Review Report

Manuscript Title:

Geometric Nonlinear Analysis of Timoshenko Beams with Variable Cross-Section Using Co-Rotational Formulation

This paper addresses the geometrically nonlinear analysis of Timoshenko beam structures with variable cross-sections, which remains a challenging problem in computational structural mechanics. The authors propose a co-rotational finite element framework that incorporates an analytically derived displacement-based Timoshenko beam element for variable cross-sections, with the aim of improving both accuracy and computational efficiency in modeling large-deformation behavior.

The work is generally well developed and addresses a relevant research topic, with references that are consistent with the adopted methodology. Nonetheless, some concerns remain and should be adequately addressed.

- Although the modeling of Timoshenko co-rotational beams with tapered or variable cross-sections has been addressed in previous studies using multiple successful approaches, it remains unclear what constitutes the specific novelty or methodological advancement in the present work. Could the authors clarify how their formulation provides a substantive improvement or development over existing models?
- What is the practical advantage of incorporating analytically derived displacement shape functions within the co-rotational Timoshenko beam formulation, and how does this choice improve accuracy, computational efficiency, or overall performance compared to existing approaches?
- While the methodology presented in Section 2 attempts to account for geometric variability through Gaussian integration (Equation 16), there is a fundamental concern regarding the mathematical consistency of the local stiffness matrix derivation. The formulation in Equations (14 and 15) utilizes analytical shape functions originally developed for prismatic members;

however, applying these functions to non-prismatic elements without incorporating the spatial derivatives of the sectional properties (EI'(x) and GA'(x)) introduces a known field inconsistency. For elements with significant tapering, the omission of these gradient terms may lead to an inaccurate representation of the internal equilibrium, potentially affecting the overall robustness of this co-rotational framework.

- While the tangent stiffness formulation in Section 3 explicitly identifies the K_m and K^g matrices (as seen in Equation 42), the derivation lacks a detailed discussion on how these components are specifically adapted to the element's variable cross-section.
- Although the proposed formulation is applied to six numerical examples, the study does not provide a comparative assessment against existing methods for variable cross-section beams, which limits the demonstration of the approach's relative effectiveness and advantages.
- In Section 4, while the first two examples address 3D configurations, they are limited to constant cross-sections, whereas the subsequent four examples that account for variable sections are restricted to 2D analyses. The absence of a 3D example with a variable cross-section represents a significant gap, as it leaves the element's performance unverified in cases where spatial geometric nonlinearity and sectional tapering are coupled. Consequently, the authors must explicitly define the intended scope of this formulation and specify the categories of structural problems it is reliably applicable to, as the current results do not yet justify its robustness for complex, non-prismatic 3D applications.
- The abstract and introduction must explicitly define the research's novel contribution, clearly justifying the necessity of the proposed method instead of overemphasizing well-established classical theories

Recommendation:

Major revision is required.