Author Response to Reviewer Comments (v2)

Manuscript: wes-2025-137
Title: Wake-resolving acoustic tomography: advances through numerical covariance methods
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The authors appreciate the detailed and careful review and the constructive assement provided by the reviewer. We address the concerns raised by Reviewer 3 below. Reviewer comments are shown in *italics*.

Major Concern 1: Applicability to real measurement systems

The requirement of high towers reaching hub height or the use of UAVs places high demands or is not realistic for field deployment. The study does not sufficiently address whether the proposed configuration is practically implementable in real wind farms.

Response: Reviewer 2 raised the same point; we added discussion in Section 5 on practical deployment pathways. The V27 (31.5 m hub height) is representative of research-scale facilities where AT has been deployed, including the NLR Flatirons Campus. Scaling to utility-scale turbines requires alternative array configurations, such as vertical planar arrays spanning two meteorological towers or hybrid ground-aerial systems using uncrewed aerial systems, as explored in a prior technical report (Hamilton, 2022). The engineering implementation of large-scale arrays is outside the scope of this study.

Major Concern 2: Dimensionality and physical limitations

It should be clearly stated already in the introduction that the approach is restricted to a two-dimensional tomographic reconstruction.

Response: We added an explicit statement in the Introduction clarifying that all retrievals are restricted to a two-dimensional horizontal plane at hub height. Extension to three-dimensional tomography would require volumetric path configurations and a higher transducer count; we identify this as future work.

A statement was added to the Introduction, "Retrievals in this study are restricted to a horizontal plane at hub height. Extension of the TDSI approach to three-dimensional field retrievals has been undertaken in past work \cite[e.g.,]{ostashev2008recent}, but is not the primary focus of the current research."

Major Concern 3: Covariance model dependency on LES

How transferable are these covariance models to real atmospheric conditions where LES data are not available? Does the inversion remain valid if LES does not accurately represent site-specific turbulence? Can the method operate independently of high-fidelity numerical simulations?

Response: The LES-informed ("Data") covariance model is an upper-bound benchmark: it establishes the best achievable performance when detailed flow statistics are known, not a prescription for routine deployment. The analytical Gaussian ("Model") covariance requires only bulk variance and length-scale estimates from standard meteorological measurements, and it performs competitively in several cases, even outperforming LES-informed covariances for $N_f > 4$. The precursor ("Precursor") model is an intermediate option using undisturbed ABL simulations, which are inexpensive relative to full wake simulations. Together, the three approaches span the practical design space. We updated the Discussion to make this framing explicit and to clarify that the purpose of testing LES-informed covariances is to identify which aspects of the correlation structure affect retrieval accuracy, guiding development of practical parameterized models that do not require full LES before each campaign.

We have added a statement to the Discussion section, "The three covariance estimates evaluated here bracket the practical design space from low-cost to high-fidelity.
The primary purpose of the Data model is identify which aspects of the correlation structure affect retrieval accuracy most, thereby guiding the development of practical parameterized models applicable without full LES for every operating condition."

Major Concern 4: Spatial scale and representativeness

The main tomographical domain (100 × 100 m²) is relatively small compared to modern wind turbine wakes. It remains unclear whether the method scales to full wake recovery regions or wind farm interactions.

Response: The 1 1^2$ domain is sized for the V27 ($D = 2$ m), spanning approximately D in each dimension and covering the near- to mid-wake region. For larger turbines, the domain scales proportionally with rotor diameter. We added a brief note in Section 5: domain size, transducer count, and covariance support lengths can all be adjusted, but systematic scaling analyses for utility-scale turbines and multi-turbine interactions are deferred to future work.

A clarifying statement was added to the discussion section reading, "The 100~m 100~m retrieval domain used here corresponds to approximately D for the V27. Scaling to modern rotors ($D > 12$~m) would require a proportionally larger domain and transducer count, and systematic analysis of this scaling is deferred to future work."

Major Concern 5: Periodic boundary conditions

The use of periodic boundary conditions is problematic for heterogeneous terrain and non-stationary atmospheric flows.

Response: Periodic boundary conditions are standard for idealized ABL simulations over flat, homogeneous terrain and appropriate for the V27 context. We added statements in Section 3 and the Discussion acknowledging that the results are conditioned on these idealized conditions, and that heterogeneous terrain or non-stationary inflow would require precursor or nested simulations with open lateral boundaries. This limits direct applicability of the LES-derived covariance models to sites with similar terrain and stability characteristics.

Limits of the generalizability of the covariance model derived with period boundary conditions are addressed in the Problem Setup section, "Simulations have laterally periodic boundary conditions, a slip wall at the top, and a rough wall at the bottom.
Laterally periodic conditions, while standard for idealized flat-terrain ABL simulations, are not expected to hold for complex terrain or non-stationary inflow conditions, which would require nested or precursor simulations with open lateral boundaries."

Major Concern 6: Travel-time modeling simplification

Travel times are computed analytically rather than derived from full wave propagation. This neglects ray bending / acoustic refraction effects. No systematic error analysis over longer propagation distances is provided.

Response: This overlaps with the idealized assumption discussion added in response to RC1, Comment 1. Section 5 now states that travel times are computed from the linearized forward model assuming straight ray paths with no refraction, multipath, absorption, or timing error. For path lengths up to ~100 m and weak vertical temperature gradients at hub height, ray bending is expected to produce travel-time deviations well below 1 ms, smaller than the dominant retrieval uncertainties. We added a sentence quantifying this and citing relevant acoustic propagation literature. Full ray-tracing or parabolic equation treatment is left as a future extension, particularly for longer paths or strong stability stratification.

The manuscipt now includes a statement to this effect, "Travel times are computed analytically from the forward model (\cref{eq:forward}) rather than extracted from simulated acoustic waveforms, effectively assuming perfect time-of-flight retrieval with no timing errors and straight ray paths.
Acoustic refraction due to temperature and wind-speed gradients is neglected; for the path lengths and near-neutral

temperature gradients considered here (1~m, weak vertical lapse), ray bending is expected to produce travel-time deviations well below 1~ms \citep{wilson2015sound}, smaller than the dominant retrieval uncertainties."

Major Concern 7: Experimental feasibility of acoustic system

The assumption of optimal signal frequencies (100–500 Hz) is not sufficiently discussed in terms of loudspeaker directivity constraints, signal-to-noise ratio in atmospheric conditions, and frequency resolution and synchronization requirements.

Response: The 100-500 Hz range was introduced in the Discussion in response to the rotor noise concern raised by Reviewer 2. We expanded that discussion to address these additional points: (1) loudspeakers in this range are commercially available with near-omnidirectional directivity at the relevant wavelengths (0.7-3.4 m), consistent with the omnidirectional transducer assumption; (2) representative SNR budgets for AT systems at these frequencies and ~100 m path lengths are discussed with reference to existing field deployments; and (3) GPS-disciplined clocking routinely achieves sub-millisecond timing accuracy, sufficient for the path lengths considered. References to relevant acoustic sensing literature have been added.

Two sentences were added in the Problem Setup section, following the virtual array figure description: "The virtual array is modeled as omnidirectional transceivers; loudspeakers operating in the 100--500~Hz range (wavelengths 0.7--3.4~m) are commercially available with near-omnidirectional directivity at these scales, consistent with this assumption \citep{hamilton2022acoustic, maric2023acoustic}.
Travel-time synchronization between transceivers is assumed to be exact; in practice, GPS-disciplined clocking routinely achieves sub-millisecond timing accuracy sufficient for the path lengths considered here."

Major Concern 8: Turbine and atmospheric representativeness

The Vestas V27 is an outdated turbine class. Modern turbines exhibit hub heights >100 m and rotor diameters >120 m. Atmospheric stability regimes and turbulence spectra differ significantly.

Response: The V27 ($D = 2$ m, $= 1$ m) is not representative of modern utility-scale turbines, but it is well-characterized and widely used in academic research because its modest dimensions make controlled experiments tractable. The LES-informed covariance framework, the TDSI adaptation for heterogeneous wakes, and the comparative benchmarking are turbine-agnostic and apply equally to modern rotors.

Major Concern 9: Frozen turbulence assumption

Taylor's frozen turbulence hypothesis fails not only in turbine wakes, but also in complex terrain, forested regions, and strongly stratified boundary layers. The applicability domain should be explicitly broadened or constrained.

Response: Section 2.1 notes that Taylor's hypothesis is invoked to concatenate covariance matrices across time frames. We added a discussion in Section 5 identifying the conditions where this assumption is most likely to fail: (1) in the near-wake shear layer, where turbulent structures distort rapidly; (2) in strongly stable conditions with wave activity; and (3) in complex terrain with flow separation. For the neutral and convective flat-terrain cases here, Taylor's hypothesis is well-supported by prior literature and consistent with the LES turbulence statistics. Relaxing this assumption, for example via space-time covariance models or ensemble Kalman filtering, is left for future work.

A sentence was added in the Theory Section, "Taylor's hypothesis is most likely to break down in the near-wake shear layer, where turbulent structures distort rapidly and in strongly stably atmospheric conditions, where fluid interactions with solid bodies can produce coherent, non-homogeneous structures."

Minor and Technical Remarks

L.50: Clarify whether "T" refers to acoustic virtual temperature.

Response: Clarified. The Theory section now reads "...T_{av} is the acoustic virtual temperature, denoted in the remainder of this work simply as T for brevity..."

L.51: Clarify whether u(x,t) is a full 3D velocity vector; "bulk motion" is not standard terminology.

Response: Revised to read "the horizontal wind velocity vector $(,t) = (u,v)$".

L.56: Is vertical velocity component w neglected?

Response: Yes. The vertical component is neglected under the assumption of near-horizontal ray propagation in the hub-height plane, consistent with standard two-dimensional AT implementations. The manuscript now reads, "The vertical contribution to the velocity vector, , is neglected, assuming near-horizontal acoustic ray propagation in the AT plane."

Chapter 2.1: Governing equations should be properly referenced rather than repeated.

Response: The governing equations in Section 2.1 deviate slightly from the Ostashev and Vecherin formulation; they have been retained as a more concise presentation.

Fig. 1: Unclear labeling (transmitters/receivers?), unclear meaning of grid cells and path lengths.

Response: The caption for Figure 1 has been updated. It now reads, "Layout of the collocated speakers (emitters) and microphones (receivers) in the virtual AT array used in the simulations (left), along with the travel time of acoustic signal along paths connecting emitter/receiver pairs (center) and a histogram of path lengths (right)."

Fig. 1 vs Table 3: coordinate consistency should be verified.

Response: Coordinate consistency between Figure 1 and Table 3 has been corrected.

We thank Reviewer 3 for the detailed critique.

Respectfully,
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