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Comment on "Glauert's optimum rotor disk revisited – a calculus of variations solution and exact integrals for thrust and bending moment coefficients" by Tyagi and Schmitz (2025)

~~by~~
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The paper by Tyagi and Schmitz (2025a) presents a reformulation of the classical momentum theory optimum rotor problem using the calculus of variations and closed-form integrations for thrust and bending moment coefficients. This ~~paper (Tyagi & Schmitz, 2025a) revisits the~~comment examines these derivations within the physical and mathematical limits of the one-dimensional momentum ~~theory problem for a windmill (i.e., a wind turbine) with apparent~~

Several of the mathematical ~~rigor and pedagogical eloquence. While it ultimately offers a seemingly mathematically rigorous reformulation, it contributes limited new insight to the wind energy field. Recasting Glauert's "optimum rotor" formulation using the calculus of variations and~~developments consist of algebraic manipulations of Glauert's induction relations. These include polynomial expansions, rational simplifications, and repeated applications of L'Hôpital's ~~rule may suggest an air of analytical sophistication. Yet, it is little more than a formal re-derivation that does~~rule. These manipulations change the algebraic form of the standard relations but do not ~~clarify~~modify the structure of the underlying ~~physical principles for engineering use. The authors introduce no new physical interpretation, favoring abstract derivations over engineering context. Although mathematically elaborate, their derivation of "exact" integrals for thrust and bending moment coefficients has limited practical relevance and does not materially advance rotor theory or wind turbine engineering.~~momentum equations.

The ~~authors first examine the condition where the tip speed ratio~~paper begins by examining the behavior of the model equations in the limit $\lambda \to \infty$~~, and then find the power coefficient~~and evaluates the resulting expressions for C_P

In the low tip speed ratio limit ($\lambda \to 0$), the authors next derive exact integrals for C_P, C_T, and C_{Be} ~~using repeated applications of L'Hôpital's rule~~under that limiting assumption. In this limit, the Glauert relations give $a' \to 0$. Because torque is proportional to ~~resolve singularities. While these derivations may be formally correct, the physical context~~a', the torque also approaches zero. The mechanical power coefficient C_P is ~~again fundamentally flawed. In~~proportional to torque multiplied by the rotational speed, so C_P approaches zero when a' approaches zero within the axial plus swirl momentum model. Any expression predicting nonzero power in this ~~regime, the turbine~~limit is ~~either stationary, operating~~inconsistent with ~~blade stall, or in a transient startup phase,~~the angular momentum balance on which the model is based. This inconsistency arises from evaluating the model outside the conditions ~~under~~for which ~~no meaningful power is extracted, and the aerodynamic loads~~its assumptions are ~~dominated by separated and unsteady flow. In effect, the turbine behaves more as a high-drag device~~valid, rather than ~~an energy-extracting turbine; it operates in the turbulent wake state, which means that it obstructs the flow rather than extracting energy~~ from ~~it, thereby violating key assumptions of the simple one-dimensional flow model they have used. Under these conditions, the flow field is highly three-dimensional, non-uniform, and often dominated by large blade section angles of attack, rendering the momentum theory assumptions of steady, axisymmetric, and uniform flow with a constant pressure jump entirely inapplicable. Moreover, from a practical standpoint, modern turbines are designed to avoid operation in this regime altogether, typically idling or feathering at low wind speeds. The analytical effort invested by the authors in characterizing this physically irrelevant limit offers no guidance for turbine design, control strategy, or performance optimization.~~the algebraic manipulations themselves.

~~Using the calculus of variations to rederive Glauert's third-order polynomial equation offers a modest pedagogical novelty.~~ The paper next derives limiting expressions for C_P, C_T, and C_{Be} ~~may be formally "new"~~ in ~~closed form. Still, they~~the limit $\lambda \to 0$ by repeated differentiation of the numerator and denominator terms. These limits follow from the mathematical continuation of the steady one-dimensional momentum equations. When λ is small, the flow around the rotor is typically separated, unsteady, and non-axisymmetric, and the assumptions of the actuator disk model, namely steady one-dimensional uniform flow with a constant pressure jump, are ~~derived~~not satisfied. The limiting values obtained from the continued equations, therefore, describe the algebraic behavior of the continued model but do not represent the behavior of a rotor operating under ~~conditions so idealized as to be irrelevant to any form of practical wind turbine engineering. Furthermore, the paper fails even to acknowledge the real possibility of tip losses, finite blade count, profile drag, wake expansion, or non-uniform and yawed inflow. These are factors of higher priority in wind engineering practice. Indeed, in Glauert's original theory, the limiting behavior has not generally been regarded as a theoretical or practical limitation in wind energy research. Their so-called "math problem," therefore, is only one of their invention.~~such conditions.

~~Additional issues arise~~The calculus of variations derivation in the ~~presentation of their results. Several equations (e.g., Eqs. 35, 48, and 50) include numerical constants such as 2.5457 and −13.3272 without explanation or derivation. While not strictly erroneous,~~paper reproduces Glauert's optimum loading condition. The resulting third-order relation is identical to that obtained by extremizing the power coefficient subject to the one-dimensional momentum constraints. This equivalence follows directly from the structure of the classical actuator disk formulation. Because this ~~practice reduces transparency and reproducibility. Providing explanations or citing how these constants were computed would have improved the clarity and independent reproducibility of their results. Without clearly stated methods or symbolic groundwork, the derivations appear procedural rather than intellectually motivated, making it more difficult to assess and reproduce the results independently. Indeed, despite the apparent technical competence of the derivations, their work remains disconnected from the practical challenges and standards of modern wind turbine research. There is~~formulation assumes an infinite number of blades, no ~~comparison to empirical data, computational results, or reconciliation with other findings. Their model assumes a wind turbine with an infinite number of blades that have a continuously optimal span loading at any λ. This is a mathematical abstraction that has~~tip losses, no ~~place in any realistic wind turbine analysis, particularly in the high- and low-λ regimes that the authors specifically emphasize.~~drag, and steady uniform inflow, the resulting expressions apply only within those idealized assumptions.

~~While their mathematical work appears internally consistent within the constraints of an idealized model, the model's assumptions physically break down~~The numerical constants appearing in ~~precisely those regions where their work attempts~~several equations, such as 2.5457 and −13.3272 in Eqs. 35, 48, and 50 arise from evaluating the closed-form antiderivatives at the lower limit $x = 1 - 3a$ with $a = 1/4$. Substituting $x = 1/4$ into the polynomial and logarithmic terms yields the reported constants. Providing intermediate symbolic steps would have allowed readers to ~~provide insight. The authors do not extend Glauert's theory in a way that adds engineering value or new physical understanding. While the work may be of limited academic interest to those studying the historical development of rotor theory, it certainly falls short of the novelty, applicability, and physical relevance expected of contributions to *Wind Energy Science*. Instead, by clinging to an idealized and largely irrelevant theoretical framework, the authors ultimately diverge from the practical context of Glauert's original contributions. Their work most certainly does not "unlock new possibilities in wind turbine design that Hermann Glauert did not consider" (Sliman, 2025). Therefore, their framing is not aligned with established physical understanding and practical engineering considerations.~~reproduce these values directly.

~~Finally, it~~

The high and low λ regimes considered in the paper correspond to conditions where at least one assumption of the one-dimensional momentum model does not hold. At high λ, the swirl component of the momentum model gives zero torque, so the model predicts zero power. At low λ, the flow is ~~important to note that the central~~

There is substantial overlap between the derivations, ~~coefficient~~polynomial expressions, and ~~conclusions~~

~~of~~integration procedures in this ~~article closely match~~paper and those in a previously published and publicly accessible conference paper by the same authors~~ in a publicly accessible conference paper~~ (Tyagi ~~&~~and Schmitz, 2025b). ~~That~~The earlier ~~published~~paper contains many of the same mathematical developments. Citing this prior work ~~contains essentially the same analytical development, including the polynomial forms, integration techniques, convergence analysis, and variational formulation. This 2025 *Wind Energy Science* article does not cite or acknowledge the prior conference version, which is a notable omission that warrants editorial attention. Of greater concern is that both papers contain substantial verbatim reuse of text, structure, and phrasing. The core mathematical derivations are essentially the same, and the conclusions are restated with only minor editorial variation. This unacknowledged repetition represents substantial overlap with a prior publication; clarification of~~would clarify the relationship between the two ~~works and appropriate citation would have been helpful. While this journal article appears more refined on the surface, the failure to cite a substantially overlapping publication, particularly one with broad public accessibility, suggests a need for clarification regarding attribution and transparency per journal policy.~~publications and help readers follow the development of the results.

Tyagi, D., and Schmitz, S., "Glauert's optimum rotor disk revisited – a calculus of variations solution and exact integrals for thrust and bending moment coefficients," Wind Energ. Sci., 10, pp. 451–460, available at https://doi.org/10.5194/wes-10-451-2025, 2025.

Kevin Sliman, "Student refines 100-year-old math problem, expanding wind energy possibilities, Penn State News, Feb. 21, 2025, available at https://news.engr.psu.edu/2025/schmitz-sven-wind-energy-math-problem.aspx

Tyagi, D., and Schmitz, S., "Amendment to Glauert's Optimum Rotor Disk Solution," AIAA Conference Publication, DOI: 10.2514/6.2024-84552, available at https://arc.aiaa.org/doi/pdf/10.2514/6.2024-84552.