

Referee:2

General comments:

This paper presents a novel set of measurements characterising the near wake of a yawed, small-scale wind turbine in a wind tunnel. By combining these measurements with existing literature data, the authors compare several analytical velocity deficit models coupled with yaw models. A model is proposed and evaluated alongside established approaches. Before comparison, all models are calibrated using non-yawed wind turbine wake data, ensuring a consistent methodology across the analysis. The proposed paper is interesting and includes a new dataset for model validation. However, this first submission suffers from limitations that will be detailed later. Also, the potential impact of this study and its innovative character are not clear to me.

We thank the reviewer for providing valuable positive feedback on our manuscript. In the following document we address each of their comments, enhancing the revised version to meet the standards for publication. Any modifications made to the manuscript are highlighted in blue and, when relevant, have been included below to address the reviewer's specific comment.

The authors would like to take a moment to highlight the novelty of this submission, which comes in three forms:

- The inclusion of a high resolution near-wake dataset.
- A comparison of current analytical models using three experimental datasets (including full-scale), where previous literature would have mostly only included one experimental dataset.
- The proposition of an analytical model as the combination of two previous models.

Specific comments:

The analysis is restricted to lateral profiles, which may be misleading. Wake centers are not consistently located near hub height, contrary to the paper's assumptions. Expanding measurements to include 2D y - z planes would provide a more comprehensive understanding of wake behaviour.

We thank the reviewer for the indication on where greater analysis can be conducted and where the authors have made an oversight in their assumptions. The authors agree with the reviewer that work by [3] (Figure 5) shows the wake centre deviates beyond the hub height plane. From work by [3] an assumption is made in their wake model that "The wake centre in (5.1) is assumed to remain at hub height z_h as the vertical displacement of the wake centre is rather small for lower yaw angles (see figure 5)." The assumption in the manuscript has been modified to reflect the reviewers astute comment, shown below, in blue.

The authors agree that additional planes would offer a more comprehensive understanding of the wake behaviour, however, some of the models analysed in this work are strictly two dimensional. Seeing as 2 dimensional models are still used as a starting point within the industry, the comparison is valid.

Work by Bastankhah and Porté-Agel [3] states "the vertical displacement of the wake centre is rather small for lower yaw angles" hence, when implementing the wake centre deflection model from Bastankhah and Porté-Agel [3] there is an assumption that the wake centre y_c remains at hub height.

The study's focus on the near wake is limiting in view of validating models. Extending measurements to cover the full wake would offer deeper insights into wake evolution and model applicability.

We thank the reviewer for their comment and agree that a purely near-wake focus is limiting. Hence the reason for the two additional datasets used within this investigation, both of which cover the far-wake region. To emphasise this point, the manuscript has been updated below, shown in blue.

The first dataset is a near-wake high-resolution planar PIV wind tunnel experiment at a working TSR of $\lambda = 5.5$. The necessity to accurately predict the near-wake comes not only from the possible interaction with downstream turbines, but, recently, the interaction with other objects within a farm, such as transmission lines or bird clusters ([1] and [17]). As a purely near-wake focus would be limiting, two additional datasets are included in this investigation.

The calibration process is unclear and requires significant clarification. Specifically:

- Which parameters are calibrated, and how?
- Do parameters vary with streamwise location, or are predefined functions assumed (e.g., a linear approach for the Gaussian width sigma or another function for the super-Gaussian order n)?

TABLE 1: Tuned parameters for each analytical model. X_0 represents the models standard calibration.

Model	Parameter	Near Wake	Far Wake	Full Scale	X_0
BPA					
	α^*	0.733	2.05	4.16	2.32
	k	0.00560	0.0292	0.0293	0.0220
Sh					
	k_w	0.0103	0.0948	0.0644	0.0834
	σ_0/D	0.283	0.223	0.323	0.235
Bl					
	c_s	0.109	0.175	0.179	0.195
	k	0.0344	0.0301	0.0461	0.0270
	b_f	-0.354	-1.06	-0.775	-1.15
Ba					
	U^*	0.341	0.493	0.525	0.216
	α	0.629	0.248	0.695	0.600
Pr					
	α	0.0613	0.0338	0.990	-
	n	0.174	0.937	0.567	-
	r'_{min}	0.467	0.416	0.477	-
	d'_e	1.06	0.881	1.02	-

- If parameters are set for each streamwise location, how are integrations performed with the super-Gaussian model?

We thank the reviewer for their comments regarding the calibration process. The authors agree that more details can be included for the calibration of the models, hence an additional Table 1 is included within the manuscript, shown below. The authors reached out to Frédéric Blondel to obtain their super-Gaussian model, hence the implementation of the integration was consistent with the published article ($x_1=0$). Additional investigations were conducted as to how the initial streamwise location affected the integrations, it was concluded that there was no difference in the result of the super-Gaussian model with starting locations beyond $x_1=0$, consistent with code by Frédéric Blondel.

The process used to tune these parameters is outlined in Sect. 2.2 and the resulting parameters are displayed in Table 1. The results from the tuning procedure are discussed in the following section.

Was the presented model implementation validated in their original form, before calibration?

We thank the reviewer for their comment and confirm that the wake centre was validated with results from [3] and the velocity deficit was validated with results from [8] shown in Fig. 1.

The presented results for the super-Gaussian model deviate from those in other studies using the same dataset (e.g., wind tunnel data from Bastankhah & Porté-Agel). This discrepancy must be addressed, possibly by validating the implementation. Additionally, reporting calibrated values and comparing to accepted model calibration (i.e., <https://doi.org/10.5194/wes-8-141-2023> in the present case) would provide some insight. A sensitivity study on the calibration constant could also provide insight.

The authors thank the astute comment made by the reviewer concerning the validity of the implementation of the super-Gaussian model. The authors must apologise for an error when implementing the Upper Incomplete Gamma Function converting between C++ and MATLAB. The model was initially validated at $\gamma = 0^\circ$ (as it was wrongfully assumed the deflection code sent from the author had been correctly converted), however under further inspection and correction, the model has now been fully validated under $\gamma = 10^\circ$, $\gamma = 20^\circ$ and $\gamma = 30^\circ$, shown with blue crosses in Fig. 2. These corrections have made changes to the deflection of the super-Gaussian model, hence figures 8, 10, 11, 9, 12 and relevant parts of the results section have been modified, shown below, in blue.

In terms of the prediction of the wake centre, all models capture the deflection well, other than the super-Gaussian model (Bl) which overpredicts the wake centre at both yaw angles.

...

Similarly to the near-wake results, the super-Gaussian model (Bl) overpredicts the yawed wake position at 10° . This overprediction becomes more pronounced at a yaw angle of 20° . Similarly, the single-Gaussian model (QI) by Qian and Ishihara [11] overpredicts the wake centre at 20° .

...

In the near-wake, the single-Gaussian model (QI) developed by Qian and Ishihara [11] performs the worst, followed by the super-Gaussian model (Bl) developed by Blondel et al. [5]. The single-Gaussian model (BPA) by Bastankhah and Porté-Agel [3], the vortex sheet model (Ba), and lifting line model (Sh) show similar behaviour, with MAE values between 4.8 % and 5.9 %. The double-Gaussian model (Pr) closely matches the experimental data achieving a MAE of just 2.2 %.

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For the full-scale case all models display comparable predictive accuracy, with a MAE within $\pm 0.3\%$. When averaging across all datasets, it is evident that the single-Gaussian model (QI) by Qian and Ishihara [11] has the poorest predictive capability, as expected given its lack of tunable parameters. The super-Gaussian (Bl), single-Gaussian model (BPA) by Bastankhah and Porté-Agel [3], lifting line (Sh), and vortex sheet (Ba) methods exhibit similar performance, with MAE values ranging between 3.6 % and 4.6 %.

...

The double-Gaussian model (Pr) performs best at all angles in the near-wake exhibiting MAE values below 2.5 %. In contrast, the super-Gaussian model (Bl) performs the worst at large yaw angles ($> 10^\circ$), akin to the underprediction of the wake deficit and overprediction of the wake centre. This suggests not only that a more complex wake shape than the single-Gaussian is required, but also an accurate description of the wake centre.

Missing information to reproduce the results: based on the information provided in the paper, it is not possible to reproduce the test cases. At least the wind turbine thrust coefficients must be provided, together with other relevant flow and wind turbine operation variables.

We thank the reviewer for their valuable insight and agree that there is information lacking to reproduce the results within the paper. To correct this, Table 2 has been inserted into the manuscript to ensure reproducibility and to give a better understanding of the test conditions for the three experimental datasets.

The information necessary to reproduce the results from the near-wake, far-wake and full-scale datasets are presented in Table 2.

The method used to estimate the yawed thrust coefficient (CT, γ with γ the yaw angle) is unclear. Different approaches exist (e.g., $CT \cos(\gamma)$, $CT \cos^2(\gamma)$, $CT \cos^3(\gamma)$), and consistency across models is essential for fair comparison. A clear presentation of all models and their assumptions is needed.

We thank the reviewer for their insightful feedback on estimating yawed thrust coefficient. Aside from the full-scale dataset all thrust coefficients are measured, hence no additional estimation is required. In the case of the full-scale dataset the \cos^2 approach is utilised. These changes have been updated in the manuscript, shown below, in blue.

It must be noted that all yawed thrust coefficients are measured for the near- and far-wake datasets, however, for the full-scale dataset only the zero yaw angle thrust

TABLE 2: Flow and turbine characteristics.

Dataset	Yaw angle [°]	C_T	U_∞ [ms ⁻¹]	TI [%]	D [m]	λ
Near-wake						
	-30	0.542	7.8	5.3	0.5	5.5
	-25	0.579	7.8	5.3	0.5	5.5
	-20	0.618	7.8	5.3	0.5	5.5
	-15	0.632	7.8	5.3	0.5	5.5
	-10	0.657	7.8	5.3	0.5	5.5
	-5	0.674	7.8	5.3	0.5	5.5
	0	0.682	7.8	5.3	0.5	5.5
	5	0.672	7.8	5.3	0.5	5.5
	10	0.653	7.8	5.3	0.5	5.5
	15	0.628	7.8	5.3	0.5	5.5
	20	0.602	7.8	5.3	0.5	5.5
	25	0.557	7.8	5.3	0.5	5.5
	30	0.508	7.8	5.3	0.5	5.5
Far-wake						
	0	0.820	4.88	7.5	0.15	3.8
	10	0.780	4.88	7.5	0.15	3.8
	20	0.730	4.88	7.5	0.15	3.8
Full-scale						
	0	0.780	8.6	12.5	131	7
	11	$0.750 = 0.78 \cos(11)^2$	8.6	12.5	131	7

coefficient is known, hence the thrust coefficient is estimated using $\cos^2(\gamma)$, shown in Table 2

The "new" approach combines two existing models, which raises questions about its novelty and the space devoted to it in the paper. If something new is introduced here, please mention it clearly.

We thank the reviewer for their comment on the novelty of the proposed method and agree that it is a combination of two methods, which is now stated more clearly in the manuscript. When reviewing what equations are required from (2)-(20), equations 2,3,4,5,6,7,8,9,10,11,16,17,18,19&20 are necessary. We suggest that equations (12)-(14) are also required to understand the main assumptions and formulation of the model by [8]. These changes are now more clearly highlighted in the manuscript, alongside a modified definition of the novelty of the proposed method, shown below, in blue.

An additional model, developed with the combination of the double-Gaussian formulation by [8] coupled with the yaw model from [3] is proposed.

...

an additional model, developed with the combination of the double-Gaussian formulation by [8] coupled with the yaw model from [3]

...

Following this, Sect. 3 describes the proposed analytical model based on a combination of work by Keane [8] and Bastankhah and Porté-Agel [3].

...

This section proposes a wake model using a combination of existing models, developed to improve the accuracy of predictions in the near-wake under yaw misalignment.

...

Any novel modifications to the equations are highlighted in bold and described in the section below.

...

where D_{\pm} is modified from work by [8] with the addition of the wake centre y_c , and multiplication of r_{min} by $\cos^2(\gamma)$, given by

...

The equation for the wake centre y_c is dependant on the normalised length of the potential core x_0/D given below from work by Bastankhah and Porté-Agel [3]

...

Work by Keane [8] uses the actuator disc model. Keane employs work by Tennekes and Lumley [13] to show the mean momentum flux across a disk is given by

...

This study compared six analytical yaw wake models, including a proposed double-Gaussian formulation as a combination of the work by Keane [8] and Bastankhah and Porté-Agel [3]

While mean streamwise velocities and turbulence intensity are compared to ISO standards, further details are required:

- Clear definition of turbulence intensity (TI)
- Assessment of the lateral velocity component
- Uniformity of velocity profiles in the lateral direction
- Comparison of integral length scales to expectations

- Provision of both lateral and vertical profiles

We thank the reviewer for drawing attention to these important points. A clear definition of turbulence intensity and integral length scale has been added to the manuscript, shown below, in blue.

The mean lateral velocity components (V and W) were measured and are $0 \pm 2\%$ of U at all heights. Additionally, the lateral variation in the mean streamwise velocity and streamwise turbulence intensity is less than 2% at hub height, shown in Fig. 3. Finally, the streamwise integral length scale at hub height was determined to be $L_u = 0.31$ m via the integration of the autocorrelation, and confirmed using the central peak frequency method ($L_u = 0.29$ m), a fit to the von Kármán spectrum ($L_u = 0.30$ m), and the zero-frequency spectral intercept method ($L_u = 0.35$ m). These points are reflected in the manuscript, shown below, in blue.

All measurements within this campaign are taken at a hub-height velocity of $U_h = 7.8 \text{ ms}^{-1}$ with a hub height turbulence intensity of $T_i = 5.3\%$ and a hub height streamwise integral length scale of $L_u = 0.31$ m. The streamwise turbulence intensity is calculated using local mean velocity at the specified height with the equation shown below

$$T_i = \frac{\sqrt{u'^2}}{U}, \quad (1)$$

where u' is the fluctuations in streamwise velocity. The streamwise integral length scale $L_u = U_h T_l$ is determined via the integration of the autocorrelation, and confirmed using the central peak frequency method, a fit to the von Kármán spectrum, and the zero-frequency spectral intercept method

$$T_l = \int_0^\infty \rho_x(\tau) dt, \quad (2)$$

where τ is the time lag and ρ_x is the autocorrelation coefficient defined as

$$\rho_x(\tau) = \frac{R_x(\tau)}{R_x(0)}, \quad (3)$$

where R_x is the autocorrelation function in time.

...

Figure 1 presents the normalised mean velocity U/U_h and streamwise turbulence intensity T_i profiles

...

At all measured heights, the mean spanwise and vertical velocity components are approximately zero, with magnitudes within $\pm 2\%$ of the streamwise velocity component (i.e., $V = 0 \pm 0.02U$ and $W = 0 \pm 0.02U$). Additionally, the variation in the streamwise velocity component U_h and streamwise turbulence intensity T_i at hub height with spanwise location ($\pm 0.6D$) is less than 2%.

Beyond the new dataset, the study's novelty is unclear. Recent advancements in modelling complex wake shapes under yaw and secondary steering are not addressed, limiting the paper's contribution to the field.

We thank the reviewer for their insight. The novelty of this submission comes in three forms: the inclusion of a high resolution near-wake dataset, a comparison of current analytical models using three experimental datasets (including full-scale), previous literature would mainly have at most one experimental dataset, and the proposition of an analytical model (albeit the combination of previous models). A mention of the recent advancements in complex wake shapes and secondary steering have been included within the manuscript, shown below in blue.

The wake of a turbine under yawed conditions is complex (and shown as non-Gaussian), hence the vortex sheet model, proposed by Bastankhah et al. [4] predicts the wake shape by treating the wake edge as a vortex sheet.

...

Recent models, such as work by King et al. [9] and Howland and Dabiri [6] are capable of predicting secondary steering, where the wake of a downstream turbine is influenced by the wake of an upstream turbine.

Surprisingly, the widely used Jimenez model is omitted from the comparison. Including it would provide a more comprehensive benchmark for the proposed and existing models.

We are grateful to the reviewer for this thoughtful comment on including the classic Jimenez model. It is of the authors opinion that the inclusion of the Jimenez model, although regularly seen in previous literature, detracts from the current manuscript. In preliminary analyses, the Jiménez model exhibited substantially larger errors under yawed conditions than the other models, which led to a compression of the error scale and obscured the relative performance differences among the remaining models. For this reason, and to preserve the readability and interpretability of the comparative figures, the Jiménez model was omitted from the final comparison.

Technical corrections:

1. L.2: The use of "2D" is ambiguous. Please clarify.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

This study compares six yawed wake models capable of predicting the streamwise velocity component in the hub height plane. The yaw models evaluated include

2. L.3: The claim that a "new" double-Gaussian model is proposed is misleading. The model used is that of Keane et al., coupled with the Bastankhah & Porte-Agel yaw model. Please clarify what is new in the proposed approach.

We thank the reviewer for this comment. The authors have modified the manuscript to ensure there is no confusion with the proposed model, shown below, in blue.

An additional model, developed with the combination of the double-Gaussian formulation by [8] coupled with the yaw model from [3] is proposed.

3. L.5: The assertion that measurements serve as an undebatable reference overlooks the complexity of full-scale measurements, which are subject to uncontrolled environmental biases. Are these measurements based on neutral atmospheric conditions? A more critical discussion is needed.

We thank the reviewer for this comment, a more critical discussion is included in the manuscript, shown below, in blue.

However, all models struggle to predict the full-scale dataset under yawed conditions, possibly from uncontrollable environmental biases, but none the less emphasising the necessity for validating models against a wide range of turbine operating conditions.

4. L.11: The term "more" is vague. Please reformulate for precision and provide references to support the claim.

We thank the reviewer for this comment, additional references have been included in the manuscript, shown below, in blue.

Although less detailed than numerical simulations, recent analytical models are able to accurately predict the entire wake region under large yaw misalignments, thereby increasing their use in the development of farm-wide control algorithms ([2, 16]).

5. L.17: While the model depends on a single tuning parameter (σ , I assume), σ itself is usually assumed to be a linear function with coefficients dependent on turbine operating and environmental conditions. This should be made explicit.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

With just a single tuning parameter (the wake expansion) determined from the turbine operating and environmental conditions, the model successfully predicts the far-wake behind a host of turbine geometries subjected to different (non-yawed) inflow conditions.

6. L.29: The mention of wind shear in the Ishihara & Qian paper is unclear. Shear is not a parameter in the models, and previous models (e.g., Bastankhah & Porte-Agel) are compatible with non-uniform vertical streamwise velocity profiles as input. Clarify the intended meaning.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

In recent years three-dimensional models are developed based on the idea of axisymmetric self-similarity assumptions ([7, 12, 10, 14]). Some of which include the introduction of an incoming Atmospheric Boundary Layer (ABL) and wind shear.

7. L.37: Insert “that” between “setting” and “a wake steering” for grammatical correctness.

We thank the reviewer for this comment and have modified the manuscript.

8. L.43: The statement “requires no tunable parameters” is misleading, as the model depends on multiple calibration constants. Furthermore, these constants could also be tuned in this study.

We thank the reviewer for this comment and have modified the manuscript.

9. A discussion on wake deformation (curled-wake effect) and the complex wake shape behind yawed turbines should be included in the introduction.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

The wake of a turbine under yawed conditions is complex (and shown as non-Gaussian), hence the vortex sheet model, proposed by Bastankhah et al. [4] predicts the wake shape by treating the wake edge as a vortex sheet.

...

Recent models, such as work by King et al. [9] and Howland and Dabiri [6] are capable of predicting secondary steering, where the wake of a downstream turbine is influenced by the wake of an upstream turbine.

10. L.81: please use the international system of units

We thank the reviewer for this comment, the $10' \times 5'$ wind tunnel is the name of the facility here at Imperial College London.

11. L.84: Define turbulence intensity (TI) clearly: is it based on hub-height velocity or local velocity $u(z)$?

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

All measurements within this campaign are taken at a hub-height velocity of $U_h = 7.8 \text{ ms}^{-1}$ with a hub height turbulence intensity of $T_i = 5.3 \%$ and a hub height streamwise integral length scale of $L_u = 0.31 \text{ m}$. The streamwise turbulence intensity is calculated using local mean velocity at the specified height with the equation shown below

$$T_i = \frac{\sqrt{u'^2}}{U}, \quad (4)$$

where u' is the fluctuations in streamwise velocity. The streamwise integral length scale $L_u = U_h T_l$ is determined via the integration of the autocorrelation, and confirmed using the central peak frequency method, a fit to the von Kármán spectrum, and the zero-frequency spectral intercept method

$$T_l = \int_0^\infty \rho_x(\tau) dt, \quad (5)$$

where τ is the time lag and ρ_x is the autocorrelation coefficient defined as

$$\rho_x(\tau) = \frac{R_x(\tau)}{R_x(0)}, \quad (6)$$

where R_x is the autocorrelation function in time.

12. L.85: Both positive and negative yaw angles were considered. Did you observe any non-symmetric behavior as claimed in some studies (<https://doi.org/10.5194/wes-6-1521-2021>)? This should be discussed.

We are grateful to the reviewer for this thoughtful comment. When mirroring the PIV data around the vertical plane perpendicular to the turbines blades, and then taking the absolute error between each symmetric angle. For example, mirroring $\gamma = -30^\circ$ so it is consistent with $\gamma = 30^\circ$ and taking the absolute difference, there is limited non-symmetric behaviour observed in the mean velocity. More specifically, for all angles a maximum error of 5% of U_h is observed with a mean difference between each angle of 1% of U_h . A comment has been added to the manuscript, shown below in blue.

To investigate the symmetry of the PIV data, it is found that when mirroring the data around the vertical plane perpendicular to the turbines blades, and then taking the absolute error between each symmetric angle, there is limited non-symmetric behaviour observed in the mean velocity. More specifically, for all angles a maximum error of 5% of U_h is observed with a mean difference between each angle of 1% of U_h .

13. Figure 1.: Define TI and include V / U_h , T_{iv} , T_{iw} , and turbulent length scales for completeness.

We appreciate the reviewer's thoughtful and constructive comment, however it is of the authors opinion to keep Fig. 1 the same, and instead include the relevant information regarding turbulence parameters within the manuscript, as there is little perceived change in the vertical or lateral directions. The changes to the manuscript are shown below, in blue.

At all measured heights, the mean spanwise and vertical velocity components are approximately zero, with magnitudes within $\pm 2\%$ of the streamwise velocity component (i.e., $V = 0 \pm 0.02U$ and $W = 0 \pm 0.02U$). Additionally, the variation in the streamwise velocity component U_h and streamwise turbulence intensity T_i at hub height with spanwise location ($\pm 0.6D$) is less than 2%.

14. Section 2.1.2: Please provide thrust coefficients for both unyawed and yawed cases. Is the current controller representative of real-scale turbines, especially regarding yaw misalignment? L.119: Did you verify the statistical convergence of the procedure? L.123 to L.126: Are these filtering operations standard? If so, provide references.

We thank the reviewer for these comments. As mentioned above Table 2 has been inserted into the manuscript. For the current measurements the tip speed ratio was kept constant, which for an upwind turbine is representative of the operating conditions. The PIV data are well converged: convergence to within $\pm 1\%$ of the mean is achieved within 2000 samples, and since 3000 samples are used in the present study, the mean can be considered fully converged within $\pm 1\%$. The filtering methods are outlined in [15]. The manuscript has been updated below, in blue.

Finally, a min-max filter, which can be interpreted as a binary filter, is then applied (Tropea et al. [15]).

15. What is the main motivation for focusing on near-wake characteristics in these measurements?

We thank the reviewer for drawing attention to this important point. It is of the authors opinion that we require accurate near wake modelling because:

- Under certain circumstances (U_h, T_i) the near-wake extends beyond 5D, therefore in some farms, the near-wake will influence other turbines.
- Recent work has been conducted to understand the influence of objects within the near-wake of a turbine, for example transmission lines (challenging the current view and guidelines that overhead transmission lines should not be installed within 3D of a wind turbine; see [1]) and bird clusters [17].

The first dataset is a near-wake high-resolution planar PIV wind tunnel experiment at a working TSR of $\lambda = 5.5$. The necessity to accurately predict the near-wake comes not only from the possible interaction with downstream turbines, but, recently, the interaction with other objects within a farm, such as transmission lines or bird clusters ([1] and [17]). As a purely near-wake focus would be limiting, two additional datasets are included in this investigation.

16. Section 2.2: The calibration process is unclear. Specify which parameters are tuned, and whether they are tuned independently for each streamwise location and test case. Explain how integration is performed in the super-Gaussian model if the parameters are not continuous.

We thank the reviewer for their comments regarding the calibration process. The authors agree that more details can be included for the calibration of the models, hence an additional Table. 1 is included within the manuscript.

17. Section 2.2: It would be informative to compare tuned models to their standard calibration, possibly in an appendix.

We thank the reviewer for this helpful observation, the authors have included a column in Table 1 to reflect the difference between the tuned parameters and their standard calibration.

18. Section 3: The derivation appears to use the double-Gaussian formulation of Keane et al. with the Bastankhah & Porté-Agel yaw model. If no new models are proposed, the extensive derivation should be justified or condensed. If new elements are introduced, they should be clearly highlighted.

We thank the reviewer for their comment on the novelty of the proposed method and agree that it is a combination of two methods, which is now stated more clearly in the manuscript, shown below, in blue.

An additional model, developed with the combination of the double-Gaussian formulation by [8] coupled with the yaw model from [3] is proposed.

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Following this, Sect. 3 describes the proposed analytical model based on a combination of work by Keane [8] and Bastankhah and Porté-Agel [3].

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This section proposes a wake model using a combination of existing models, developed to improve the accuracy of predictions in the near-wake under yaw misalignment.

...

Any novel modifications to the equations are highlighted in bold and described in the section below.

...

where D_{\pm} is modified from work by [8] with the addition of the wake centre y_c , and multiplication of r_{min} by $\cos^2(\gamma)$, given by

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The equation for the wake centre y_c is dependant on the normalised length of the potential core x_0/D given below from work by Bastankhah and Porté-Agel [3]

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Work by Keane [8] uses the actuator disc model. Keane employs work by Tennekes and Lumley [13] to show the mean momentum flux across a disk is given by

...

This study compared six analytical yaw wake models, including a proposed double-Gaussian formulation as a combination of the work by Keane [8] and Bastankhah and Porté-Agel [3]

19. Eq.2: Explicitly state that γ corresponds to the yaw angle. Ensure all quantities in the derivation are clearly defined.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

To ensure a consistent and unbiased analysis of the wake models under yaw misalignment, each model's parameters are tuned at $\gamma = 0^\circ$, where γ is the yaw angle, with the exception of the model by Qian and Ishihara [11] as no tuning parameters are required.

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where r is the distance from the centre of the turbine in the spanwise direction.

...

where x is the distance from the centre of the turbine in the streamwise direction and $\epsilon = (d'_e - r'_{min})/6$.

20. L.152: σ is not a Gaussian function; please correct this phrasing.

We thank the reviewer for this astute comment. The authors have modified the manuscript, shown below, in blue.

The single Gaussian cross-section

21. Eq.6: Provide a reference to the appropriate source for this model.

We thank the reviewer for this comment and have modified the manuscript, shown below, in blue.

where D_{\pm} is modified from work by [8] with the addition of the wake centre y_c , and multiplication of r_{min} by $\cos^2(\gamma)$, given by

22. L.158: Please clearly distinguish between non-yawed and yawed CT throughout the paper.

We thank the reviewer for this comment and have modified the manuscript.

23. L.166: The claim that the wake center is aligned with hub-height is invalid. Bastankhah & Port'e-Agel (Figure 5) clearly shows otherwise.

We thank the reviewer for the indication on where the authors have made an oversight in their assumptions. The authors agree with the reviewer that work by [3] (Figure 5) shows the wake centre deviates beyond the hub height plane. From work by [3] an assumption is made in their wake model that "The wake centre in (5.1) is assumed to remain at hub height z_h as the vertical displacement of the wake centre is rather small for lower yaw angles (see figure 5)." The assumption in the manuscript has been modified to reflect the reviewers astute comment, shown below, in blue.

Work by Bastankhah and Porté-Agel [3] states "the vertical displacement of the wake centre is rather small for lower yaw angles" hence, when implementing the wake centre deflection model from Bastankhah and Porté-Agel [3] there is an assumption that the wake centre y_c remains at hub height.

24. Figure 4.: The x-axis should not be the streamwise distance, x/D , but rather $x/D + (u_{-uh})/u_h$ (or similar).

We thank the reviewer for their comment and agree it would be helpful to have additional axis, this has been modified in the manuscript, and shown below in Fig. 5,6,7,8,10,11.

25. Figure 5. the poor agreement between Ishihara & Qian's model and measurements is surprising. Was the implementation validated against the original paper's test cases?

We thank the reviewer for their comment and confirm that the wake model from [11] was validated shown in Fig. 4.

26. L.240: the experimental data also contain a test case at 30 degrees. Why was this not considered in the analysis?

We thank the reviewer for this perceptive observation however in work by [3], specifically Figure 21, the spanwise distribution of the streamwise velocity for 30° are not published, hence were not used within the current investigation.

27. L.253: The use of a square cosine function to modify r_{min} requires justification. A simple cosine might be more appropriate for a purely trigonometric transformation.

We appreciate the reviewer's insightful comment and agree that as a projection it should be cosine, however the data suggests that the relationship between the distance of the Gaussian peaks decreases with the cosine squared. Hence, is the implementation within the paper.

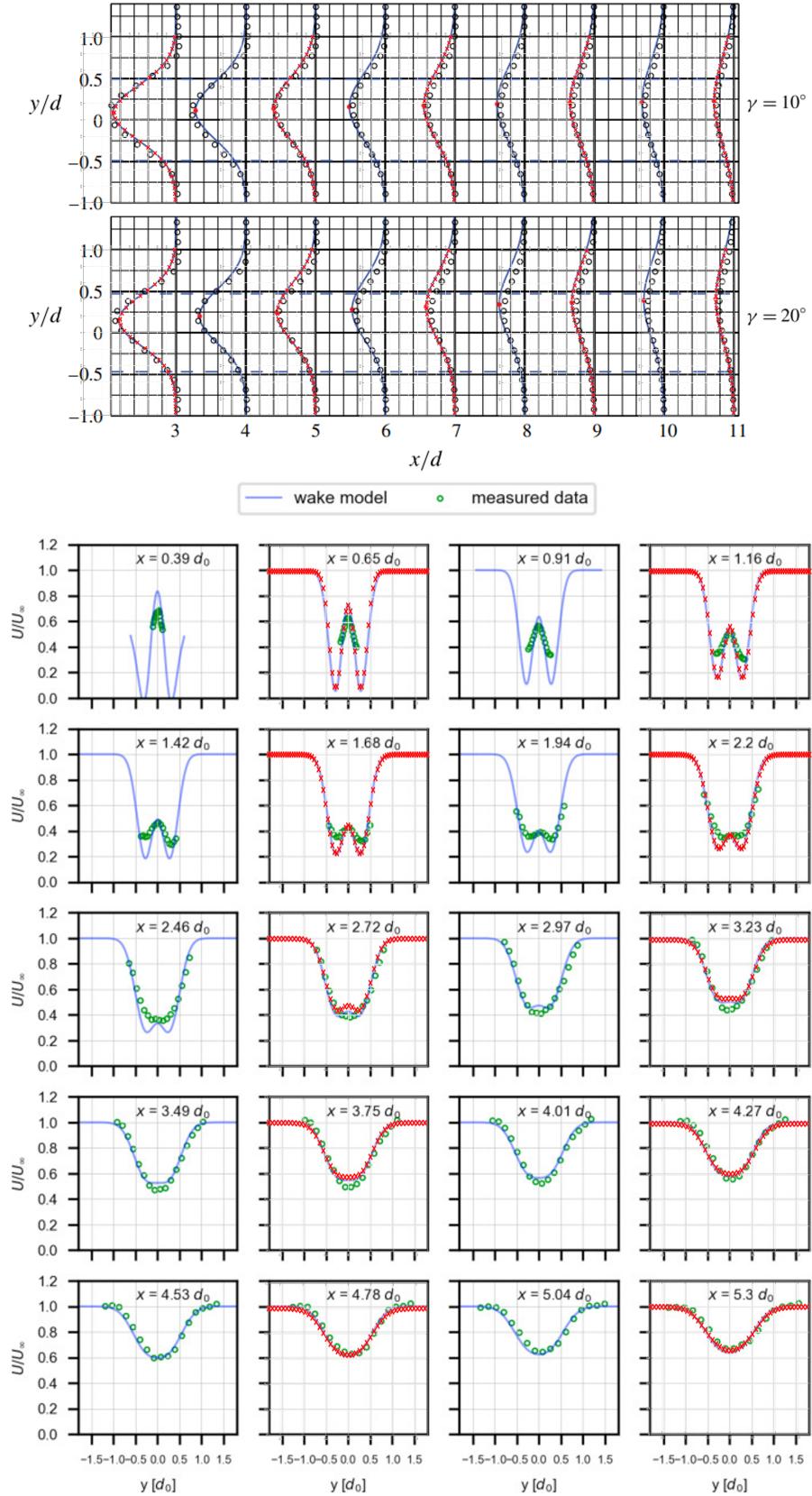


FIGURE 1: Validation of Present model (red crosses) with results from [3] (Top) at $\gamma = 10^\circ, 20^\circ$ at $x/D = 3, 5, 7, 9, 11$ and results from [8] (Bottom) at $x/D = 0.65, 1.16, 1.68, 2.2, 2.72, 3.23, 3.75, 4.27, 4.78, 5.3$.

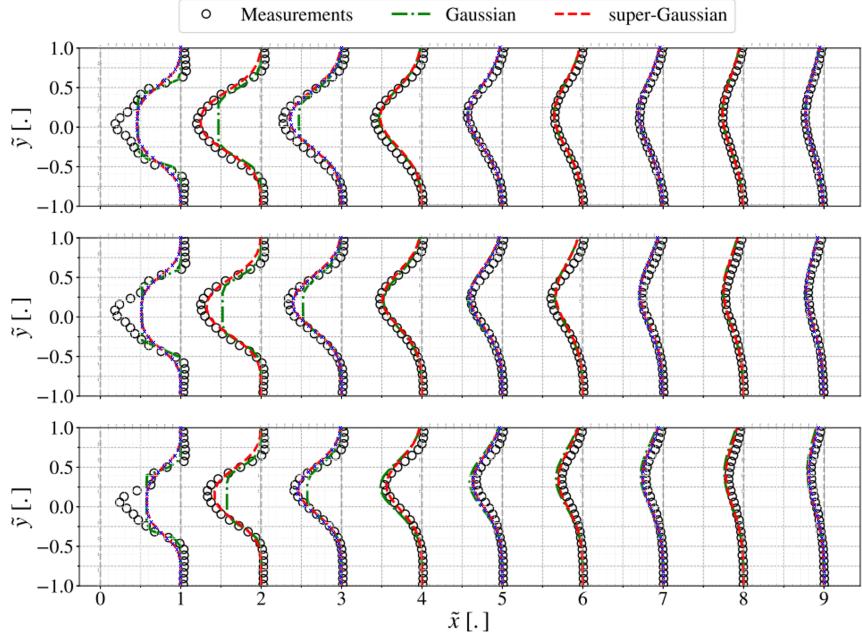


FIGURE 2: Validation of Bl model with results from [5] at $\gamma = 10^\circ, 20^\circ, 30^\circ$ for $x/D = 1, 3, 5, 7, 9$. The model from the authors is shown with blue crosses.

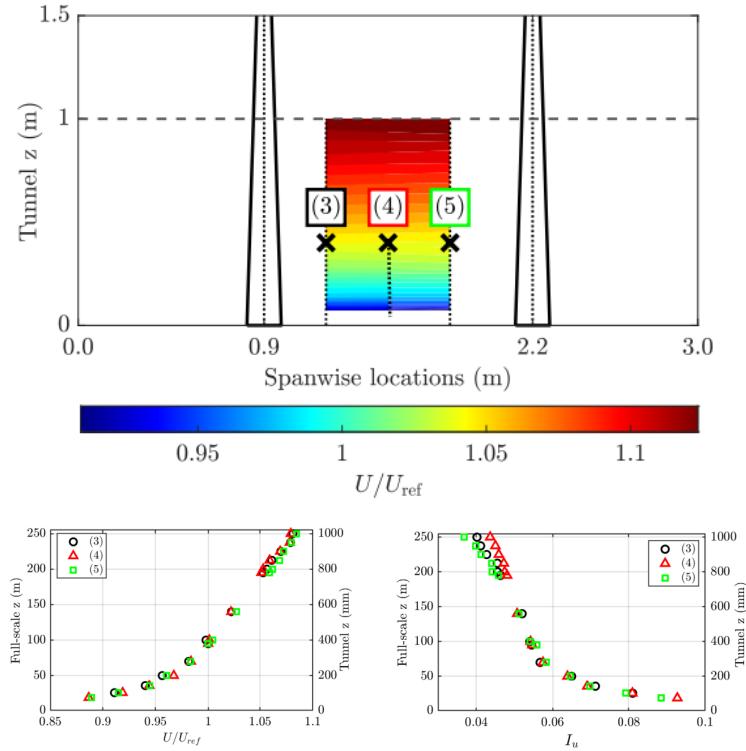


FIGURE 3: (Top) Contour plot of the variation in streamwise velocity profile in the lateral direction, (Bottom left) variation in streamwise velocity profile in the lateral direction and (Bottom right) variation in streamwise turbulence intensity profile in the lateral direction.

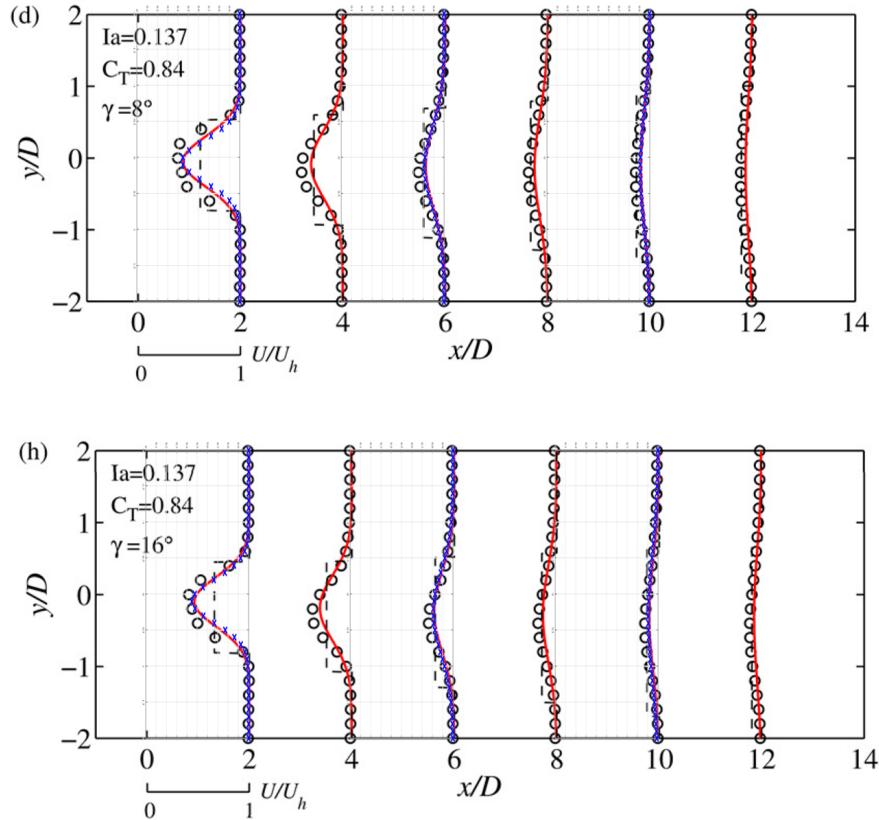


FIGURE 4: Validation of model in blue crosses with results from [11] at $x/D = 2, 6, 10$ for $\gamma = 8^\circ, 16^\circ$.

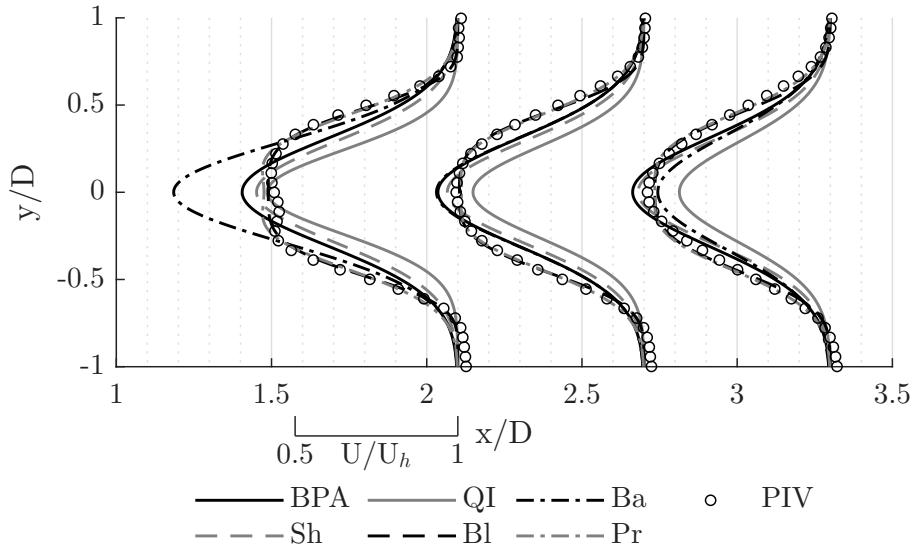


FIGURE 5: Spanwise profiles of the normalised streamwise velocity at $x/D = 2.1, 2.7$ and 3.3 , comparing near-wake experimental data with tuned models at a yaw angle of zero degrees. The solid vertical grey lines indicate $U/U_\infty = 0.5$.

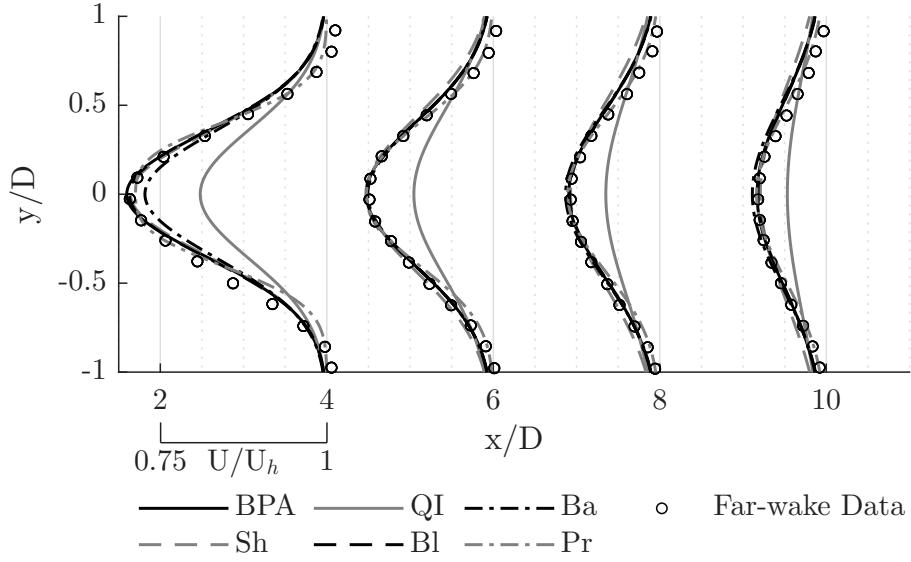


FIGURE 6: Spanwise profiles of the normalised streamwise velocity from $x/D = 4$ to 11, comparing far-wake experimental data with tuned models at a yaw angle of zero degrees. The solid vertical grey lines indicate $U/U_\infty = 0.25$.

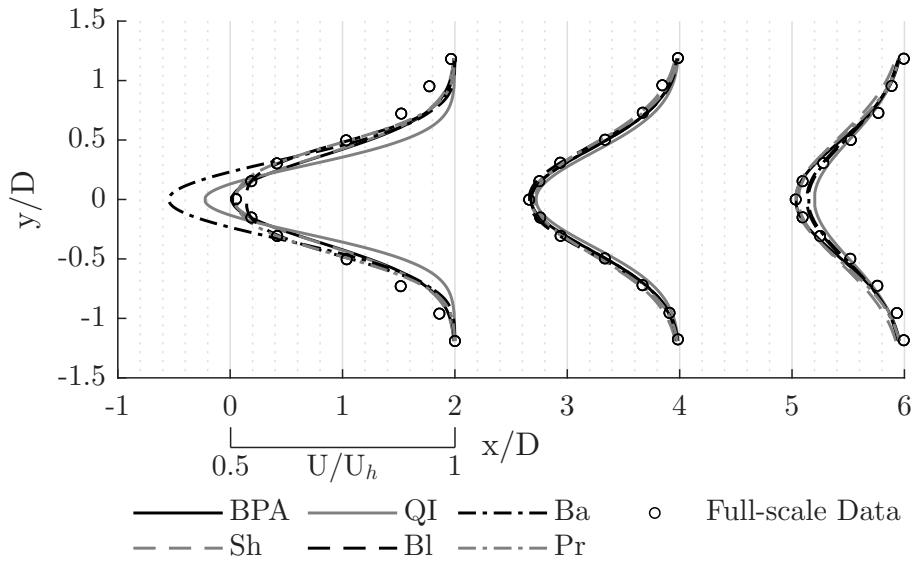


FIGURE 7: Spanwise profiles of the normalised streamwise velocity at $x/D = 2, 4$ and 6, comparing full-scale experimental data with tuned models at a yaw angle of zero degrees. The solid vertical grey lines indicate $U/U_\infty = 0.25$.

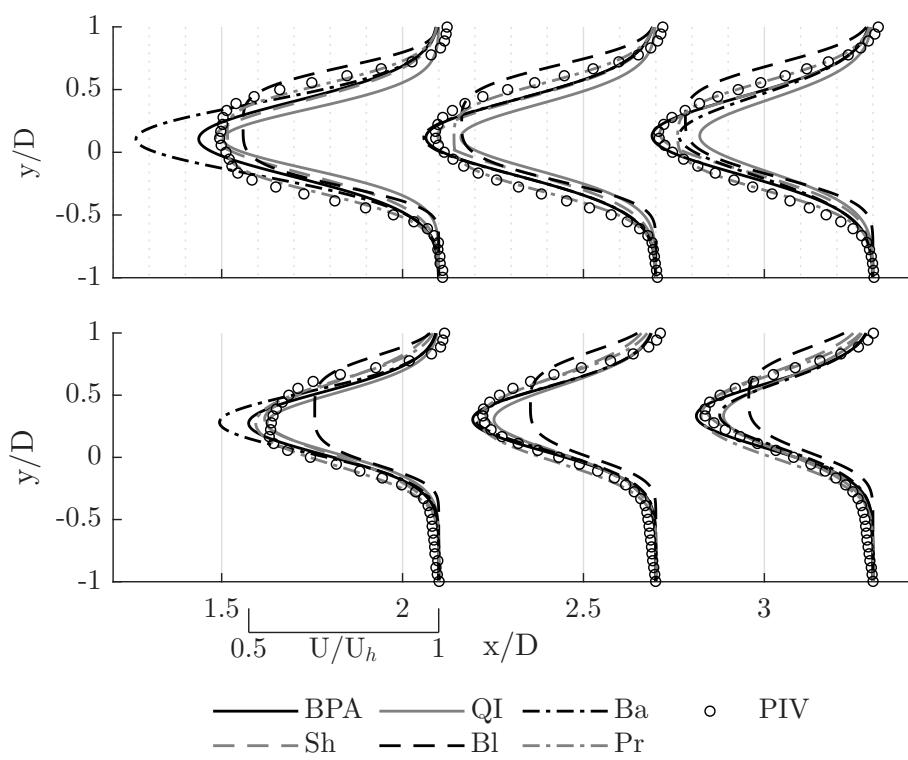


FIGURE 8: Spanwise profiles of normalised streamwise velocity at $x/D = 2.1, 2.7$ and 3.3 , comparing near-wake experimental data with tuned models at (Top) $\gamma = 10^\circ$, and (Bottom) $\gamma = 30^\circ$. The solid vertical grey line marks $U/U_\infty = 0.5$.

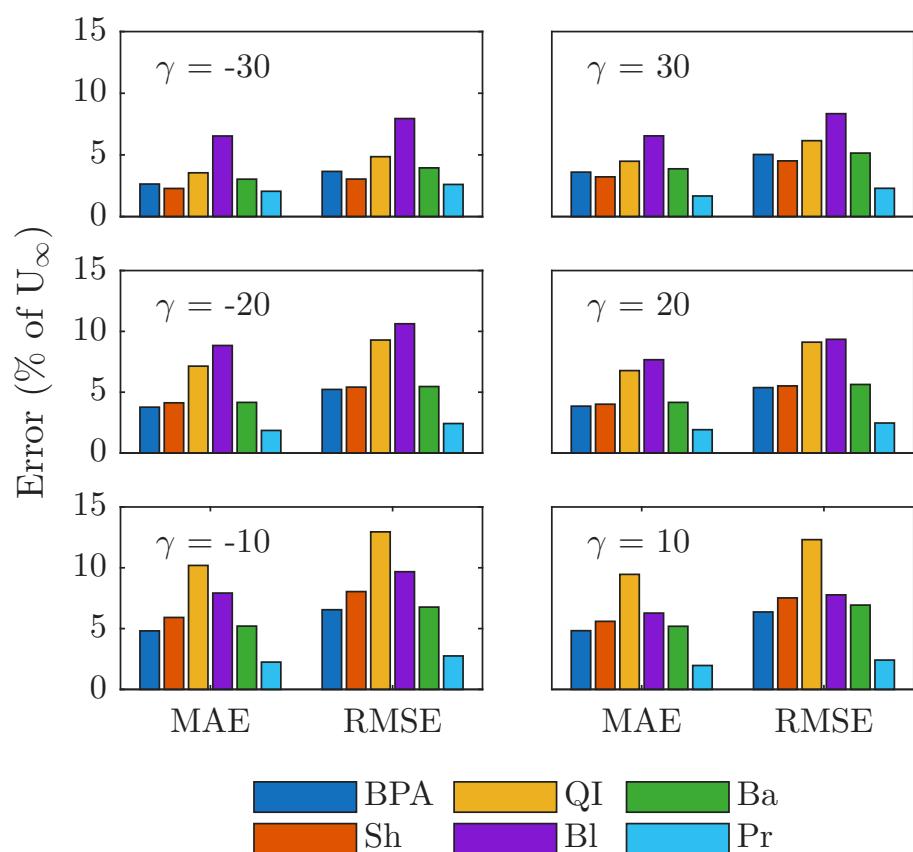


FIGURE 9: MAE and RMSE, expressed as percentages of the free-stream velocity, for each analytical model compared with near-wake velocity data at yaw angles ranging from $\gamma = -30^\circ$ to $\gamma = 30^\circ$

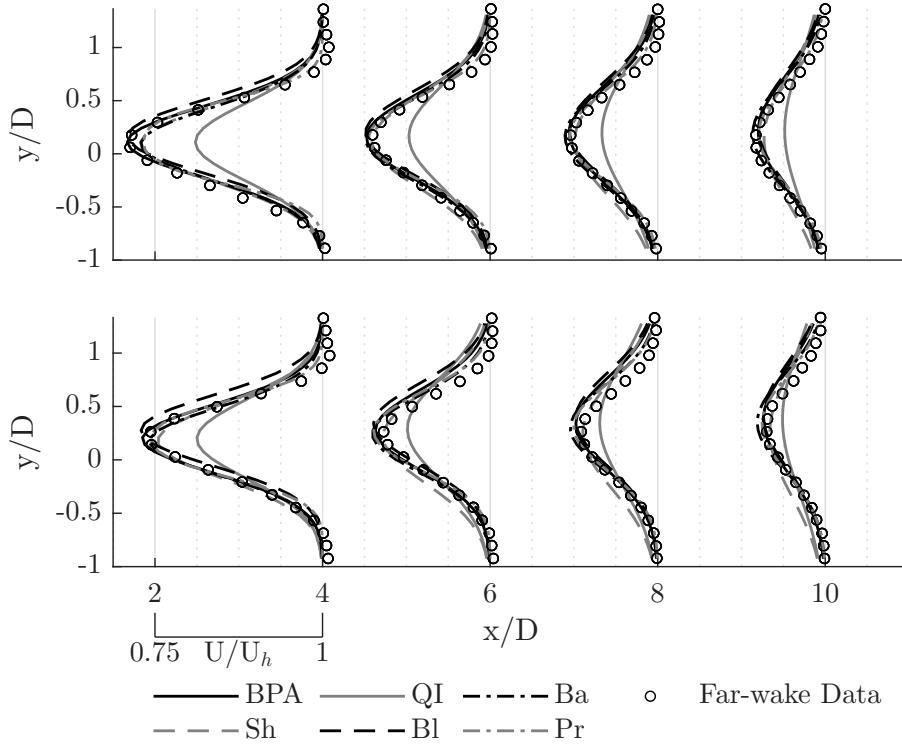


FIGURE 10: Spanwise profiles of normalised streamwise velocity from $x/D = 4$ to 11, comparing far-wake experimental data with tuned models at (Top) $\gamma = 10^\circ$, and (Bottom) $\gamma = 20^\circ$. The solid vertical grey line marks $U/U_\infty = 0.25$.

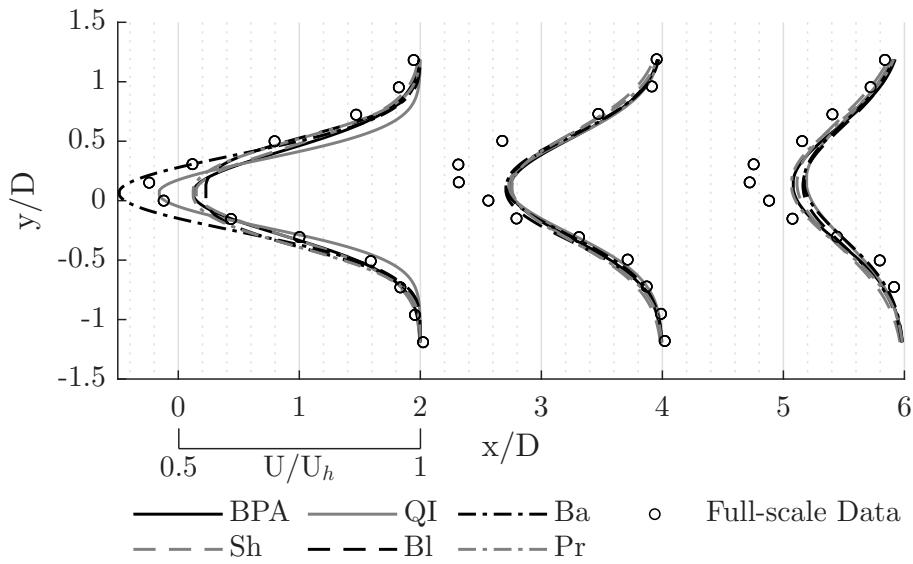


FIGURE 11: Spanwise profiles of normalised streamwise velocity at $x/D = 2, 4$ and 6, comparing full-scale experimental data with tuned models at $\gamma = 11^\circ$. The solid vertical grey line marks $U/U_\infty = 0.25$.

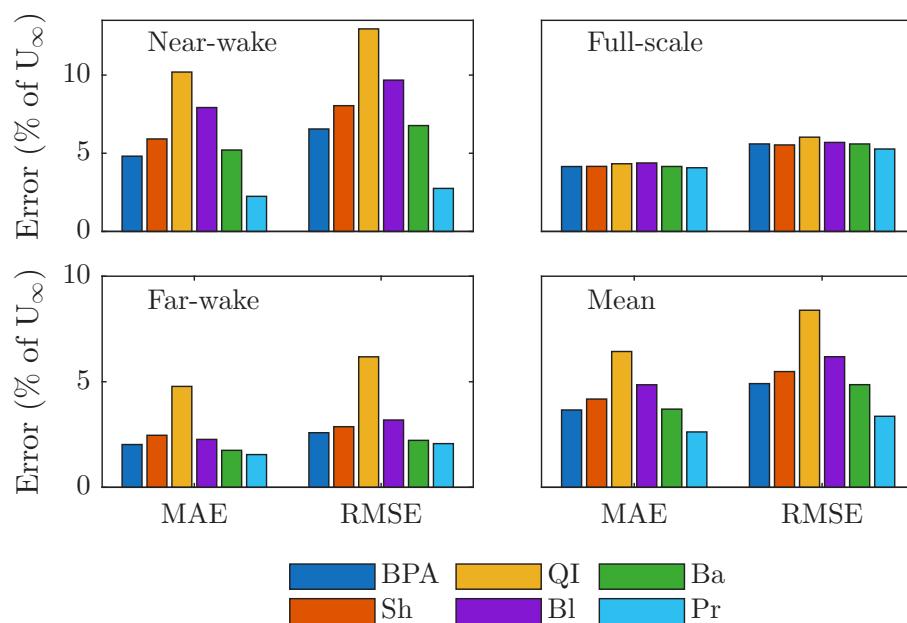


FIGURE 12: MAE and RMSE, expressed as percentages of the free-stream velocity, for each analytical model compared with near-wake ($\gamma = 10^\circ$), far-wake ($\gamma = 10^\circ$), and full-scale ($\gamma = 11^\circ$) velocity data, along with the mean of all three datasets.

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