by 2.20%, and combined uncertainty yields a modest 1.38% increase, highlighting the need for probabilistic assessments in operational planning. Realistic offshore wind conditions, modeled using the Youwind platform with wake effects, provided seasonally representative inputs. The proposed framework effectively adapts to seasonal variability, ensuring operational efficiency and economic performance while maintaining computational tractability for future hybrid AC/DC energy island applications.

Potential directions for future research may include:

i) Integrate detailed techno-economic assessments of subsystems into the planning framework for hybrid energy islands to evaluate their impact on overall system performance and cost-effectiveness.

ii) Apply stochastic optimization techniques to extend the proposed strategies, enabling robust planning of hybrid energy islands across both representative days and long-term horizons under uncertainty.

iii) Extending the analysis to other energy island projects, such as Denmark's Energy Islands and the Bornholm Energy Island, to validate the proposed methodology across different geographical and operational contexts, leveraging project-specific data and configurations to enhance the generalizability of the model.

2 Second reviewer

Reviewer Comment 2.1

The authors have created an interesting test system, and they have effectively written a "tutorial" paper on how to create your own test system, along with some simulation outputs to show that their model works. However, the research value of the paper is unclear. A non-linear optimization model is proposed and implemented, but it is not clear whether such an approach is well justified. For example, ac power flow is implemented, and voltage limits are monitored, but it is unclear if these are binding constraints, and hence whether a simpler dc power flow approach could have been implemented to produce "similar" costs, but with a reduced computational burden. (The presented results focus on "dispatch" variables rather than "network" variables.) It also seems that perfect forecasting of electricity prices and wind power is assumed. How do forecast errors impact the methodology and the results and conclusions? Representative days are convenient for showing that a model is working, but otherwise they have limited value, particularly when the BESS start and end state of charge is fixed, despite day to day variations in wind speed and electricity price. The electrolyzer produces and accumulates hydrogen across a representative day, but what happens to the hydrogen, and are there "downstream" constraints associated with the hydrogen production? The term "predictive" is used, but what does it mean? The authors seem to assume perfect knowledge of wind power and electricity price, so what does "predicted" refer to?

Authors' response 2.1

We thank the reviewer for their insightful feedback, which has prompted substantial revisions to the manuscript. Given the breadth of concerns raised in this comment, we address each point individually below, incorporating clarifications, justifications, and enhancements to strengthen the paper's contributions.

Reviewer's Concern 2.1.1

A non-linear optimization model is proposed and implemented, but it is not clear whether such an approach is well justified. For example, ac power flow is implemented, and voltage limits are monitored, but it is unclear if these are binding constraints, and hence whether a simpler dc power flow approach could have been implemented to produce "similar" costs, but with a reduced computational burden. (The presented results focus on "dispatch" variables rather than "network" variables.)

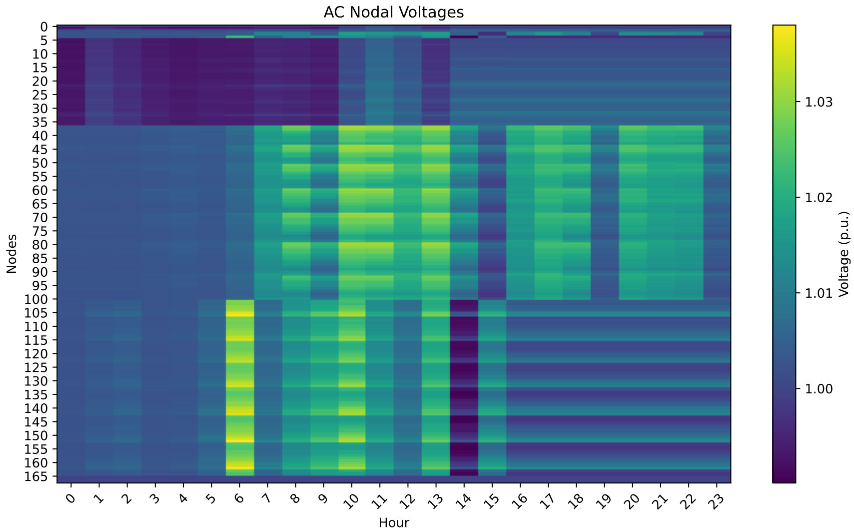
Authors' response 2.1.1

In response, we would like to clarify that the nonlinear formulation was purposefully selected to capture the full range of operational capabilities of the energy island concept, particularly with respect to reactive power control. Unlike DC power flow models, the AC formulation enables the explicit representation and dispatch of reactive power from both the wind turbines and the BESS (Battery Energy Storage System). This modeling choice is instrumental in leveraging voltage support functionalities and minimizing active power losses—capabilities that are especially relevant in offshore AC systems with medium to long collector distances.

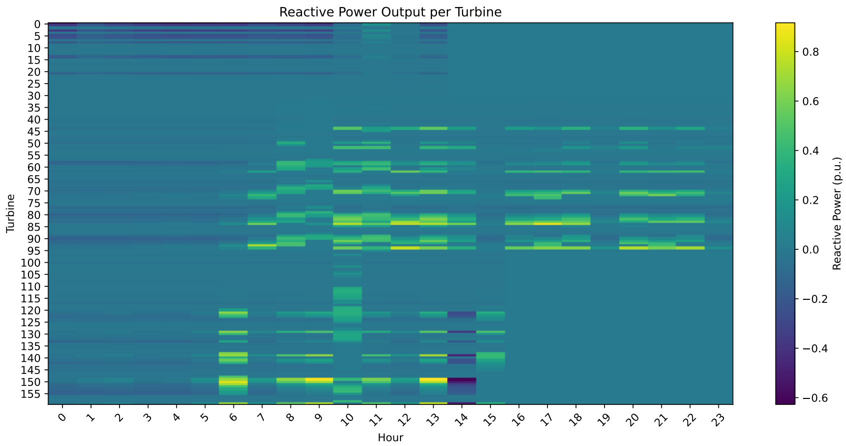
To further illustrate the impact of reactive power control, we have included Figure 1, which presents the nodal voltages and the corresponding reactive power injections of each wind turbine for the autumn scenario. The figure clearly shows the correlation between turbine-level reactive power dispatch and voltage magnitudes. This correlation highlights the role of reactive power in managing voltages within safe operational limits, especially during periods of high wind power generation or power export.

Additionally, Nonlinear power flow analysis enables reactive power dispatch to reduce system power losses, achieving approximately a 1% reduction in total generated energy in our case study. Unlike DC power flow approximations, which neglect voltage magnitudes, reactive power flows, and losses, this approach provides finer granularity. Thus, our nonlinear optimization framework offers a more accurate representation of the energy island's operations while supporting system-level benefits, such as reduced losses and improved voltage regulation.

Furthermore, the proposed nonlinear optimization framework takes advantage of the high sparsity of the network admittance matrix, which contributes significantly to computational efficiency. For instance, the simulations for the representative autumn daily profile were completed in approximately 27 seconds on a standard laptop equipped with an Intel Core i5-1235U processor and 16 GB of RAM, using Python 3.11. This demonstrates that the proposed AC-based formulation is computationally tractable, making it suitable for extended multiperiod and seasonal studies despite its increased modeling detail compared to DC approximations. This is included in the manuscript, as detailed in



(a) AC Nodal Voltages heatmap



(b) Reactive power per Turbine heatmap

Figure 1: Heatmaps of nodal voltage magnitudes and reactive power injection of wind turbines for the autumn scenario.

the following paragraph of Subsection 5.2:

> *5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island*
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> Fig. 10 presents the nodal voltages and turbine-level reactive power injections for the representative autumn scenario, clearly evidencing the correlation between reactive power dispatch and local voltage magnitudes. This relationship is fundamental in offshore AC networks, where voltage regulation plays a critical role in ensuring system stability—particularly during periods of high wind generation or significant power export. Capturing this interaction requires a modeling framework capable of representing both voltage magnitudes and reactive power flows, which is not possible in simplified DC power flow approximations where reactive power is neglected and voltages are assumed constant. To address this, the proposed nonlinear AC power flow formulation explicitly models and optimally dispatches reactive power from both the wind turbines and the Battery Energy Storage System (BESS), enabling coordinated voltage support and reduction of active power losses. In our case study, dispatching reactive power from the wind turbines and the BESS reduced total system losses by approximately 1% compared to an equivalent scenario without such reactive power support. Furthermore, the high sparsity of the network admittance matrix ensures competitive computational performance despite the increased modeling detail: for example, the autumn representative day simulation was completed in approximately 27 seconds on a standard laptop (Intel Core i5-1235U, 16 GB RAM, Python 3.11). This demonstrates that the AC-based nonlinear formulation not only delivers a more accurate and operationally meaningful representation of the energy island but also remains computationally tractable for extended multiperiod and seasonal analyses.

Reviewer's Concern 2.1.2

It also seems that perfect forecasting of electricity prices and wind power is assumed.

Authors' response 2.1.2

We thank the reviewer for this important observation. In response, additional simulations have been conducted to include the uncertainty associated with forecasting errors. A probabilistic power flow forecasting approach has been applied, and the Monte Carlo method was used to evaluate the impact of forecast uncertainty on system performance. This is explained in the following subsections:

3.6 Uncertainty Modelling in Energy Island Operation

Accurate forecasting of wind power generation and electricity market prices is critical for the optimal operation of AC/DC energy islands. However, forecast errors are inevitable due to the inherent variability of wind and the stochastic nature of electricity markets. Ignoring these uncertainties can lead to suboptimal decisions and potential economic losses. Therefore, it is essential to evaluate the robustness of the proposed optimization framework under realistic conditions that account for forecast inaccuracies.

To this end, a probabilistic power flow (PPF) analysis was performed to assess the impact of uncertainty on the system. A Monte Carlo simulation approach was applied, where multiple realizations of the uncertain input variables—namely, wind power generation and energy prices—were generated according to their probabilistic distributions and used as inputs to the nonlinear optimization model. This methodology enables a systematic evaluation of how variability in forecasts affects the operational performance and economic outcomes of the energy island. The detailed methodology and results of this analysis are presented in Subsection 5.4.

5.4 Impact of Wind and Price Uncertainty using a Monte Carlo Analysis

This subsection presents a probabilistic assessment of the energy island's operation under uncertainties in wind power and electricity prices, using Monte Carlo simulations to evaluate variability in key performance indicators, as detailed in Subsection 3.6. In this study, wind power generation and electricity market prices were modeled with Gaussian-distributed uncertainty, using the forecasted value as the mean and a standard deviation of 10%, consistent with the approaches in [RefA] and [RefB].

A Monte Carlo simulation approach was applied to evaluate the impact of these uncertainties on the operational performance of the energy island. Multiple realizations of the uncertain input variables were generated and used as inputs to the nonlinear optimization model. A total of 1,000 simulation runs were performed, and the statistical distribution of the resulting economic benefits was analyzed to quantify the impact of forecast uncertainty. Finally, a statistical analysis of the output values is performed, studying key indexes to evaluate and visualize the probability distribution of the resulting effect of the uncertainty of energy price and wind power forecasting on the operation of the energy island. Three scenarios were considered: (i) uncertainty in wind power forecasts, (ii) uncertainty in electricity price forecasts, and (iii) combined uncertainty in both wind power and prices. Table 4 summarizes the key statistical indicators of the economic profit obtained for the simulations of each scenario.

The following points highlight the primary findings from the Monte Carlo analysis of wind power and price uncertainty impacts on the energy island's economic performance:

- Wind power uncertainty alone has a minor effect on economic profits, as indicated by the very low coefficient of variation (0.14 %) and narrow percentile range. The mean and median are almost identical, highlighting the symmetry of the distribution. Relative to the deterministic case, this scenario reduces the mean profit by about 2.20%, with risk metrics like the Conditional Value at Risk (CVaR) showing a slightly larger drop of 2.48%.

- Price uncertainty introduces substantially higher variability (CV ≈1.86 %), with wider percentiles and a more pronounced impact on the Value at Risk. This indicates that economic performance is more sensitive to market price fluctuations than to wind forecast errors. Compared to deterministic, the mean increases by 3.58%, but the CVaR decreases slightly by 0.41%, suggesting potential downside risks in extreme cases.
- Combined uncertainty produces results dominated by price variability, with slightly higher dispersion than the wind-only scenario but slightly lower than price-only, reflecting interactions between both sources of uncertainty. Relative to deterministic, the mean rises by 1.38%, but risk metrics vary: p95 increases by 4.41% (upside potential), while CVaR drops by 2.34% (downside risk).
- In all scenarios, the deterministic profit (€7.26 M) lies within the interquartile ranges, but the VaR and CVaR metrics reveal that extreme realizations can significantly reduce profits. The percentage comparisons emphasize that wind uncertainty tends to bias profits downward, while price uncertainty can boost averages but introduces two-tailed risks. This highlights the importance of probabilistic assessment for robust operational planning.

Table 4: Statistical indicators of economic profit under different forecast uncertainty scenarios. All values are in millions of euros.

Scenario	Mean	Median	Std Dev	CV (%)	p5	p95	$VaR_{5\%}$	$CVaR_{5\%}$	Comparison vs Deterministic (%)
Wind Power	7.10	7.10	0.01	0.14	7.09	7.12	7.09	7.08	-2.20
Price	7.52	7.52	0.14	1.86	7.29	7.75	7.29	7.23	3.58
Combined	7.36	7.36	0.13	1.81	7.14	7.58	7.14	7.09	1.38

[RefA] Yang, Y., Wan, C., Yang, D., and Wang, J., Deep ensemble learning based probabilistic load forecasting in smart grids, *Energy*, Volume 189, 2019, 116324, ISSN 0360-5442, doi: `10.1016/j.energy.2019.116324`.
[RefB] Mingxu Xiang, Juan Yu, Zhifang Yang, Yan Yang, Hongxin Yu, He He, Probabilistic power flow with topology changes based on deep neural network, *International Journal of Electrical Power & Energy Systems*, Volume 117, 2020, 105650, ISSN 0142-0615, doi: `10.1016/j.ijepes.2019.105650`.

Reviewer's Concern 2.1.3
Representative days are convenient for showing that a model is working, but otherwise they have limited value, particularly when the BESS start and end state of charge is fixed, despite day to day variations in wind speed and electricity price.

Authors' response 2.1.3
We thank the reviewer for their comment regarding the use of representative days and the fixed state of charge (SoC) for the battery energy storage system (BESS). In our study, representative days were selected to capture typical intraday variations in wind power generation and electricity prices, rather than relying on averaged data that could obscure these dynamics. As detailed in Subsection 5.1 ("Wind Power Profile Analysis of the Princess Elisabeth Energy Island"), we selected

representative days for each season by identifying daily profiles with average total power closest to the seasonal median. This approach, illustrated in Figure 7, ensures that the profiles reflect realistic intraday generation behavior under seasonal wind conditions, providing robust input scenarios for the operational optimization of the hybrid energy island. The fixed SoC at the start and end of each representative day was chosen to maintain consistency across scenarios, enabling a clear evaluation of the proposed framework's performance under typical operating conditions.

Reviewer's Concern 2.1.4

The electrolyzer produces and accumulates hydrogen across a representative day, but what happens to the hydrogen, and are there "downstream" constraints associated with the hydrogen production?

Authors' response 2.1.4

Addressing the reviewer's insightful comment on the downstream use of hydrogen from the electrolyzer and its constraints, we clarify that our model does not include an explicit hydrogen market due to the current lack of a mature trading framework. However, the inclusion of hydrogen production is justified by its critical role in decarbonizing hard-to-abate sectors, such as steelmaking, ammonia production, and heavy transport, where its high energy density and versatility make it a vital energy carrier [International Energy Agency, 2021; European Commission, 2020]. To tractably represent this anticipated demand, we assume fixed daily hydrogen production, reflecting policy-driven or contractual obligations to supply industrial end-users.

To meet this daily hydrogen production and address downstream logistics, we assume hydrogen is transported offshore via marine carriers rather than pipelines, motivated by three factors:

(i) Geographical flexibility – Marine transport enables hydrogen delivery to multiple ports across Europe, aligning with the vision of a cross-border hydrogen economy, as outlined in the European Hydrogen Strategy [European Commission, 2020]. (ii) Operational flexibility – Decoupling production from delivery allows hydrogen production during periods of high wind generation, reducing curtailment and enhancing renewable integration. (iii) Infrastructure scalability – Ship-based transport avoids large upfront investments in dedicated pipelines, offering a modular solution for future hydrogen value chains.

This concern is addressed in the manuscript with the following paragraph added to Subsection 4.1:

4.1 Test System Definition: Setup Based on the Princess Elisabeth Energy Island

The system includes a green hydrogen production unit and a Battery Energy Storage System to utilize surplus wind energy for industrial decarbonization. Hydrogen is expected to play a critical role in decarbonizing hard-to-abate sectors, such as steelmaking, ammonia production, and heavy transport, due to its high energy density and versatility. Given the current absence of a mature hydrogen market, a fixed daily hydrogen production quota is assumed to represent anticipated demand, reflecting policy-driven or contractual obligations to supply industrial end-users. For downstream logistics, hydrogen is transported offshore via marine carriers rather than pipelines, motivated by: (i) geographical flexibility, enabling delivery to multiple European ports, aligning with the cross-border hydrogen economy vision, as highlighted by International Energy Agency (2021); European Commission (2020).; (ii) operational flexibility, allowing production during high wind generation periods to reduce curtailment; and (iii) infrastructure scalability, avoiding large upfront investments in dedicated pipelines while offering a modular solution for future hydrogen value chains.

References

International Energy Agency: Net Zero by 2050: A Roadmap for the Global Energy Sector, Report, International Energy Agency, `https://www.iea.org/reports/net-zero-by-2050`, 2021.

European Commission: A Hydrogen Strategy for a Climate-Neutral Europe, `https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52020DC0301`, 2020.

Reviewer's Concern 2.1.5

The term "predictive" is used, but what does it mean? The authors seem to assume perfect knowledge of wind power and electricity price, so what does "predicted" refer to?

Authors' response 2.1.5

We thank the reviewer for raising this important point regarding the use of the term "predictive." In our work, the term refers to the application of model predictive optimization, a methodology inspired by the principles of Model Predictive Control (MPC). Specifically, we solve a time-coupled optimization problem over a finite prediction horizon, using available forecasts of wind generation and electricity prices as inputs. The goal is to determine an optimal operational plan that accounts for the intertemporal dynamics of system components such as BESS and hydrogen production. The "predictive" nature lies in the use of time-varying forecasts to inform decisions across a rolling or fixed horizon—anticipating future conditions rather than optimizing in a myopic or static manner. This approach enables a more realistic and coordinated dispatch, especially in systems with energy storage and production constraints that unfold over time. We have clarified the meaning of "predictive" in this context and revised the manuscript accordingly.

Reviewer Comment 2.2

Remove the full stop at the end of the paper title

Authors' response 2.2

The full stop at the end of the paper title has been removed as requested.

Reviewer Comment 2.3
Line 20 – a long list of references is given, but no details are provided on the individual references.

Authors' response 2.3
We have incorporated a new paragraph that elaborates on the specific findings and relevance of each cited work. The added paragraph is as follows:

> *1. Introduction*
>
> ⋮
>
> ··· In particular, the study by Teng et al. (2019) proposes a coordinated optimization to improve flexibility within multi-energy systems by integrating hydrogen and energy storage systems, thereby reducing the curtailment of renewable energy in the power grid. The report by Williams and Zhao (2023) emphasizes the role of Power-to-X technologies in the energy transition, positioning energy islands as strategic locations for green hydrogen production while enhancing power grid flexibility through wind-to-hydrogen systems and hydrogen storage, thereby reducing curtailments imposed by grid restrictions. Likewise, Yang et al. (2023) explores a multi-energy system with storage and hydrogen supply, optimizing combined power and hydrogen delivery to enhance energy utilization and reduce curtailment. Finally, the works by Østergaard et al. (2023); Useche-Arteaga and et al. (2024); Lüth et al. (2024) discuss the pivotal role of energy islands in the future of power systems, highlighting their potential as offshore energy hubs that integrate renewable generation, hydrogen production, and energy storage to enhance system flexibility, address grid integration challenges, and support long-term energy transition goals.

Reviewer Comment 2.4
Line 220 – forcing the BESS start and end state to be the same is not optimal, noting, for example, wind variability from one day to the next

Authors' response 2.4
We thank the reviewer for the comment regarding the imposed constraint that forces the BESS to end the day with the same state of charge as at the start. This modeling choice was made intentionally to evaluate the system's performance over a single representative day, in a way that is independent of initial or final conditions from adjacent days. By enforcing equal initial and final BESS states, we avoid artificial advantages (e.g., starting the day fully charged) or disadvantages (e.g., starting from an empty state), allowing for a fair assessment of the system's capability under typical daily wind variability. Furthermore, this assumption facilitates the application of a rolling optimization framework, where each day can be optimized independently without requiring inter-day coupling of the BESS state. We have clarified this rationale in the manuscript accordingly.

Reviewer Comment 2.5
The paper only considers energy revenues. What about potential revenues from providing system services?

Authors' response 2.5

We clarify that the current study focuses on energy revenues to streamline the analysis of core operational aspects, such as active and reactive power dispatch, and optimal management of BESS and hydrogen production. While we recognize the potential of energy islands to provide system services, such as frequency regulation and voltage support, their rigorous inclusion would require a substantial extension of the framework, incorporating stability theory and converter control modeling. As this would significantly expand the scope and length of the current work, these aspects will be addressed in future research.

Reviewer Comment 2.6

The figures (2 of) spell electrolyzer incorrectly

Authors' response 2.6

We thank the reviewer for pointing this out. The spelling of "electrolyzer" has been corrected in Figures 1 and 5 accordingly.

Reviewer Comment 2.7

GB and UK terminology are both used – GB is correct

Authors' response 2.7

We appreciate the reviewer's observation. The terminology has been corrected throughout the manuscript to consistently use "GB" instead of "UK", including in Figures 4 and 5.

Reviewer Comment 2.8

Table 1 – are the values presented here publicly known, or have they been chosen by the authors? If the latter, how have the values been chosen, and do they lead to revenue maximization?

Authors' response 2.8

We thank the reviewer for the insightful comment. Technical details, including the specifications of array cables, transformers, and export cables, have been added in the Annex section for transparency and completeness. We also clarify that the majority of the values presented in Table 1 are based on real data from the official documentation of the project, which includes the location of the energy islands and wind power plants, the installed capacities of each wind farm, and their interconnections with the countries BE, DK, and UK. However, some subsystems such as energy storage and hydrogen production are not yet considered in the initial stages of the project. Therefore, the corresponding parameters were selected based on values commonly reported in the literature. Specifically, the battery storage parameters are referenced from peer-reviewed journal papers on energy storage systems. For the electrolyzer, the parameters were derived from the electrolysis model described in Section 4.2, using the parameter estimation methodology detailed in Section 4.3. This is explained in the following paragraph of the subsection 4.1:

4.1. Test System Definition: Setup Based on the Princess Elisabeth Energy Island

The Princess Elisabeth Energy Island is planned for construction by the Belgian Transmission System Operator (TSO) Elia in the Belgian sector of the North Sea, approximately 45 km offshore, as described in n Williams and Zhao (2023); Viaene et al. (2022); der Straeten (2022), and illustrated in Fig. 4. The energy island is designed to integrate three future offshore wind power plants: a 700 MW installation (PE-I), expected to be operational by 2028, and two additional wind power plants (PE-II and PE-III), each with a capacity of up to 1400 MW, planned for 2029, as shown in Fig. 5a.
The infrastructure will incorporate both AC and DC technologies, where AC cables will be used for wind farm collection, while high-voltage direct current (HVDC) connections will facilitate interconnections. The HVDC links with Denmark and the GB are scheduled for commissioning in 2030. Given that these wind farms are yet to be developed, this study defines and simulates their layout within the designated areas, employing commercially available tools from Youwind (2025). Assuming full capacity utilization, the turbine layout is designed using the IEA-22 MW reference wind turbine defined in Zahle et al. (2024), applying a staggered grid arrangement with optimized row orientation and spacing to minimize wake losses, as shown in Fig. 5a. Wind Power Plant PE-I contains 32 turbines, while Wind Power Plants PE-II and PE-III contain 64 turbines.
To support the analysis, Fig. 5b presents the single-line diagram of the test system, conceptually based on the Princess Elisabeth Energy Island. The diagram illustrates the topological structure, including offshore wind farms, internal AC collection systems, HVDC converters, storage options, and export transmission links to multiple regions. This configuration serves as the foundation for simulation scenarios used to validate the proposed optimization framework. The main technical parameters are summarized in Table 1, including the parameters of the BESS, which are based on the study by Pozo (2022), and the parameters of the green hydrogen system, calculated as explained in Subsections 4.2 and 4.3.

Additionally, the collector system and the array cables were designed using the Youwind platform, a professional tool specialized in offshore wind energy system design. To ensure clarity and comprehensiveness, detailed technical specifications have been included in the Annex section. Finally, we would like to emphasize that the main focus of this paper is the operation of energy islands. Future work will address the planning and optimal sizing of the energy island subsystems, as mentioned in the section of conclusions.

6. Conclusions

⋮

Potential directions for future research may include:

i) Integrate detailed techno-economic assessments of subsystems into the planning framework for hybrid energy islands to evaluate their impact on overall system performance and cost-effectiveness.

ii) Apply stochastic optimization techniques to extend the proposed strategies, enabling robust planning of hybrid energy islands across both representative days and long-term horizons under uncertainty.

iii) Extending the analysis to other energy island projects, such as Denmark's Energy Islands and the Bornholm Energy Island, to validate the proposed methodology across different geographical and operational contexts, leveraging project-specific data and configurations to enhance the generalizability of the model.

⋮

Reviewer Comment 2.9

Line 299 – the word "degradation" normally relates to a reduction in performance over the lifetime of a component, but here it looks as if the term is being used in relation to a change in electrolyzer output. The authors' definition is unexpected.

Authors' response 2.9

Thank you for your comment. We agree that the term "degradation" is typically associated with long-term deterioration in component performance and could be misleading to the reader in this context. To avoid any confusion, we have revised the text to eliminate the use of the term "degradation", as follows:

4.3 Measurement-Based Optimization for Estimating Green Hydrogen Production Models

A significant challenge in modeling hydrogen production systems is accurately estimating the parameters of the model presented in the subsection 3.4. This subsection presents a methodology for estimating the parameters of the green hydrogen production model through an optimization approach. Accordingly, the following optimization problem is proposed:

⋮

Reviewer Comment 2.10

Line 299 – in relation to Figure 6b, it would be helpful for the reader to understand the most likely electrolyzer output, and how that might vary across the different seasons, in order to better appreciate whether a "larger" error at low outputs is of minor or major significance

Authors' response 2.10

We sincerely thank the reviewer for this insightful comment. As shown in Figure 6b, the largest discrepancies in hydrogen estimation occur when the electrolyzer operates at low power levels. However, as illustrated in Figure 15, the hydrogen system operates most of the time close to its nominal capacity, which mitigates the practical impact of estimation errors in the hydrogen parameters. This clarification has been incorporated into the manuscript through the following paragraph:

> *5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island*
>
> ⋮
>
> As observed in Fig. 6b, the largest discrepancies in hydrogen production estimation occur when the electrolyzer operates at low power levels. However, as shown in Fig. 15 in subsection 4.5, the hydrogen production system operates most of the time close to its nominal capacity across different seasons. Consequently, the practical impact of estimation errors at low power levels is limited, as the system predominantly operates in levels where the linear model demonstrates high accuracy.

Reviewer Comment 2.11

The test results focus on seasonal "normal" days, but "less normal" days are also important, and may well influence equipment sizing.

Authors' response 2.11

We sincerely thank the reviewer for their valuable feedback regarding the focus on seasonal "normal" days in the test results. To address this comment, a new subsection, the Subsection 5.3, 'Enhancing Curtailment Mitigation through BESS and Hydrogen Systems under Limited Transmission Scenarios', has been incorporated to assess the model's effectiveness in a contingency case involving constrained transmission conditions where High-Voltage Direct Current (HVDC) link capacities are reduced to 33% of their nominal value.

Additionally, we clarify that the selection of representative days for each season was deliberate to illustrate how the operation of the energy island varies throughout the year. This approach allows us to demonstrate the adaptability of our proposed methodology to different seasonal conditions, while also providing readers with clear insights into the expected operational performance for each season. Finally, we have included a note in the Conclusions section to highlight the importance of analyzing "less normal" days for equipment sizing as a direction for future work. This addition underscores our commitment to exploring these scenarios in subsequent studies to further enhance the robustness of the energy island design.

6. Conclusions

⋮

Potential directions for future research may include:

i) Integrate detailed techno-economic assessments of subsystems into the planning framework for hybrid energy islands to evaluate their impact on overall system performance and cost-effectiveness.

ii) Apply stochastic optimization techniques to extend the proposed strategies, enabling robust planning of hybrid energy islands across both representative days and long-term horizons under uncertainty.

iii) Extending the analysis to other energy island projects, such as Denmark's Energy Islands and the Bornholm Energy Island, to validate the proposed methodology across different geographical and operational contexts, leveraging project-specific data and configurations to enhance the generalizability of the model.

Reviewer Comment 2.12

Page 16 – lots of details are given on modelling individual components, and basic details of the test data, but very little information is given on the nonlinear optimization methodology.

Authors' response 2.12

We thank the reviewer for highlighting the need for more details on the nonlinear optimization methodology. To address this, we have revised Sections 3 and 5 to describe the methodology and implementation using Pyomo and the IPOPT solver, which employs a primal-dual interior-point algorithm with a filter line-search method. Key references have been added to guide further reading. The updated paragraphs are as follows:

3. Mathematical formulation for the optimal operation of AC/DC energy islands

⋮

This section describes the mathematical programming models for the subsystems of the AC/DC energy island, details the objective function, and presents the complete optimization problem governing its operation. Note that the primary sources of nonlinearity in our model arise from the power flow equations for both AC and DC systems, where the state variables are the voltages. In the AC system, voltages are represented in polar form as $V = \nu\angle\theta$, where ν denotes the voltage magnitude and θ represents the voltage angle. In the DC system, node voltages are denoted by u. Specifically, the mainly nonlinearities stem from the trigonometric expressions in the AC power flow equations and the model of the HVDC converter station. Furthermore, quadratic terms in the AC and DC power flow equations introduce additional nonlinearities.

5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island

⋮

In this subsection, we evaluate the performance of the optimal operation strategies proposed for the Princess Elisabeth Energy Island, focusing on representative days that capture typical seasonal variations in wind power availability and electricity prices. The nonlinear mathematical programming model, developed in Section 3, are implemented using the Python-based optimization modeling library Pyomo, as described by Bynum et al. (2021) , and solved with the IPOPT solver, developed by Wächter and Biegler (2006), which employs a primal-dual interior-point algorithm with a filter line-search method to efficiently handle the nonlinearities inherent in the system.

Reviewer Comment 2.13

How does pyflow-acdc work, and what are the key differences with the authors' approach? The paper doesn't provide sufficient information to judge the comparison, and pyflow-acdc results are not shown in the paper.

Authors' response 2.13

pyflow-acdc is an open-source Python library designed for power flow analysis and optimal power flow (OPF) studies in hybrid AC/DC networks. However, pyflow-acdc does not support the modeling of energy storage systems or green hydrogen production systems, which are key elements in the operation of energy islands.

In this work, pyflow-acdc was used to validate the accuracy of the power flow and dispatch results obtained from our proposed nonlinear optimization framework in nominal conditions without considering BESS and green hydrogen production. For this reason, the results from pyflow-acdc were not shown in the paper, as they serve only as an internal verification tool for the electrical consistency of our model, and not for the full operational optimization problem addressed in this work.

5.2 Energy Management and Optimization for Representative Days on the Princess Elisabeth Energy Island

⋮

However, before proceeding with the multiperiod and seasonal analyses, a preliminary validation of the proposed optimization approach was conducted to ensure its reliability under nominal operating conditions. Specifically, a cross-check was performed using the Python-based power flow tool pyflow_acdc, developed by Valerio et al. (2025). pyflow-acdc is an open-source library designed for power flow and optimal power flow (OPF) studies in hybrid AC/DC networks. However, it does not support the modeling of energy storage systems or green hydrogen production units, which are essential for the comprehensive operation of energy islands. Therefore, the validation was limited to a simplified case excluding BESS and hydrogen production. The comparison shows that both approaches yield very similar results, with the proposed method achieving a 1.5% improvement in the objective function value compared to pyflow-acdc, thereby confirming the accuracy and robustness of the proposed optimization framework.

Reviewer Comment 2.14

Line 350 – it seems that the optimization process assumes perfect (day ahead) knowledge of wind power daily profiles and the day ahead electricity prices, but the validity of this assumption is not justified.

Authors' response 2.14

We thank the reviewer for this important observation. This point is addressed in our response to the first comment from Reviewer 2. In particular, additional simulations have been conducted to incorporate the uncertainty associated with forecasting errors in wind power daily profiles and day-ahead electricity prices. A probabilistic power flow forecasting approach has been applied, and the Monte Carlo method has been used to evaluate the impact of forecast uncertainty on system performance.

Reviewer Comment 2.15

A variety of x-axis scales are used to show a 24 hour day – use one style for all figures, either the style from Fig. 10 or from Fig. 11.

Authors' response 2.15

We sincerely thank the reviewer for their insightful comment regarding the inconsistent x-axis scales used to represent a 24-hour day across the figures. To address this concern, we have revised Figures 8, 9, and 10 to adopt a consistent x-axis style, aligning them with the format used in Figure 10, as suggested.

Reviewer Comment 2.16
Reference is made to hydrogen targets. What are these, and are they an optimized variable?

Authors' response 2.16
We thank the reviewer for their query regarding the hydrogen targets and their role in the optimization framework. The term "hydrogen targets" refers to a fixed daily hydrogen production, which represents the anticipated demand for green hydrogen to support industrial decarbonization. This is addressed in the response to the first comment from Reviewer 2 and explained in Subsection 4.1 of the manuscript. In this context, the optimized variables are the demanded power of the electrolyzer $P^e_{i,t}$, the hydrogen produced $h_{i,t}$ and the cumulative hydrogen M_{i,t_i}