Comment on"Glauert's optimum rotor disk revisited – a calculus of variations solution and exact integrals for thrust and bending moment coefficients" by Tyagi and Schmitz (2025)

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The paper by Tyagi and Schmitz (2025a) presents a reformulation of the classical momentum theory optimum rotor problem using the calculus of variations and closed-form integrations to compute thrust and bending-moment coefficients. This comment examines these derivations within the physical and mathematical limits of the one-dimensional momentum model. Several of the mathematical developments consist of algebraic manipulations of Glauert's induction relations. These include polynomial expansions, rational simplifications, and repeated applications of L'Hôpital's rule. These manipulations change the algebraic form of the standard relations but do not modify the structure of the underlying momentum equations.

The paper begins by examining the behavior of the model equations in the limit $\lambda \to \infty$ and evaluates the resulting expressions for C_P and C_{Be} under that assumption. In this limit, the Glauert relations give $a' \to 0$, so that the circumferential induction and the associated aerodynamic torque vanish. The generalized momentum formulation therefore reduces to the axial momentum theory, in which power extraction arises from the streamwise pressure jump across the actuator disk rather than from angular-momentum transfer. Any interpretation of swirl-related terms as contributing finite power in this limit would be inconsistent with the angular-momentum balance and should be regarded as a mathematical artifact of the algebraically continued equations. The issue lies not in the algebraic manipulations themselves, but in the interpretation of swirl-based expressions outside the model's range of physical applicability.

The paper next derives limiting expressions for C_P, C_T, and C_{Be} in the limit $\lambda \to 0$ by repeated differentiation of the numerator and denominator terms. These limits follow from the mathematical continuation of the steady one-dimensional momentum equations. When λ is small, the flow around the rotor is typically separated, unsteady, and non-axisymmetric, and the assumptions of the actuator disk model, namely steady one-dimensional uniform flow with a constant pressure jump, are not satisfied. The limiting values obtained from the continued equations, therefore, describe the algebraic behavior of the continued model equations but not the behavior of a rotor operating under such conditions.

The calculus of variations derivation in the paper reproduces Glauert's optimum loading condition. The resulting third-order relation is identical to that obtained by extremizing the power coefficient subject to the one-dimensional momentum constraints. This equivalence follows directly from the structure of the classical actuator disk formulation. Because this formulation assumes an infinite number of blades, no tip losses, no drag, and steady uniform inflow, the resulting expressions apply only within those idealized assumptions. As such, the results are only of theoretical interest under the classical actuator-disk assumptions

The numerical constants appearing in several equations, such as 2.5457 and -13.3272 in Eqs. 35, 48, and 50 arise from evaluating the closed-form antiderivatives at the lower limit $x = 1 - 3a$ with $a = 1/4$. Substituting $x = 1/4$ into the polynomial and logarithmic terms yields the reported constants. Providing intermediate symbolic steps would have allowed readers to reproduce these values directly. It is also noted that the limiting values of the coefficients follow directly from algebraic manipulation of the governing relations and can equivalently be obtained without reliance on L'Hôpital's rule. The apparent indeterminate forms arise from the particular parameterization chosen for the integrals, rather than from any intrinsic singular behavior of the momentum equations. Therefore, the resulting limits are algebraic consequences of the continued equations, independent of the specific limiting procedure employed.

Several sections of the paper contain extended symbolic manipulations, including multi-step rational simplifications and repeated differential reductions. Clarification of the computational tools used, if any, would assist reproducibility of the intermediate steps and verification of the closed-form expressions. This observation concerns only reproducibility and does not bear on the physical interpretation of the results.

The high and low λ regimes considered in the paper correspond to conditions where at least one assumption of the one-dimensional momentum model does not hold. At high λ, the swirl-related contribution to torque vanishes, and the generalized momentum formulation asymptotically reduces to the axial momentum limit. At low λ, the flow is unsteady and separated and does not satisfy the assumptions of a uniform steady actuator disk. In these regimes, expressions obtained from algebraic continuation of the model equations should be interpreted strictly as mathematical consequences of the continued equations and not as predictions of rotor performance under physical conditions outside the validity of the model.

There is substantial overlap between the derivations, polynomial expressions, and integration procedures in this paper and those in a previously published and publicly accessible conference paper by the same authors (Tyagi and Schmitz, 2025b). The earlier paper contains many of the same mathematical developments. Citing this prior work would clarify the relationship between the two publications and help readers follow the development of the results.

References

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