Review of the revised manuscript "Large Eddy Simulation of Thermally Stratified Atmospheric Boundary Layers with a Lattice Boltzmann Method" by Henry Korb, Henrik Asmuth, Martin Schönherr, Martin Geier, and Stefan Ivanell submitted for publication in Wind Energy Science. In the revised manuscript the authors addressed adequately my comments except for one related to the Boussinesq approximation and Equation (5). Equation (5) is not correct. Boussinesq approximation is obtained by using constant background density and not horizontally averaged density. The authors claim that the buoyancy term defined in Equation (5) is the same as in the PALM model, however, checking the technical documentation, "Governing equations" for PALM model that can be found at: https://palm.muk.uni-hannover.de/trac/wiki/doc/tec/gov it is clear that this is not how the buoyancy term is implemented in PALM.

First, in the PALM technical documentation the same mean potential temperature is in both the numerator and denominator (notice angle brackets), unlike in the Equation (5) where horizontally averaged potential temperature is in the numerator while a constant, reference potential temperature is in the denominator. Unfortunately, in the PALM documentation the averaging represented by angle brackets is not defined.

Second, Boussinesq approximation is defined as, for example: "The Boussinesq approximation is a special case of the anelastic approximation, valid for cases where the vertical scale of circulation is much less than the scale height of the atmosphere (shallow circulation). With the Boussinesq approximation,density (ρ_a) is replaced with a constant boundary-layer-averaged value ($\langle\rho\rangle_a$) in all the equations of atmospheric dynamics, except in the buoyancy term of the vertical momentum equation." (Jacobson, 2005: "Fundamentals of Atmospheric Modeling." Cambridge Univ. Press., pp. 813). This form is used in numerous codes and peer reviewed publications (Moeng 1984,..., Stipa et al. 2024, etc.) By defining the buoyancy term as it is in defined in Equation (5) the following forcing is omitted: $(\langle\theta\rangle(z) - \theta_r)/\theta_r$. This term impacts the mean vertical velocity, which in a horizontally homogeneous boundary layer vanishes and this term is balanced by the vertical pressure gradient. The authors state that "Preliminary studies with a constant reference temperature led to spurious oscillations," which would indicate that there may be a problem with the implementation of the Lattice Boltzman method. Considering that in a horizontally homogeneous boundary layer the vertical velocity must vanish, it is justified to set the mean horizontal velocity to zero, and therefore, in effect, solve vertical momentum equation with the buoyancy term defined by Equation (5), however, this should be explicitly stated. This, however, is not a valid justification to claim that Equation (5) represents Boussinesq approximation.

The manuscript can be accepted for publication after the reason for using Equation (5) is explicitly stated as outlined above.

We thank the reviewer for their continued review. We have clarified the derivation of Eq(5) as per the reviewer's request.