



Improved modeling of flow curvature effects and actuator line method with aerodynamic moment, with application to vertical-axis turbines

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Abstract. We revisit the modeling, using actuator line methods (ALM), of flow-curvature effects on airfoils and their application to vertical-axis turbines (VAT); these effects being important when the ratio between the airfoil chord c and the arm length R is not small. The modeling can only use as input the aerodynamic coefficients of the airfoil in uniform flow and as function of the angle of attack (obtained here using wall-resolved CFD simulations). It then consists of analytical modifications of those coefficients. The models for the normal force and moment coefficients are based on the analogy with potential flow with curved streamlines past an airfoil; they are not new, although many authors have neglected the contribution of the aerodynamic moment. Their expression depends on both the airfoil pitch angle and the location of its attachment to the arm, and we develop and validate models that cover all possibilities, up to angles near stall. The model for the tangential force is new (the analogy with that of potential flow being flawed): its inviscid part correspond to thrust (i.e; negative drag) and is required to compensate for the aerodynamic moment.

The correction models are first validated against CFD data, obtained using wall-resolved simulations, of a steady flow: a rotating NACA0015 airfoil with $\frac{c}{R} = \frac{2}{7}$ at various pitch angles and up to stall; also for two attachment points of the airfoil to the arm: at mid-chord and at quarter-chord.

The ALM with the improved models is then also implemented in the CFD framework, and is used to simulate the unsteady flow corresponding to a VAT configuration: the rotating NACA0015 airfoil without pitch placed in a free stream and operating at optimal tip speed ratio, and simulated for both attachment points. For that, a novel method is also developed to explicitly enforce the moment in the ALM. The various components of the ALM results are compared with those of the reference CFD data, and are found to be in good agreement throughout the rotation cycle.

1 Introduction

The pursuit of sustainable development encourages the development of environmentally responsible technologies. One such innovation is the vertical-axis turbine (VAT), which allows for the extraction of energy from wind, river and tidal currents.



This study specifically focuses on this technology. As highlighted in several studies (Paraschivoiu, 2002; De Tavernier et al., 2022), VATs offer several advantages, including the ability to generate energy regardless of flow direction, ease of installation, and adaptability to various environments. This makes them a reliable and consistent energy source. Furthermore, Dabiri (2011) demonstrated that VATs can increase the energy density of turbine farms, further enhancing their potential to improve overall efficiency.

A precise approach for predicting and analyzing VAT performances is through high-fidelity numerical simulations, such as time-accurate wall-resolved (WR) simulations. These simulations can enable turbine optimization with parametric studies and design guideline recommendations (Balduzzi et al., 2021). For instance, Villeneuve et al. (2020) demonstrated that incorporating detached end-plates significantly enhances the efficiency of VATs, while a follow-up study (Villeneuve et al., 2021) further improved VATs efficiency by modifying blade support structures. However, WR simulations are computationally expensive, as they require resolving the flow within the boundary layers. This region requires fine computational cells to accurately capture the strong gradients. One approach to reducing computational costs is to use a moderate-fidelity approach. The Actuator Line Method (ALM), which has been widely applied in rotorcraft studies (Merabet and Laurendeau, 2022) and horizontal-axis turbines (Churchfield et al., 2017), is a moderate fidelity approach that enables realistic predictions of aerodynamic performance (Trigaux et al., 2024a) and accurately replicates wake behavior (Richard et al., 2018). Originally introduced by Shen et al. (2002, 2009), the ALM replaces the body-meshed lifting surfaces with equivalent volumetric forces in the Navier-Stokes equations. These forces are determined for each blade section by estimating the local upstream velocity and angle of attack, then applying precomputed aerodynamic airfoil coefficients to compute the corresponding loads and moment. Thus, this method allows for bypassing the resolution of the boundary layers on the lifting surface and significantly reduces the required mesh resolution and overall computational cost while still maintaining realistic results. Consequently, once the ALM has been suitably adapted to VATs, it becomes particularly valuable for performing multi-turbine cluster simulations, which would be unrealistic using WR simulations (Mohamed et al., 2024).

Recently, several ALM models have been specifically developed for VATs (Bachant et al., 22 September 2018; Melani et al., 2019; Zhao et al., 2020; Melani et al., 2021). All these studies emphasize the importance of considering flow curvature effects in the calculation of volumetric forces.

1.1 Corrections for flow curvature

As mentioned in the work of Mohamed et al. (2022), the primary challenge of VAT modeling using the ALM lies in the quality of the airfoil's aerodynamic coefficient data. Since the blades of a VAT operate in a curved flow, the curvature of the streamlines modifies the aerodynamic coefficients of the airfoil compared to those in uniform flow. As a result, predicting these aerodynamic coefficients in curved flow becomes the main challenge. The effects of flow curvature were first studied by Migliore et al. (1980). In their work, they explore these effects using the concepts of “virtual camber” and “virtual incidence”. They demonstrated that, concerning the lift and the moment, the aerodynamics of a symmetric airfoil in curved flow can be treated similarly to the aerodynamics of a cambered airfoil with a non-zero incidence in uniform flow. This conclusion was reached through a purely geometric transformation, which modifies the airfoil profile by converting the curved flow into a



uniform flow, thereby unfolding the curved streamlines while preserving the local angles between the streamlines and the airfoil surface. Rainbird et al. (2015) and Bianchini et al. (2016) demonstrated the effects of virtual camber and virtual incidence by comparing the experimental results of an airfoil in uniform flow, and with a camber similar to the virtual camber of a symmetric airfoil in curved flow, with the CFD results of a symmetric airfoil in rotation.

60 Since flow curvature effects are important when the ratio between blade chord c and arm length R is not small, and since the ALM relies on accurate aerodynamic coefficients to replicate the forces and moments exerted by the blade on the flow, it is essential that the modeled aerodynamic coefficients used in the ALM accurately account for the effects of flow curvature.

As stated in Section 1.1, the curvature effects modify the aerodynamic coefficients; also the aerodynamic moment coefficient. For symmetric airfoil in uniform flow, the ALM does not need to account for the moment around the aerodynamic center, x_{ac} ,
65 as it is essentially zero (since x_{ac} coincides closely with the center of pressure). However, in curved flow, the moment m_{ac} is no longer negligible (Ruiz-Husmann et al., 2023). The ALM must therefore also be able to impose the aerodynamic moment to account for its effects on the flow. We will hence further develop a novel method for imposing a moment in the ALM, in addition to imposing forces.

2 Objectives and outline

70 This paper focuses on developing and validating improved models to accurately account for flow curvature effects on airfoils. The modeling approach can only use the aerodynamic coefficients as a function of the angle of attack (for lift, drag and moment), as obtained when considering the airfoil in a uniform flow. We here consider a NACA0015 airfoil operating at a high Reynolds number of $Re_c = 6.0 \times 10^6$, and the aerodynamic coefficients are obtained using wall-resolved CFD, up to the stall angle. The curves obtained are then also fitted using polynomials; see Appendix A.

75 The correction models are used to adapt the aerodynamic coefficients obtained in uniform flow to the curved flow conditions. Thus, the ALM user only needs to apply the corrections to usual aerodynamic coefficient data, by adjusting them to the flow curvature based on the dimensions of the modeled VAT (the critical ratio being c/R) and on the attachment point of the airfoil to the arm.

The models for the normal force and moment are based on the analogy with the potential flow past an airfoil with curved
80 streamlines; they have long been known, although many authors have neglected the contribution of the aerodynamic moment. Moreover, the model for the normal force is further improved here to remain valid for a wide range of angles, up to stall. The model for the tangential force is new; here, the analogy with a curved potential flow is shown to be flawed (i.e., the correct tangential force does not correspond to adding the parasitic drag to the potential flow prediction).

The models for the forces and the moment are developed in Section 3, also for various attachment points of the blade to the
85 arm: at mid-chord in Section 3.3 and at quarter chord in Section 3.4. For each case, the validity of the models is then also assessed by comparing the model predictions to the reference results of CFD for a rotating NACA0015 airfoil with $c/R = 2/7$, put at various pitch angles; and here up to the two stall angles (which are indeed different due to flow curvature). The models are then further extended to an arbitrary location of the attachment point in Section 3.5.



The actuator line method (ALM) used to impose forces in the unsteady flow solver is then presented in Section 4. A new
90 method for also imposing the aerodynamic moment into the ALM is then developed in Section 5.

The improved flow curvature correction models are then used in actual ALM simulations of a simplified vertical-axis turbine
(VAT) configuration in Section 6; for the purpose of further validating the adapted ALM in a more complex and unsteady flow.
The case considered is that previously studied by Villeneuve et al. (2021): a single-blade VAT configuration with $c/R = 2/7$
operating at $TSR = 3.25$ (corresponding to its maximum power production). Two cases are considered: Section 6.1 for the
95 blade attached at quarter-chord and with the ALM centered there; Section 6.2 for the blade attached at mid-chord and for two
sub-cases: ALM centered at quarter-chord and ALM centered at mid-chord. For each case, the predictions obtained using the
adapted ALM are validated against those of corresponding reference CFD simulation.

3 Corrections of flow curvature effects for various blade attachment points

We consider a blade section of a VAT that corresponds to an aerodynamic profile without camber. Its chord is noted c and
100 the arm that links the blade to the mast has a length noted R . The airfoil is attached to the arm at some distance x_p from its
leading edge. The case where the attachment point is at mid-chord corresponds to $x_p = c/2$; that where the attachment point is
at quarter-chord corresponds to $x_p = c/4$.

We consider cases where flow curvature effects are important. In particular, we compare the prediction of the simplified
models developed below against the reference results from wall-resolved CFD of the rotating airfoil at a high Reynolds number
105 of $Re_c = 6.0 \times 10^6$ and for a case with strong flow curvature effects corresponding to $c/R = 2/7$; those CFD simulations being
performed in the rotating frame.

3.1 Curved potential flow past a flat plate

We consider a (x, y) coordinate system with a flat plate of chord c and centered at the origin. The plate is tilted by an angle α_p
relative to the horizontal. To produce curved streamlines, we put a point vortex of circulation $\Gamma_0 < 0$ (i.e., inducing a clockwise
110 velocity field) below the plate, at the position $(0, -R)$ (i.e., at $z = -iR$). The exact solution for the curved potential flow past
the flat plate can be obtained analytically, and so can the force exerted by the flow on the plate: this is done in Appendix E.
The streamlines of the potential flow are shown in Fig. 1 for the case $c/R = 2/7$ and for different values of α_p . The Kutta-
Joukowski condition being satisfied, the streamline emanating from the trailing edge is tangent to it and the velocity is finite
there. The case with $\alpha_p = 0$ corresponds to a flow that is left/right symmetric; the normal force exerted by the flow acts inward.
115 The case with $\alpha_p = 4$ deg has a normal force close to zero. The case with $\alpha_p = 8$ deg has a normal force acting outward.

One can make an analogy between the potential flow and the flow past a flat plate that is attached at its center to an arm
of length R that rotates at the angular velocity Ω . The upstream velocity seen by the plate is $U = (\Omega R)$ and its analog is the
velocity $U = \frac{\Gamma_0}{2\pi R}$ induced by the vortex in the potential flow model; this sets $\Gamma_0 = -2\pi \Omega R^2$.

However, it must be stressed that the analogy is limited. The potential flow being irrotational, the velocity induced by the
120 vortex decreases when R increases; for the plate in solid body rotation, the velocity increases with increasing R . In particular,

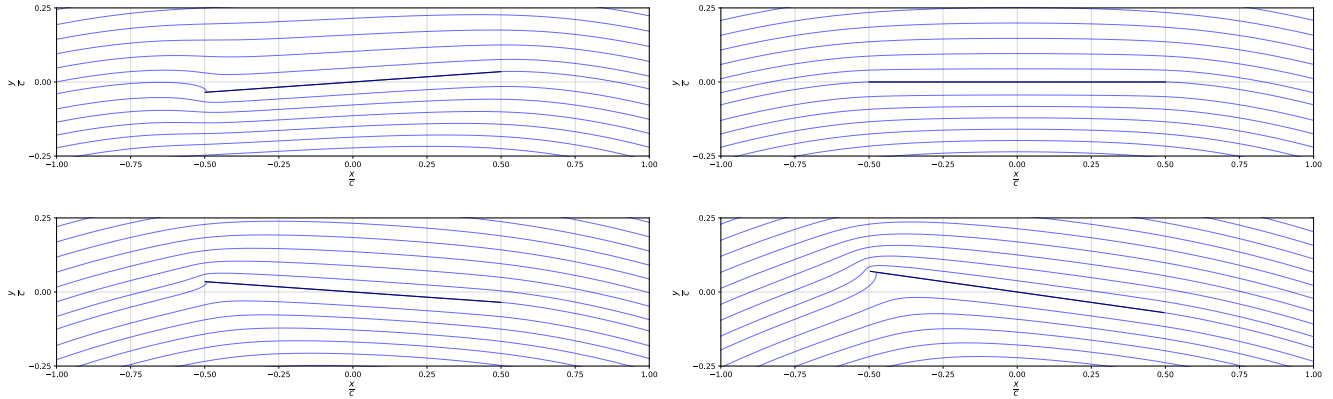


Figure 1. Streamlines (blue) of curved potential flows past a flat plate (black) with $c/R = 2/7$, and for various tilt angles: $\alpha_p = -4, 0, 4$ and 8 deg.

the analogy can only be used to obtain a model for the normal force f_n exerted by the flow on the plate, and for the moment around its aerodynamic center, m_{ac} ; and also for moderate tilt angle of the plate. Properly modeling the tangential force f_t will require another argument that is not compatible with potential flow theory, even for inviscid flows, and that will be developed in the paper.

125 When considering airfoils (thus with some thickness) instead of plates, the models for f_n and m_{ac} based on the analogy with potential flow will also be acceptable if all points of the airfoil remain such that their distance to the airfoil center is small compared to the arm length R .

3.2 Transformation used for developing the simplified model

The complex potential due to the vortex is

$$130 \quad f(z) = \phi(z) + i\psi(z) = -i \frac{\Gamma_0}{2\pi} \log\left(\frac{z + iR}{R}\right) = iUR \log\left(\frac{z}{R} + i\right). \quad (1)$$

As it is defined up to an arbitrary constant, we can subtract $\log(i) = \pi/2$ and also write

$$f(z) = iUR \log\left(1 - i \frac{z}{R}\right). \quad (2)$$

We can then consider the conformal transformation used in Akimoto et al. (2013):

$$Z(z) = iR \log\left(1 - i \frac{z}{R}\right) \quad \Leftrightarrow \quad z(Z) = iR \left(e^{-i \frac{Z}{R}} - 1\right). \quad (3)$$

135 Now, for z points with $\frac{|z|}{R}$ small, which is the case when $\frac{c/2}{R} \ll 1$, each conformal transformation can be approximated using the first term in its Taylor series:

$$Z(z) \simeq z + \frac{i}{2} \frac{z^2}{R} \quad \Leftrightarrow \quad z(Z) \simeq Z - \frac{i}{2} \frac{Z^2}{R}. \quad (4)$$



Those simplified transformations are those retained for developing the correction models accounting for flow curvature effects.

The flat plate in the z -plane is transformed into a curved plate in the Z -plane, and the potential $f(z)$ for the curved flow past the flat plate can be approximated from the potential $F(Z)$ of a uniform flow past the cambered plate: this is done by writing that $f(z) = F(Z)$ for $z = z(Z)$. We stress that this approximated potential flow is not the exact one (the exact solution for the curved flow past the flat plate is that developed in Appendix E). Nevertheless, we will see that the simplified potential flow analogy already allows to model quite well the effects of flow curvature on the force normal to the arm and on the aerodynamic moment. However, it completely fails to model the force tangential to the arm. The proper model for that tangential force will be obtained by considering the rotating airfoil attached to the arm and the net moment at the center of rotation (which should be zero in inviscid flow).

3.3 Flat plate or thin airfoil attached at its mid-chord ($x_p = c/2$)

3.3.1 Flat plate without pitch angle in an inviscid flow

We first consider a flat plate of chord c in the z -plane, which is centered at $z_c = 0$ and is also attached to the arm there: $z_p = 0$ (which corresponds to the case $x_p = c/2$). The plate is horizontal: no pitch angle ($\alpha_p = 0$): see Figure 2 (a).

Using the transformation $Z(z) = z + \frac{i}{2} \frac{z^2}{R}$, the plate is transformed into a horizontal and symmetric cambered plate in the Z plane: the attachment point $z_p = z_c = 0$ is transformed into $Z_p = Z_c = 0$, the trailing edge $z_T = c/2$ is transformed into $Z_T = c/2 + i c^2/(8R)$ and the leading edge $z_L = -c/2$ is transformed into $Z_L = -c/2 + i c^2/(8R)$; see Figure 2 (b).

Using the approximate potential flow discussed above, the normal force, f_n , exerted by the flow U on the cambered plate is obtained as

$$C_n(0) = -2\pi \frac{c}{4R}. \quad (5)$$

The force $f_n(0)$ is negative, thus oriented inward.

On the other hand, we also have the exact value of $C_n(0)$ obtained from the exact potential flow presented in Appendix E, with the final result of Eq. (E25). The amplitude of $C_n(0)$ is thus a little bit less than $2\pi \frac{c}{4R}$. Nevertheless: when $(\frac{c}{2R})^2 \ll 1$, which is most often the case and is the one considered in this paper, we can neglect this effect. We then simply associate the flow curvature effect to an effective shift of the angle of attack, by the angle α_c defined as

$$\sin(\alpha_c) = \frac{c}{4R} \quad \Rightarrow \quad C_n(0) = -2\pi \sin(\alpha_c). \quad (6)$$

The aerodynamic moment $m_{c/2}(0)$ acting at the center of the plate (i.e. at the mid-chord point) is here zero, by left/right symmetry of the flow around the plate; in the same way as for the potential flow developed in Appendix E and shown in Fig. 1.

For an inviscid flow, it would take no energy to rotate the airfoil attached to the arm: the moment m_0 around the center of rotation of the arm would therefore be zero. We stress that this is also confirmed in Appendix D by performing an inviscid

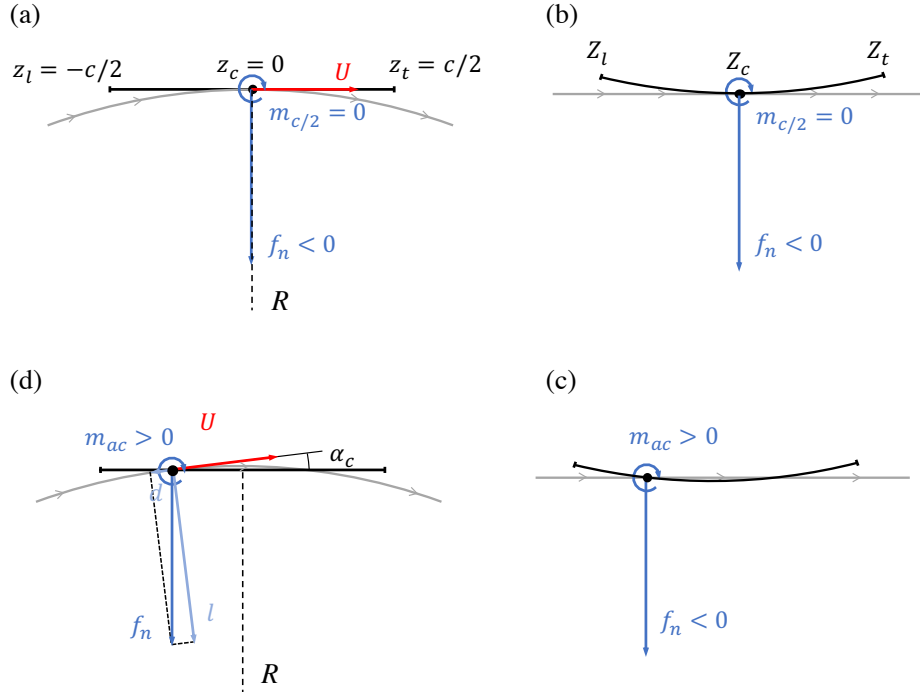


Figure 2. (a) and (d): Schematic of a flat plate attached at $x_p = c/2$ without pitch angle in the z -plane (solid black) and a background flow with curved streamlines (light grey). (b) and (c): transformed cambered plate in the Z -plane in a uniform background flow. Note: the flow curvature was here exaggerated for clarity, using a too large value of $c/R = 1/2$.

simulation of the flow past a rotating airfoil. As this moment is equal to $m_0 = m_{c/2}(0) + R f_t(0)$, and since $m_{c/2}(0) = 0$, the tangential force exerted on the plate by an inviscid flow will also be zero: $f_t(0) = 0$ and hence $C_t(0) = 0$. For a viscous flow, there will, of course, be some parasitic drag, and thus a value of $C_t(0) > 0$; we will add this effect later, in Section 3.3.3.

We already stress that, in an inviscid flow, the tangential force will no longer be zero when the plate is pitched by some angle α_p , because $m_{c/2}$ will then be non zero; see Section 3.3.2.

We also obtain the moment at the location of the aerodynamic center (which is located at a distance $c/4$ from the leading edge): since $m_{c/2}(0) = 0$, we obtain $m_{ac}(0) = m_{c/4}(0) = -(c/4) f_n(0) > 0$, see Figure 2 (c) and (d). The associated moment coefficient is thus

$$C_{m_{ac}}(0) = C_{m_{c/4}}(0) = \frac{m_{c/4}(0)}{\frac{1}{2} \rho U^2 c^2} = -\frac{C_n(0)}{4} \simeq \frac{\pi}{2} \sin(\alpha_c) \simeq \frac{\pi}{2} \frac{c}{4R}. \quad (7)$$

We note that the angle $\alpha_p = 0$ considered here is the geometrical pitch angle of the profile attached at its mid-point relatively to the horizontal; it is not the angle that the profile makes with the curved flow streamline at the location of the aerodynamic center; and that is used as the “effective angle of attack” α in an actuator line method (ALM) when the line is centered at



the aerodynamic center. At that location, the curved streamline makes an angle of $\alpha = \frac{c}{4R}$ relative to the horizontal plate (see Figure 2 (d)); thus an angle also equal to α_c .

185 The usual aerodynamic coefficients relative to the curved flow velocity vector considered at the location of the aerodynamic center are easily obtained using the angle α_c of the local velocity vector relative to the plate:

$$\begin{aligned} C_l &= C_n(0) \cos(\alpha_c) , \\ C_d &= C_n(0) \sin(\alpha_c) . \end{aligned} \quad (8)$$

As $C_n(0)$ is negative, both C_l and C_d are negative; as shown in Figure 2 (d).

3.3.2 Flat plate with pitch angle in an inviscid flow

190 We now further pitch the flat plate at a moderate angle α_p relative to the horizontal, as shown in Figure 3 (a). The normal force coefficient will be modeled using

$$C_n(\alpha_p) \simeq 2\pi \sin(\alpha_p) - 2\pi \sin(\alpha_c) . \quad (9)$$

We note that this formula is slightly different from the formula inspired from the potential flow theory past a cambered plate, and which would be

$$195 \quad C_n(\alpha_p) \simeq 2\pi \sin(\alpha_p - \alpha_c) . \quad (10)$$

Obviously, both formulas provide results that are very close for small α_c and moderate $\alpha_p - \alpha_c$; they both also provide $C_n = 0$ when $\alpha_p = \alpha_c$. The reason why we will use Eq. (9) instead of Eq. (10) will become clear when we further extend the model to thin airfoils in real viscous flows, and up to large pitch angles; even up to the stall of the airfoil, and compare the results to those of reference CFD results.

200 For $\alpha_p < \alpha_c$, $C_n < 0$ and the normal force is oriented inward; as in Figure 3 (a); for $\alpha_p > \alpha_c$, $C_n > 0$ and the normal force is oriented outward.

When $\alpha_p \neq 0$, the flow around the plate is no longer left-right symmetric (as for the potential flow shown in Fig. 1). The moment $m_{c/2}$ around the center of the plate is, therefore, non-zero.

205 For the model, we use the prediction for a potential flow past the tilted cambered plate put in an horizontal stream at velocity U :

$$C_{m_{c/2}}(\alpha_p) = \frac{m_{c/2}}{\frac{1}{2}\rho U^2 c^2} = \frac{\pi}{4} \sin(2\alpha_p) = \frac{\pi}{2} \sin(\alpha_p) \cos(\alpha_p) . \quad (11)$$

The moment around the aerodynamic center is hence obtained as $m_{c/4}(\alpha_p) = m_{c/2}(\alpha_p) - (\cos(\alpha_p) c/4) f_n(\alpha_p)$, and thus the associated moment coefficient is

$$210 \quad C_{m_{c/4}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \frac{C_n(\alpha_p)}{4} \cos(\alpha_p) \simeq \frac{\pi}{2} \sin(\alpha_c) \cos(\alpha_p) . \quad (12)$$

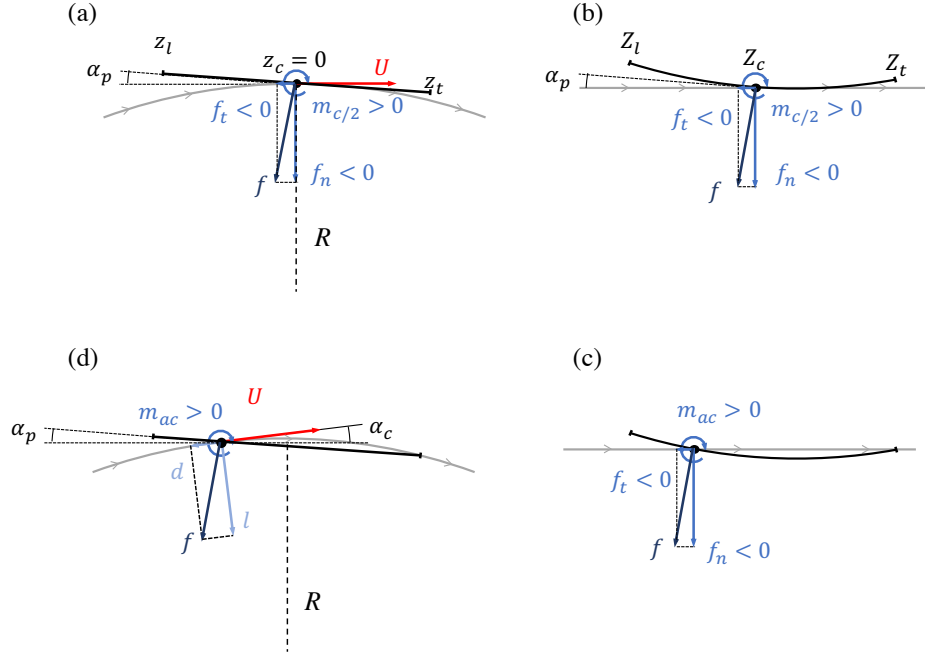


Figure 3. (a) and (d): Schematic of a flat plate attached at $x_p = c/2$ with a moderate pitch angle $0 < \alpha_p < \alpha_c$ in the z -plane (solid black) and a background flow with curved streamlines (light grey). (b) and (c): transformed tilted cambered plate in the Z -plane in a uniform background flow. Note: the flow curvature was here exaggerated for clarity, using a too large value of $c/R = 1/2$.

As the moment m_0 around the center of rotation of the arm must still be zero in an inviscid flow (see also Appendix D for a numerical confirmation of that), there must now also be a net tangential force component f_t^i due solely to the pressure distribution, and which is such that

$$m_0 = m_{c/2}(\alpha_p) + R f_t^i(\alpha_p) = 0. \quad (13)$$

215 The model for the tangential force coefficient in inviscid flow is hence obtained as

$$C_t^i(\alpha_p) = -\frac{c}{R} C_{m_{c/2}}(\alpha_p) \simeq -4 \sin(\alpha_c) C_{m_{c/2}}(\alpha_p). \quad (14)$$

Of course, there will also be a parasitic drag coefficient to add to the model for C_t when modeling viscous flow, see Section 3.3.3.

220 3.3.3 Thin airfoil with pitch angle in a real viscous flow

The correction models above, obtained assuming an inviscid flow and a flat plate, are also used for thin aerodynamic profiles without camber and in real flows: thus with the added parasitic effects associated to viscosity.



For the normal force coefficient, the model becomes:

$$C_n(\alpha_p) \simeq C_l^u(\alpha_p) - C_l^u(\alpha_c) \quad (15)$$

225 where $C_l^u(\beta)$ is the usual curve for the lift coefficient of the airfoil in a uniform flow as a function of its angle of attack β relative to that uniform flow. This curve is obtained from experiments or using CFD. In the present case, it was obtained using CFD, see Appendix A.

When the airfoil is not pitched, the moment at mid-chord, $C_{m_{c/2}}^0$, is small yet non-zero. It can be estimated by performing
 230 CFD of the airfoil deformed using the transformation $Z(z)$ and put in a uniform flow (for the deformed NACA0015 airfoil, we measure $C_{m_{c/2}}^0 \simeq -0.00746$: see Appendix C).

When the airfoil is pitched by an angle α_p , we then use, by similarity with the model obtained for the plate,

$$C_{m_{c/2}}(\alpha_p) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_p) \cos(\alpha_p). \quad (16)$$

We then obtain, for the moment around the quarter-chord point,

$$235 \quad C_{m_{c/4}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \frac{1}{4} C_n(\alpha_p) \cos(\alpha_p) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha_p). \quad (17)$$

In the same way, the moment around the true aerodynamic center, which is located at $\frac{x_{ac}}{c}$ very close to $\frac{1}{4}$ but not exactly there (for the NACA0015 airfoil, we measure $\frac{x_{ac}}{c} \simeq 0.2476$: see Appendix A), is obtained as

$$C_{m_{ac}}(\alpha_p) \simeq C_{m_{c/2}}^0 + \left(\frac{1}{2} - \frac{x_{ac}}{c} \right) C_l^u(\alpha_c) \cos(\alpha_p), \quad (18)$$

To model the tangential force coefficient C_t , we add to the inviscid flow contribution, $C_t^i = -\frac{c}{R} C_{m_{c/2}}$, the parasitic con-
 240 tribution C_t^ν associated with the viscous effects; i.e., due to the presence of boundary layers and their separation, resulting in pressure drag and friction drag:

$$\begin{aligned} C_t(\alpha_p) &= C_t^\nu(\alpha_p) + C_t^i(\alpha_p) \\ &= (C_t^{\nu,p}(\alpha_p) + C_t^i(\alpha_p)) + C_t^{\nu,f}(\alpha_p) = C_t^p(\alpha_p) + C_t^{\nu,f}(\alpha_p) \\ &\simeq \left(C_d^{u,p}(\alpha_p) - \frac{c}{R} C_{m_{c/2}}(\alpha_p) \right) + C_d^{u,f}(\alpha_p + \alpha_c). \end{aligned} \quad (19)$$

245 The term $C_t^{\nu,p} + C_t^i$ in parentheses represents the complete pressure component: we note it C_t^p .

Note that we modeled $C_t^{\nu,f}(\alpha_p)$ using $C_d^{u,f}(\alpha_p + \alpha_c)$ with $C_d^{u,f}(\beta)$ the curve for the fraction due to wall friction of the drag coefficient for the airfoil in uniform flow as a function of its angle of attack β (see Appendix A). This choice is motivated by the fact that, in the reference CFD of the rotating airfoil, $C_t^{\nu,f}$ is seen to be maximum when $\alpha_p \simeq -\alpha_c$; and not when $\alpha_p \simeq 0$. As to $C_t^p = C_t^{\nu,p} + C_t^i$, it is found, in the CFD, to be symmetric relative to $\alpha_p \simeq 0$, and hence we model $C_t^{\nu,p}(\alpha_p)$ using $C_d^{u,p}(\alpha_p)$.

250 The model for the parasitic drag is thus taken as

$$C_t^\nu(\alpha_p) = C_t^{\nu,p}(\alpha_p) + C_t^{\nu,f}(\alpha_p) \simeq C_d^{u,p}(\alpha_p) + C_d^{u,f}(\alpha_p + \alpha_c). \quad (20)$$



We also stress an important point: if the moment $m_{c/2}$ is not enforced in the ALM (using the procedure developed in Section 5), then only the parasitic tangential force f_t^ν must be enforced in the ALM, while f_t^i must be ignored.

255 Finally, the force coefficients can also be decomposed into the usual aerodynamic coefficients considered at $c/4$ by using the angle of attack $\alpha_p + \alpha_c$ that the curved flow velocity vector makes there relative to the airfoil chord:

$$\begin{aligned} C_l &= C_n(\alpha_p) \cos(\alpha_c) - C_t(\alpha_p) \sin(\alpha_c), \\ C_d &= C_n(\alpha_p) \sin(\alpha_c) + C_t(\alpha_p) \cos(\alpha_c). \end{aligned} \quad (21)$$

3.3.4 Comparison of the model predictions against CFD data of a rotating NACA0015 airfoil

260 We test the curvature correction models proposed above for a case with significant curvature effects (with $c/R = 2/7$) and for a NACA0015 airfoil (hence an airfoil that is not very thin) attached at $x_p = c/2$. We then obtain $\sin(\alpha_c) = 1/14 \simeq 0.07143$ and $\alpha_c = \arcsin(1/14) \simeq 0.07149$ rad (thus $\simeq 4.096$ deg).

We first consider the case $\alpha_p = 0$. Using the lift curve of the NACA0015, the prediction is $C_n \simeq -C_l^u(\alpha_c) \simeq -0.454$ ($C_n \simeq -2\pi(1/14) \simeq -0.449$ when using the flat plate model); to compare with $C_n \simeq -0.431$ measured in CFD of the rotating airfoil.

265 For the flat plate, we have that $C_{m_{c/2}} = 0$ and thus $C_{m_{c/4}} = C_{m_{ac}} = -C_n/4 \simeq (\pi/2)(1/14) \simeq 0.1122$. For the NACA0015 airfoil, and if $C_{m_{c/2}}^0$ is neglected, the predictions are $C_{m_{c/4}} \simeq 0.1135$ and $C_{m_{ac}} \simeq 0.1146$. The values measured in CFD of the rotating airfoil are $C_{m_{c/2}} \simeq -0.0056$, $C_{m_{c/4}} \simeq 0.10225$ and $C_{m_{ac}} \simeq 0.10327$. When using $C_{m_{c/2}}^0$, the predictions are $C_{m_{c/4}} \simeq 0.1079$ and $C_{m_{ac}} \simeq 0.1090$: indeed better.

The values measured in CFD of the rotating airfoil are also $C_t^p \simeq 0.00387$, $C_t^{\nu,f} \simeq 0.00682$ and thus $C_t \simeq 0.01069$.

270 When using $C_{m_{c/2}}^0 \simeq -0.00746$ (obtained using CFD of the airfoil deformed using $Z(z)$ and put in a uniform flow, see Appendix C), the predictions become $C_t^i \simeq 0.00211$ and thus $C_t^p \simeq 0.00401$, which compares quite well with the value measured in CFD of the rotating airfoil. The model is indeed improved when we incorporate the small, yet non-zero, value of $C_{m_{c/2}}^0$.

We present in Figure 4 the predicted values for the aerodynamic coefficients. The range of moderate pitch angles corresponds to $-12 \leq \alpha_p \leq 12$ deg. We see that the predictions are quite accurate within that range. The comparison is also made for the complete range, up to the two stall angles (they are slightly different due to the flow curvature effects). We see that the model developed above also performs well up to both stall angles, and that the predicted values (of roughly -18 deg and 18 deg) are in fair agreement with those of the reference CFD (that are roughly -19 deg and 17 deg). We stress that this would not be the case if we had used

$$280 \quad C_n(\alpha_p) \simeq C_l^u(\alpha_p - \alpha_c), \quad (22)$$

as model for C_n instead of Eq. (15). Although both choices produce similar predictions for C_n , and hence also for all other coefficients, in the range of moderate tilt angles, using Eq. (22) would predict incorrect stall angles (at roughly $-18+4 = -14$ deg and $18+4 = 22$ deg: not in line with those of the reference CFD).

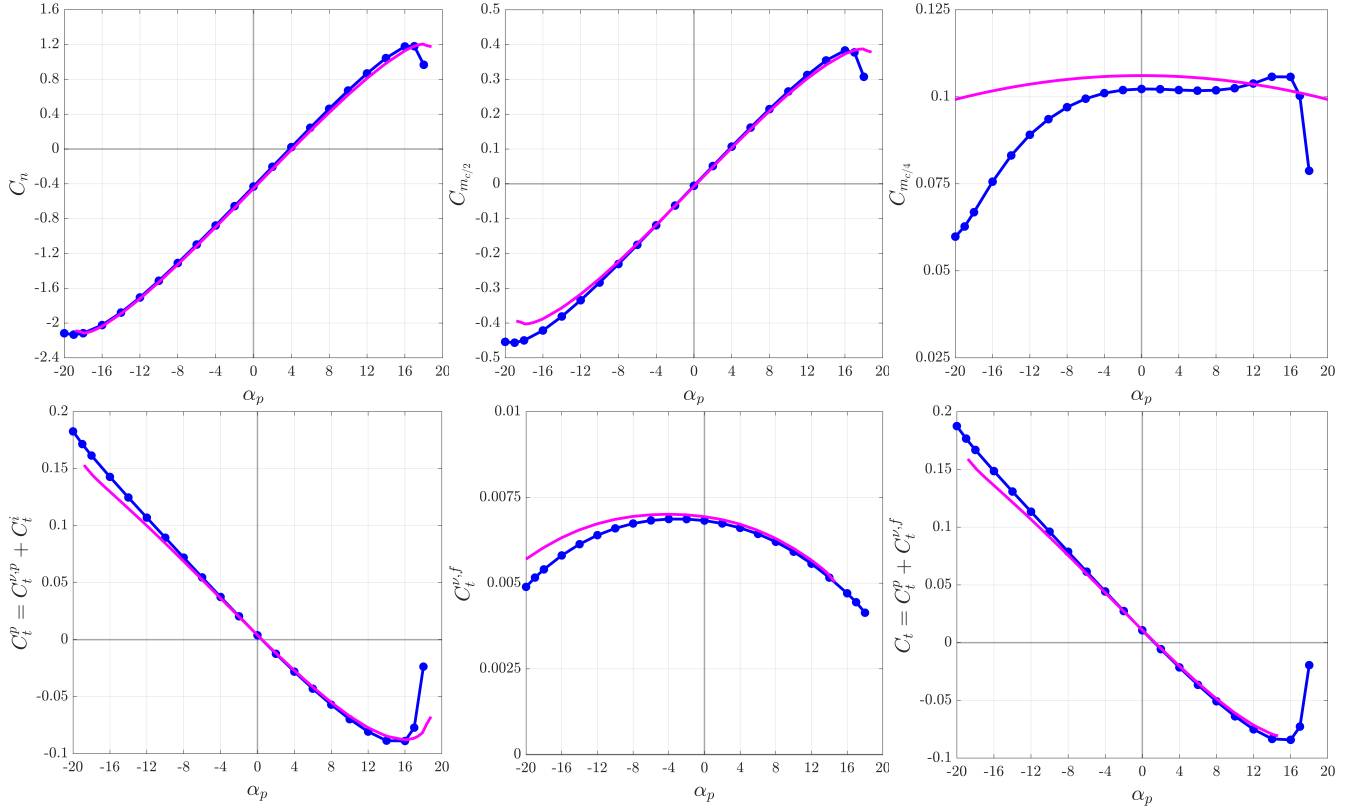


Figure 4. NACA0015 airfoil in rotation with $c/R = 2/7$ and attached at $x_p = c/2$: comparison between the model predicted values (magenta) of C_n , $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_t^p , $C_t^{\nu,f}$, C_t and the reference values obtained in CFD of the rotating airfoil (blue).

285 3.4 Flat plate or thin airfoil attached at its quarter-chord ($x_p = c/4$)

We consider next a flat plate of chord c in the z -plane, that is attached to the arm at $z_p = 0$ but has its center at $z_c = c/4$ (it corresponds to the case $x_p = c/4$). The plate is attached at its aerodynamic center, hence its pitch angle α_p is also the angle that the plate makes relative to the curved flow velocity vector at its aerodynamic center.

We first consider the case without pitch angle: $\alpha_p = 0$. The horizontal plate is transformed into a tilted and cambered plate in
 290 the Z plane: its attachment point $z_p = 0$ is transformed into $Z_p = 0$, its center $z_c = c/4$ is transformed into $c/4 + ic^2/(32R)$, its trailing edge $z_T = 3c/4$ is transformed into $Z_T = 3c/4 + i9c^2/(32R)$ and its leading edge $z_L = -c/4$ is transformed into $Z_L = -c/4 + ic^2/(32R)$; see Figure 5.

The slope of the tilted cambered plate at its center is then obtained as $(dY/dX)_c = c/(4R)$. This is the angle that the cambered plate makes relative to the uniform horizontal velocity U . We note that this angle also corresponds to α_c .

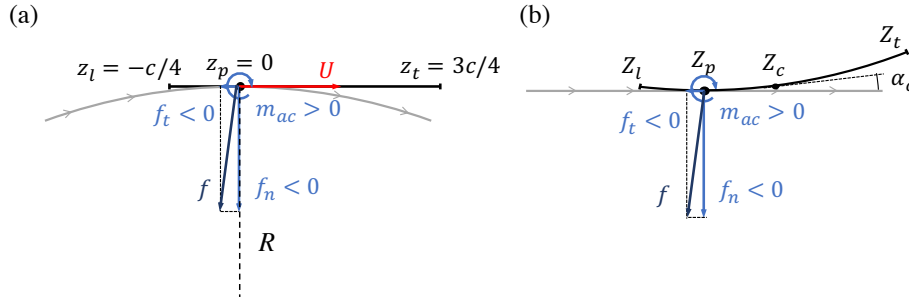


Figure 5. (a) and (b): Schematic of a flat plate attached at $x_p = c/4$ and without pitch angle in the z -plane (solid black) and a background flow with curved streamlines (light grey). (b) and (c): transformed tilted cambered plate in the Z -plane in a uniform background flow. Note: the flow curvature was here exaggerated for clarity, using a too large value of $c/R = 1/2$.

295 When $\alpha_p = 0$, the normal force coefficient is obtained as

$$C_n(0) \simeq 2\pi \sin(0 - \alpha_c) - 2\pi \sin(\alpha_c) = -4\pi \sin(\alpha_c). \quad (23)$$

When we pitch the flat plate by an angle of attack α_p , the model becomes

$$C_n(\alpha_p) \simeq 2\pi \sin(\alpha_p - \alpha_c) - 2\pi \sin(\alpha_c). \quad (24)$$

We stress again that this formula is not exactly that obtained from the potential flow theory for the curved plate in uniform flow
 300 and pitched by the angle $-\alpha_c$, and which would be

$$C_n(\alpha_p) \simeq 2\pi \sin(\alpha_p - 2\alpha_c). \quad (25)$$

Both formula provide results that are very close for moderate angles. The reason why we use Eq. (24) instead of Eq. (25) is again that it provides better results when we extend the model to thin airfoils in viscous flows and up to large angles, including stall; as will be shown in the next section.

305

The moment coefficient at the plate center is again obtained using the angle of attack $\alpha_p - \alpha_c$ of the cambered plate in the uniform flow:

$$C_{m_{c/2}}(\alpha_p) = \frac{\pi}{4} \sin(2(\alpha_p - \alpha_c)) = \frac{\pi}{2} \sin(\alpha_p - \alpha_c) \cos(\alpha_p - \alpha_c). \quad (26)$$

This moment is indeed zero, as it should, when $\alpha_p = \alpha_c$ as the cambered plate of the transformed flow is then horizontal. The
 310 moment coefficient around the aerodynamic center is hence also obtained as

$$C_{m_{c/4}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \frac{C_n(\alpha_p)}{4} \cos(\alpha_p - \alpha_c) \simeq \frac{\pi}{2} \sin(\alpha_c) \cos(\alpha_p - \alpha_c). \quad (27)$$

In an inviscid flow, the net pressure force is not purely normal: there must also be a net tangential component in order to compensate the moment at the attachment point, so that the moment around the center of the arm is zero, see Appendix D:

$$m_0 = m_{c/4} + R f_t^i = 0 \quad \Rightarrow \quad C_{m_{c/4}}(\alpha_p) + \frac{R}{c} C_t^i(\alpha_p) = 0. \quad (28)$$



315 The inviscid tangential force coefficient is thus obtained as

$$C_t^i(\alpha_p) = -\frac{c}{R} C_{m_{c/4}}(\alpha_p) \simeq -4 \sin(\alpha_c) \frac{\pi}{2} \sin(\alpha_c) \cos(\alpha_p - \alpha_c) = -2\pi \sin^2(\alpha_c) \cos(\alpha_p - \alpha_c). \quad (29)$$

The extension of the prediction models to thin airfoils in viscous flow is then also obtained as follows. For the lift coefficient, the model becomes

$$C_n(\alpha_p) \simeq C_l^u(\alpha_p - \alpha_c) - C_l^u(\alpha_c). \quad (30)$$

320 For the moment coefficient at mid chord, the model is obtained as

$$C_{m_{c/2}}(\alpha_p) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_p - \alpha_c) \cos(\alpha_p - \alpha_c). \quad (31)$$

The moment coefficients $C_{m_{c/4}}$ and $C_{m_{ac}}$ are then obtained as

$$C_{m_{c/4}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \frac{1}{4} C_n(\alpha_p) \cos(\alpha_p - \alpha_c) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha_p - \alpha_c) \quad (32)$$

and

$$325 \quad C_{m_{ac}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \left(\frac{1}{2} - \frac{x_{ac}}{c} \right) C_n(\alpha_p) \cos(\alpha_p - \alpha_c). \quad (33)$$

We note that the constant term $C_{m_{c/2}}^0$ is the same as that used previously when modeling an airfoil attached at $x_p = c/2$. Indeed, the airfoil attached at $x_p = c/4$ and with a pitch of $\alpha_p = \alpha_c$ must have the same moment as the airfoil attached at $x_p = c/2$ without pitch ($\alpha_p = 0$).

330 We now add the parasitic drag coefficient C_t^ν to the inviscid tangential pressure force coefficient $C_t^i = -\frac{c}{R} C_{m_{c/4}}$. The model is finally obtained as

$$\begin{aligned} C_t(\alpha_p) &= C_t^\nu(\alpha_p) + C_t^i(\alpha_p) \\ &= (C_t^{\nu,p}(\alpha_p) + C_t^i(\alpha_p)) + C_t^{\nu,f}(\alpha_p) = C_t^p(\alpha_p) + C_t^{\nu,f}(\alpha_p) \\ &\simeq \left(C_d^{u,p}(\alpha_p - \alpha_c) - \frac{c}{R} C_{m_{c/4}}(\alpha_p) \right) + C_d^{u,f}(\alpha_p). \end{aligned} \quad (34)$$

335 We modeled $C_t^{\nu,f}(\alpha_p)$ using $C_d^{u,f}(\alpha_p)$ instead of using $C_d^{u,f}(\alpha_p - \alpha_c)$ because the CFD data show that $C_t^{\nu,f}$ is maximum when $\alpha_p \simeq 0$. As to $C_t^p = C_t^{\nu,p} + C_t^i$, it is found, in the CFD, to be symmetric relative to $\alpha_p \simeq \alpha_c$, and hence we model $C_t^{\nu,p}(\alpha_p)$ using $C_d^{u,p}(\alpha_p - \alpha_c)$. The model for the parasitic drag is thus taken as

$$C_t^\nu(\alpha_p) = C_t^{\nu,p}(\alpha_p) + C_t^{\nu,f}(\alpha_p) \simeq C_d^{u,p}(\alpha_p - \alpha_c) + C_d^{u,f}(\alpha_p). \quad (35)$$

We again stress an important point: if the moment $m_{c/4}$ is not enforced in the ALM (using the procedure developed in
 340 Section 5), then only the parasitic tangential force f_t^ν must be enforced in the ALM, while f_t^i must be ignored.



3.4.1 Comparison of the model predictions against CFD data of a rotating NACA0015 airfoil

We test the curvature correction models proposed above for the same case with $c/R = 2/7$ and a NACA0015 airfoil, but this time attached to the arm at $x_p = c/4$.

For the case $\alpha_p = 0$, the prediction is $C_n \simeq -0.9079$ ($C_n \simeq -0.8976$ when using the flat plate model), to compare with
 345 $C_n \simeq -0.893$ measured in CFD of the rotating airfoil.

The predictions for the moment coefficients are $C_{m_{c/2}} \simeq -0.1132$ and $C_{m_{c/4}} \simeq 0.1132$, the values measured in CFD being
 $C_{m_{c/2}} \simeq -0.1222$ and $C_{m_{c/4}} \simeq 0.1010$.

We present in Figure 6 the predicted values for the aerodynamic coefficients compared to the reference CFD values. Here, the range of moderate pitch angles corresponds to $-12 + 4 = -8 \leq \alpha \leq 12 + 4 = 16$ deg (taking into account the shift by
 350 $\alpha_c \simeq 4$ deg due to the attachment point being at $x_p = c/4$ instead of $x_p = c/2$). We see that the predictions are quite accurate within that range. The comparison is also made for the complete range, up to the two stall angles (that are here much different due to the flow curvature effects combined with the shift in attachment). We see that the models developed above also perform fairly well up to both stall angles, and that the predicted values (of roughly $-18 + 4 = -14$ and $18 + 4 = 22$ are in fair agreement with those of the reference (that are -16 deg and $20 - 21$ deg). We stress again that this would not be the case if we had used

$$355 \quad C_n(\alpha_p) \simeq C_l^u(\alpha_p - 2\alpha_c) \quad (36)$$

instead of Eq. (30). Using Eq. (36) would provide similar results for moderate angles, but would predict incorrect stall angles (at roughly $-18 + 8 = -10$ deg and at $18 + 8 = 26$ deg).

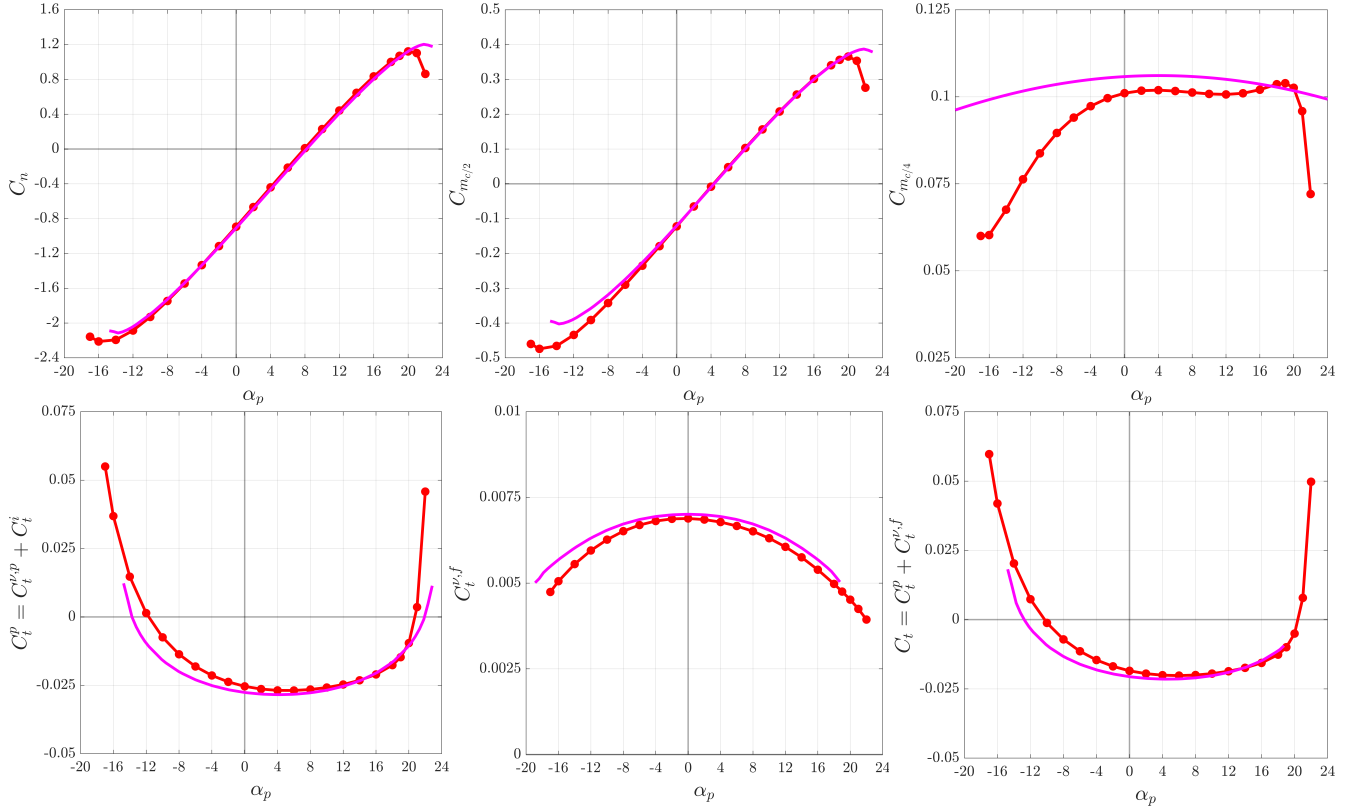


Figure 6. NACA0015 airfoil in rotation with $c/R = 2/7$ and attached at $x_p = c/4$: comparison between the predicted values (magenta) of C_n , $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_t^p , $C_t^{v,f}$, C_t and the reference values obtained in CFD of the rotating airfoil (red).

3.5 Flat plate or thin airfoil attached at an arbitrary position

The case in which the attachment point is arbitrarily chosen can also be modeled. We then write

$$x_p = (1 + \delta) \frac{c}{4} \quad (37)$$

where δ is a chosen parameter. The canonic cases $x_p = c/2$ and $x_p = c/4$ investigated and modeled so far correspond, respectively, to $\delta = 1$ and $\delta = 0$.

The opening angle, α_v , between the attachment point x_p and the airfoil center $c/2$ is obtained as

$$\alpha_v = \left(\frac{1}{2} - \frac{x_p}{c} \right) \frac{c}{R} = (1 - \delta) \frac{c}{4R} = (1 - \delta) \alpha_c. \quad (38)$$

365 We first consider the case of a flat plate without pitch angle ($\alpha_p = 0$). The horizontal plate is transformed into a tilted and cambered plate in the Z plane: its attachment point $z_p = 0$ is transformed into $Z_p = 0$, its center $z_c = (1 - \delta)c/4$ is transformed into $(1 - \delta)c/4 + i(1 - \delta)^2 c^2 / (32R)$, its trailing edge $z_T = (3 - \delta)c/4$ is transformed into $Z_T = (3 - \delta)c/4 + i(3 - \delta)^2 c^2 / (32R)$ and its leading edge $z_L = -(1 + \delta)c/4$ is transformed into $Z_L = -(1 + \delta)c/4 + i(1 + \delta)^2 c^2 / (32R)$. The slope of the tilted



cambered plate at its center is then obtained as

$$370 \quad \left(\frac{dY}{dX} \right)_{z_c} = (1 - \delta) \frac{c}{4R} = (1 - \delta) \alpha_c = \alpha_v. \quad (39)$$

The angle of the cambered plate relative to the uniform horizontal velocity U is therefore also equal to α_v .

When further pitching the flat plate by an angle α_p , the angle α that the curved flow velocity vector at $c/4$ makes relative to the plate is now

$$375 \quad \alpha = \alpha_p + (\alpha_c - \alpha_v) = \alpha_p + \delta \alpha_c. \quad (40)$$

We also obtain that

$$\alpha_p - \alpha_v = \alpha - \alpha_c. \quad (41)$$

Using the same procedure as used for the cases with $x_p = c/2$ and $x_p = c/4$, we obtain the general models: for the flat plate and for thin airfoils without camber. We write them here for the airfoils. For the normal force coefficient C_n at the attachment point, we obtain:

$$380 \quad C_n(\alpha_p) \simeq C_l^u(\alpha_p - \alpha_v) - C_l^u(\alpha_c). \quad (42)$$

The moment coefficient at the plate center is again obtained from the potential flow theory as

$$C_{m_{c/2}}(\alpha_p) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_p - \alpha_v) \cos(\alpha_p - \alpha_v). \quad (43)$$

This moment is indeed zero, as it should, for a plate (that has no $C_{m_{c/2}}^0$) when $\alpha_p = \alpha_v$ as the cambered plate of the transformed flow is then horizontal.

The aerodynamic moment coefficients, $C_{m_{c/4}}$ and $C_{m_{ac}}$, are then obtained as

$$C_{m_{c/4}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \frac{1}{4} C_n(\alpha_p) \cos(\alpha_p - \alpha_v) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha_p - \alpha_v) \quad (44)$$

and

$$C_{m_{ac}}(\alpha_p) = C_{m_{c/2}}(\alpha_p) - \left(\frac{1}{2} - \frac{x_{ac}}{c} \right) C_n(\alpha_p) \cos(\alpha_p - \alpha_v). \quad (45)$$

390 In the same way, the model for the moment coefficient around the attachment point is obtained as

$$C_{m_{x_p}}(\alpha_p) \simeq C_{m_{c/2}}(\alpha_p) - \left(\frac{1}{2} - \frac{x_p}{c} \right) C_n(\alpha_p) \cos(\alpha_p - \alpha_v). \quad (46)$$

As the moment $m_0 = m_{x_p} + R f_t^i$ around the center of rotation of the arm must still be zero in inviscid flow, the model for the tangential force coefficient in inviscid flow becomes

$$C_t^i(\alpha_p) \simeq -\frac{c}{R} C_{m_{x_p}}(\alpha_p), \quad (47)$$



395 and hence the model for viscous flow is now taken as

$$\begin{aligned} C_t(\alpha_p) &= C_t^\nu(\alpha_p) + C_t^i(\alpha_p) \\ &= (C_t^{\nu,p}(\alpha_p) + C_t^i(\alpha_p)) + C_t^{\nu,f}(\alpha_p) = C_t^p(\alpha_p) + C_t^{\nu,f}(\alpha_p) \\ &\simeq \left(C_d^{u,p}(\alpha_p - \alpha_v) - \frac{c}{R} C_{m_{x_p}}(\alpha_p) \right) + C_d^{u,f}((\alpha_p - \alpha_v) + \alpha_c). \end{aligned} \quad (48)$$

Again: if the moment m_{x_p} is not enforced in the ALM, then only f_t^ν must be enforced, while f_t^i must be ignored.

400 Finally, the force coefficients can also be decomposed into the usual aerodynamic coefficients considered at $c/4$ by using the angle of attack $\alpha_p + \delta\alpha_c$ that the curved flow velocity vector makes there relative to the airfoil chord:

$$\begin{aligned} C_l &= C_n(\alpha_p) \cos(\delta\alpha_c) - C_t(\alpha_p) \sin(\delta\alpha_c), \\ C_d &= C_n(\alpha_p) \sin(\delta\alpha_c) + C_t(\alpha_p) \cos(\delta\alpha_c). \end{aligned} \quad (49)$$

4 Actuator line method for imposing aerodynamic forces

405 We briefly recall the actuator line method (ALM) which is used here to model the effect of the airfoil forces on the flow. It was detailed in Trigaux et al. (2024b) as implemented in a fourth order incompressible flow solver using finite differences. It was also used in Trigaux et al. (2024a) for aero-elastic studies of a 15 MW horizontal-axis turbine.

At each time step, the vector force per unit span $-\mathbf{f}$ that the airfoil exerts on the flow (i.e., the opposite of the vector force $\mathbf{f}$ that the flow exerts on the airfoil) is distributed over the neighboring grid points of the flow solver. In all that follows, the term
 410 “force” is understood as “force per unit span”. This is achieved using a 2-D Gaussian kernel centered at the airfoil AC location and whose (x, y) frame is fixed relative to the airfoil.

The force field applied to the flow solver is as follows:

$$\mathbf{f}^s(x, y) = -\eta_{2D}(x, y) \mathbf{f} \quad \text{where} \quad \mathbf{f} = f_x \hat{\mathbf{e}}_x + f_y \hat{\mathbf{e}}_y \quad (50)$$

and

$$415 \quad \eta_{2D}(x, y) = \eta_{\sigma_c}(x) \eta_{\sigma_t}(y) = \frac{1}{\sqrt{\pi} \sigma_c} \exp\left(-\frac{x^2}{\sigma_c^2}\right) \frac{1}{\sqrt{\pi} \sigma_t} \exp\left(-\frac{y^2}{\sigma_t^2}\right), \quad (51)$$

with σ_c the core size in the airfoil chord direction and σ_t that in the thickness direction. Here, We use an isotropic kernel ($\sigma_c = \sigma_t = \sigma$), and with $\sigma = c/4$ as recommended by Martínez-Tossas et al. (2017). The kernel is discretized using a 2-D template that consists of $(2n+1)(2n+1)$ cells of size $h \times h$. A node is located at the center of each cell $(x_i, y_j) = (ih, jh)$, and the kernel is integrated analytically over that cell to obtain the weight. For the integration in x , we obtain, for $0 \leq i < n$:

$$420 \quad w_i = \int_{x_i-h/2}^{x_i+h/2} \eta_\sigma(x) dx = \frac{1}{2} \left[\operatorname{erf}\left(\frac{x_i+h/2}{\sigma}\right) - \operatorname{erf}\left(\frac{x_i-h/2}{\sigma}\right) \right]. \quad (52)$$

For the cell $i = n$, we include the part outside (to ensure that the sum of the weights is unity):

$$w_n = \int_{x_n-h/2}^{\infty} \eta_\sigma(x) dx = \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{x_n-h/2}{\sigma}\right) \right]. \quad (53)$$



i	0	± 1	± 2	± 3	± 4
w_i	0.27634	0.21741	0.10587	0.03189	0.00666

Table 1. Weights w_i associated to the 1-D Gaussian kernel for the case with $h = \sigma/2$ and using $n = 4$. The numbers provided have been rounded so that $\sum_{i=-4}^4 w_i = 1$ exactly.

By symmetry, we have $w_{-i} = w_i$. The weights in y are $w_j = w_i$ for $j = i$. The weight associated with the (i, j) cell is then $w_{i,j} = w_i w_j$ and the associated force is

$$425 \quad \mathbf{f}_{i,j}^s = -w_i w_j \mathbf{f}. \quad (54)$$

We use $h = \sigma/2$ and $n = 4$. The weights w_i are provided in Table 2.

In the present implementation of the ALM used for modeling a VAT configuration, an overset mesh technique is employed, in which the flow solver mesh covering the turbine region, including the AL and its template, rotates together with the blade; see Figure 9.

430 Note that, when the AL and its template move through the flow solver grid, as in Trigaux et al. (2024a), the fraction $\mathbf{f}_{i,j}^s$ of the force on each cell of the template must be further distributed to the nearest flow solver grid points. This is done using a bi-linear distribution kernel, ensuring that each fraction is distributed to the nearest grid points.

The effective velocity vector to use in the ALM is obtained by using a weighted average of the velocity vectors provided by
 435 the flow solver around the control point (= center of the Gaussian kernel, typically taken as the location of the aerodynamic center); their relative weighting being determined using the same template as that used for the force distribution. This procedure corresponds to “integral sampling” (as opposed to “pointwise sampling”) and has been shown to provide slightly better results (Merabet and Laurendeau, 2019; Caprace et al., 2020). The effective velocity vector at the control point, $\mathbf{v}$, is thus measured using:

$$440 \quad \mathbf{v} = \sum_{i=-n_x}^{n_x} \sum_{j=-n_y}^{n_y} w_i w_j \mathbf{v}(x_i, y_j). \quad (55)$$

When the airfoil is moving relative to the fixed reference frame, as is the case in the present work, we must add that relative velocity to the flow solver velocity field since the ALM requires the apparent velocity seen by the airfoil, $\mathbf{V}$.

Moreover, when the parasitic drag is significant, the apparent velocity vector, $\mathbf{V}$, is not yet the suitable vector to use in the ALM, as it also contains the spurious velocity component induced by the wake vorticity associated with the parasitic drag. That
 445 induced velocity can be estimated using the procedure described in Martínez-Tossas et al. (2017). Assuming that the wake is shed primarily in the direction of $\mathbf{V}$, the corrected apparent velocity is

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} C_d^\nu\right)}. \quad (56)$$



As C_d^v is used in the above correction, a few iterations are required to determine all values; alternatively, the value of the previous time step can be used.

450 Once we have $\mathbf{V}^{\text{corr}}$, we measure the angle that this vector makes relative to the airfoil chord. Using that angle and the norm V^{corr} of the velocity, we can compute, using the airfoil polar model, the lift and drag forces exerted by the flow on the airfoil.

5 Actuator line method for imposing an aerodynamic moment

The previous section was devoted to presenting our particular implementation of the ALM aimed at efficiently distributing the aerodynamic force acting on the airfoil onto the nearby flow solver grid points, using a 2D template corresponding to the
 455 discretization of a Gaussian kernel. The same template is also used to evaluate the effective velocity required to compute the aerodynamic forces.

As such, however, the ALM cannot handle any aerodynamic moment. We extend the actuator line method to enable the distribution of an aerodynamic moment. Of course, a pure aerodynamic moment cannot be imposed, as the ALM uses a distribution kernel of some size σ . We hence replace the aerodynamic moment by a force field that has the same net effect, as
 460 presented in Figure 7; and we then discretize this force field using the 2D template.

Assume that the flow exerts a pitching moment per unit span m_{ac} around the airfoil aerodynamic center. Recall that the pitching moment is, by convention, positive when it acts to pitch the airfoil in the nose-up direction. The airfoil hence exerts a moment $-m_{ac}$ on the flow. This effect can be achieved by imposing the following annular force field:

$$\mathbf{g}^s(x, y) = g(r) \hat{\mathbf{e}}_\theta m_{ac} \quad (57)$$

465 with

$$g(r) = -\frac{1}{2\pi\sigma^2} \frac{d}{dr} \left(\exp\left(-\frac{r^2}{\sigma^2}\right) \right) = \frac{1}{\pi\sigma^3} \frac{r}{\sigma} \exp\left(-\frac{r^2}{\sigma^2}\right). \quad (58)$$

which indeed satisfies $\iint r g(r) dS = 1$ where $dS = r d\theta dr$:

$$\iint r g(r) dS = 2\pi \int_0^\infty g(r) r^2 dr = 2 \int_0^\infty \exp\left(-\frac{r^2}{\sigma^2}\right) \frac{r^3}{\sigma^3} \frac{dr}{\sigma} = 2 \int_0^\infty \exp(-s^2) s^3 ds = \int_0^\infty \exp(-p) p dp = 1 \quad (59)$$

where we have used the changes of variable $s = r/\sigma$ and $p = s^2$, and where the last integral has been evaluated by parts.

470

This force field corresponds to applying a net moment of $-m_{ac}$ to the flow, without applying any net force. The applied force field per unit m_{ac} is illustrated at the top of Figure 7; it is indeed counter-clockwise when m_{ac} is positive. It can be decomposed as

$$\mathbf{g}^s(x, y) = g(r) \hat{\mathbf{e}}_\theta m_{ac} = g(r) (-\sin\theta \hat{\mathbf{e}}_x + \cos\theta \hat{\mathbf{e}}_y) m_{ac} = \frac{1}{\pi\sigma^3} \frac{r}{\sigma} \exp\left(-\frac{r^2}{\sigma^2}\right) (-\sin\theta \hat{\mathbf{e}}_x + \cos\theta \hat{\mathbf{e}}_y) m_{ac}. \quad (60)$$



475 Since $x = r \cos \theta$ and $y = r \sin \theta$, we finally obtain that $\mathbf{g}^s(x, y) = g_x^s(x, y) \hat{\mathbf{e}}_x + g_y^s(x, y) \hat{\mathbf{e}}_y$ with

$$\begin{aligned} g_x^s(x, y) &= -\frac{1}{\pi \sigma^3} \frac{y}{\sigma} \exp\left(-\frac{x^2 + y^2}{\sigma^2}\right) m_{ac} = -\eta_\sigma(x) \left(\frac{y}{\sigma} \eta_\sigma(y)\right) \frac{m_{ac}}{\sigma}, \\ g_y^s(x, y) &= \frac{1}{\pi \sigma^3} \frac{x}{\sigma} \exp\left(-\frac{x^2 + y^2}{\sigma^2}\right) m_{ac} = \left(\frac{x}{\sigma} \eta_\sigma(x)\right) \eta_\sigma(y) \frac{m_{ac}}{\sigma}. \end{aligned} \quad (61)$$

Those components are illustrated at the bottom of Figure 7.

The above equations are then integrated analytically over each cell of the 2D template to obtain the values of the weights.

480 We obtain

$$\int_{x_i-h/2}^{x_i+h/2} \int_{y_j-h/2}^{y_j+h/2} g_x^s(x, y) dx dy = - \left(\int_{x_i-h/2}^{x_i+h/2} \eta_\sigma(x) dx \right) \left(\int_{y_j-h/2}^{y_j+h/2} \frac{y}{\sigma} \eta_\sigma(y) dy \right) \frac{m_{ac}}{\sigma}. \quad (62)$$

The first integral has already been evaluated: it is w_i . Let's evaluate the second integral, which we note q_j . For $0 \leq j < n$, we obtain

$$\begin{aligned} q_j &= \int_{y_j-h/2}^{y_j+h/2} \frac{y}{\sigma} \eta_\sigma(y) dy = \frac{1}{\sqrt{\pi}} \int_{y_j-h/2}^{y_j+h/2} \frac{y}{\sigma} \exp\left(-\frac{y^2}{\sigma^2}\right) \frac{dy}{\sigma} = \frac{1}{\sqrt{\pi}} \int_{(y_j-h/2)/\sigma}^{(y_j+h/2)/\sigma} s \exp(-s^2) ds \\ 485 \quad &= \frac{1}{2\sqrt{\pi}} \int_{(y_j-h/2)^2/\sigma^2}^{(y_j+h/2)^2/\sigma^2} \exp(-p) dp = \frac{1}{2\sqrt{\pi}} \left[\exp\left(-\frac{(y_j-h/2)^2}{\sigma^2}\right) - \exp\left(-\frac{(y_j+h/2)^2}{\sigma^2}\right) \right]. \end{aligned} \quad (63)$$

By anti-symmetry, we have that $q_{-j} = -q_j$ and $q_0 = 0$. For the weight associated to the boundary cell $j = n$, the integration again includes the part of the kernel outside:

$$q_j = \int_{y_n-h/2}^{\infty} \frac{y}{\sigma} \eta_\sigma(y) dy = \frac{1}{2\sqrt{\pi}} \left[\exp\left(-\frac{(y_j-h/2)^2}{\sigma^2}\right) \right]. \quad (64)$$

We hence finally obtain, for the (i, j) cell:

$$490 \quad \int_{x_i-h/2}^{x_i+h/2} \int_{y_j-h/2}^{y_j+h/2} g_x^s(x, y) dx dy = -w_i q_j \frac{m_{ac}}{\sigma}. \quad (65)$$

Following the same procedure, we also obtain that

$$\int_{x_i-h/2}^{x_i+h/2} \int_{y_j-h/2}^{y_j+h/2} g_y^s(x, y) dx dy = q_i w_j \frac{m_{ac}}{\sigma}. \quad (66)$$

The weights $w_i q_j$ and $q_i w_j$ are illustrated in Figure 8.

495 **In summary:** the ALM that imposes both the aerodynamic force $\mathbf{f}$ and moment m_{ac} amounts to distributing the following force on each (i, j) cell:

$$-\left(w_i w_j f_x + w_i q_j \frac{m_{ac}}{\sigma}\right) \hat{\mathbf{e}}_x - \left(w_i w_j f_y - q_i w_j \frac{m_{ac}}{\sigma}\right) \hat{\mathbf{e}}_y. \quad (67)$$

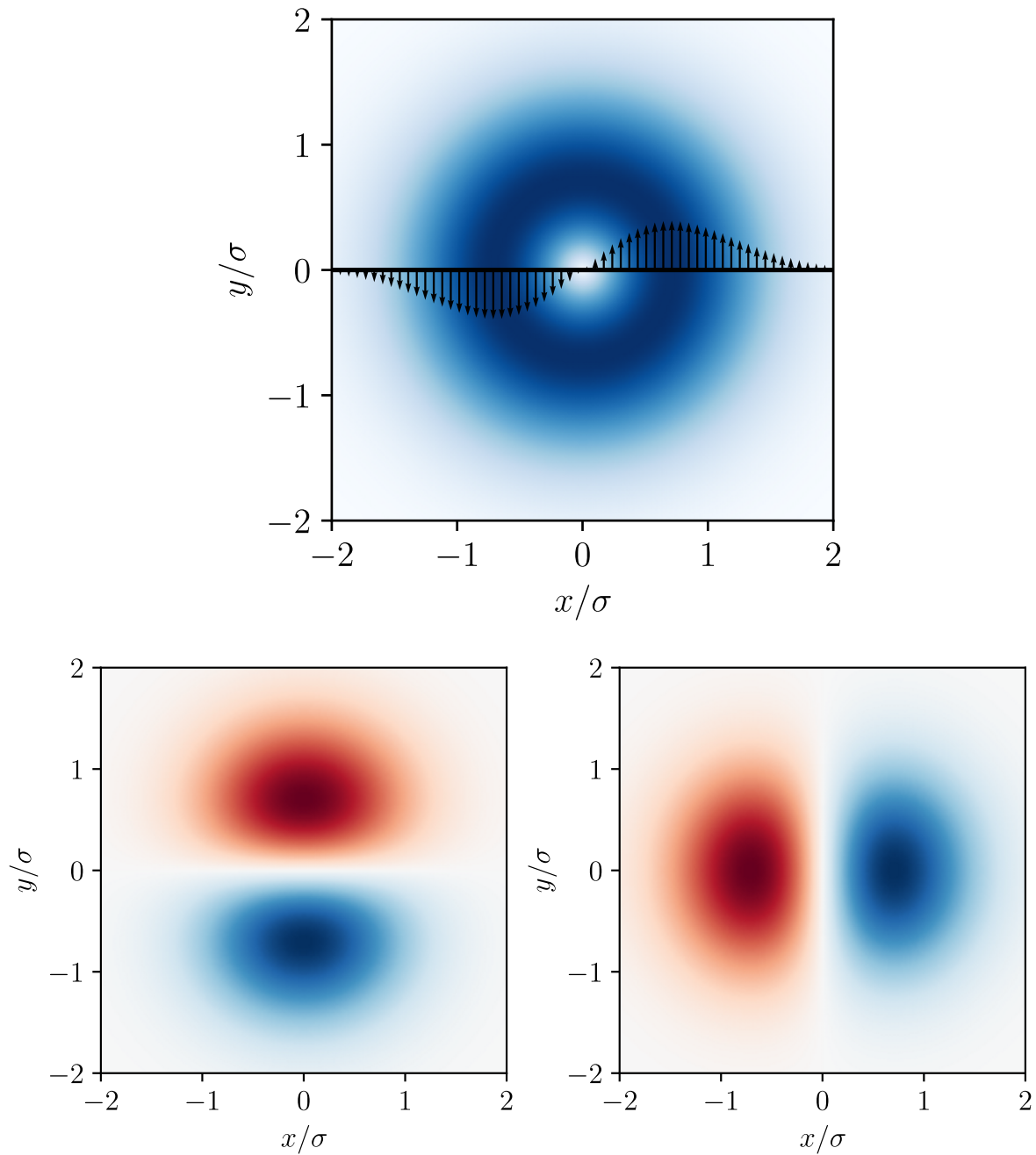


Figure 7. Vector field $\mathbf{g}^s(x, y)/m_{ac} = g(r) \hat{\mathbf{e}}_\theta$ used for the moment distribution (top), and its components (bottom): $g_x^s(x, y)/m_{ac}$ along $\hat{\mathbf{e}}_x$ (left) and $g_y^s(x, y)/m_{ac}$ along $\hat{\mathbf{e}}_y$ (right).



j	0	1	2	3	4
q_j	0	0.10427	0.10160	0.04594	0.01320

Table 2. Weights q_j for the case with $h = \sigma/2$ and using $n = 4$. By anti-symmetry, $q_{-j} = -q_j$.

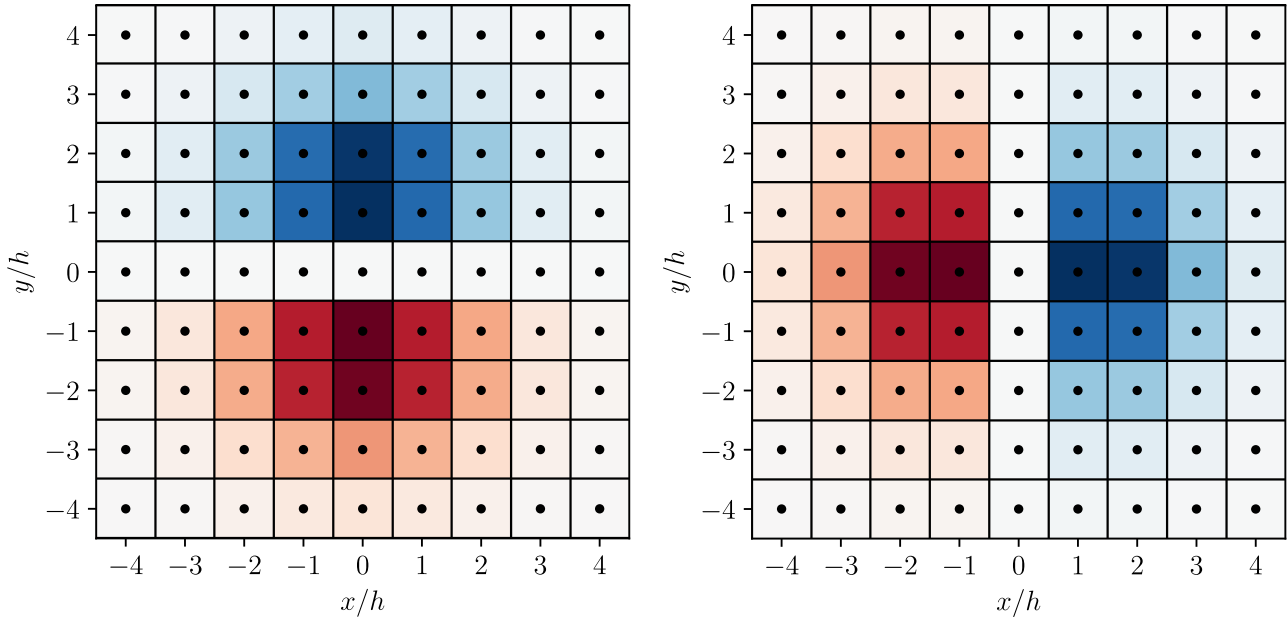


Figure 8. Template used for the moment distribution for the case with $h = \sigma/2$ and using $n = 4$. The color intensity of each (i, j) cell is proportional to its weights: $w_i q_j$ (left) and $q_i w_j$ (right).

6 Results: corrected ALM applied to a VAT configuration and comparison with CFD data

We consider the single-blade VAT configuration that was studied by Villeneuve et al. (2021). The blade profile is the NACA0015 airfoil considered previously. It is attached to the arm without pitch angle. The blade is rotating counterclockwise at the angular velocity Ω and the upstream flow, at velocity U_w , is coming from the left. The tip speed ratio is $TSR = \frac{\Omega R}{U_w}$. The simulated case corresponds to $TSR = 3.25$, which is the value at which the produced power is maximized.

6.1 Case of airfoil attached at $x_p = c/4$

We first consider the case in which the airfoil is attached to the arm at its quarter-chord.

The velocity vector $\mathbf{v}$ of the flow is measured at $x_p = c/4$ using the integral sampling; its components are (v_t, v_n) . The components of the apparent velocity vector $\mathbf{V}$ as seen by the rotating blade are thus $(V_t = \Omega R + v_t, V_n = v_n)$. The angle of that velocity vector $\mathbf{V}$ relative to the airfoil chord is noted α .



The mesh used for the ALM simulations is shown in Figure 9. A rectangular domain is used, identical to that of the blade-resolved reference case. An overset technique is implemented to maintain a hexahedral mesh with the same template for the
 510 ALM ($h = \sigma/2$) as described in Section 4. The overset approach also enables the rotation of the actuation zone. Two refinement regions are used: one for turbine region and another one for the wake. The cell size within the turbine region is dictated by the overset mesh resolution ($\sigma/h = 2$) to ensure accurate interpolation between the overset and background mesh. Notably, the blade-resolved reference case mesh is more refined ($\approx 328,000$ cells) compared to the ALM mesh ($\approx 37,000$ cells), in line with the intended efficiency of the ALM approach. The methodology is applied to all ALM cases in this work.

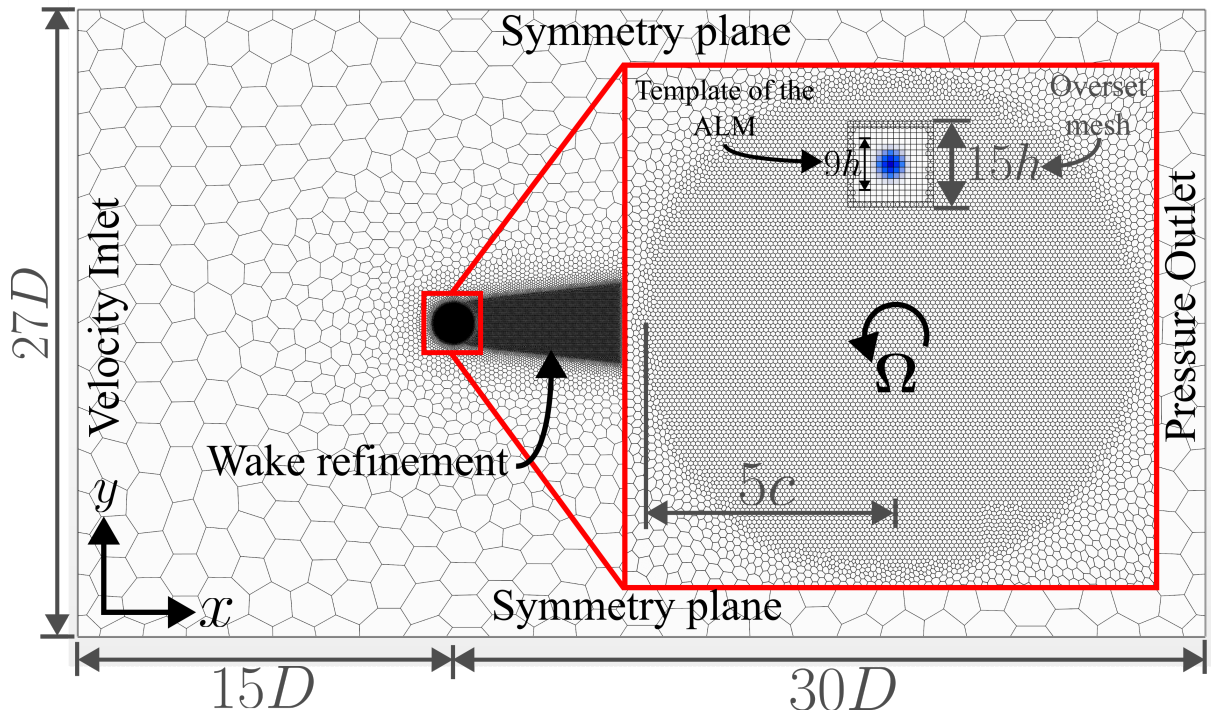


Figure 9. Computational domain and mesh used for the ALM simulations. The template of the ALM is implemented using an overset mesh approach to enable the 2-D template as described in Section 4. The template of the corresponding weights is also colored in blue.

515 6.1.1 Case without explicitly enforcing $m_{c/4}$ in the ALM

We first consider the case where we do not explicitly enforce the moment $m_{c/4}$ in the ALM; hence we also cannot enforce the inviscid tangential force f_t^i in the ALM. Only the lift and parasitic drag contributions are enforced.



The velocity vector is first corrected to compensate for the velocity induced by the shed vortex wake produced by the
 520 parasitic drag. As this wake is assumed to be shed in the direction of $\mathbf{V}$, we correct its amplitude:

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} C_d^{\nu}\right)}. \quad (68)$$

The angle that the corrected velocity vector makes relative to the chord is therefore still α .

In terms of modeling, the lift force (i.e., force in the direction orthogonal to the apparent velocity) exerted by the velocity
 525 $\mathbf{V}^{\text{corr}}$ on the airfoil attached at $x_p = c/4$ and without pitch is equivalent to the normal force (= lift force) exerted by a horizontal
 flow of velocity V^{corr} on the airfoil attached at $x_p = c/4$ and pitched at an angle $\alpha_p = \alpha$. The lift coefficient is thus modeled
 using

$$C_l(\alpha) \simeq C_l^u(\alpha - \alpha_c) - C_l^u(\alpha_c). \quad (69)$$

Likewise, the parasitic drag (i.e., force in the direction of the apparent velocity) is modeled using

$$530 \quad C_d^{\nu}(\alpha) = C_d^{\nu,p}(\alpha) + C_d^{\nu,f}(\alpha) \simeq C_d^{u,p}(\alpha - \alpha_c) + C_d^{u,f}(\alpha). \quad (70)$$

for the parasitic drag coefficient.

The moment coefficients can also be estimated. For the moment coefficient at mid-chord, the model gives

$$C_{m_{c/2}}(\alpha) = C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha - \alpha_c) \cos(\alpha - \alpha_c). \quad (71)$$

The moment coefficient $C_{m_{c/4}}$ is then also obtained as

$$535 \quad C_{m_{c/4}}(\alpha) = C_{m_{c/2}}(\alpha) - \frac{1}{4} C_l(\alpha) \cos(\alpha - \alpha_c) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha - \alpha_c) \quad (72)$$

Further adding the inviscid tangential force coefficient $C_t^i = -\frac{c}{R} C_{m_{c/4}}$, we finally obtain

$$\begin{aligned}
 C_t(\alpha) &= (C_d^{\nu}(\alpha) \cos(\alpha) - C_l(\alpha) \sin(\alpha)) - \frac{c}{R} C_{m_{c/4}}(\alpha) \\
 &= \left(C_d^{\nu,p}(\alpha) \cos(\alpha) - C_l(\alpha) \sin(\alpha) - \frac{c}{R} C_{m_{c/4}}(\alpha) \right) + C_d^{\nu,f}(\alpha) \cos(\alpha), \\
 C_n(\alpha) &= C_d^{\nu}(\alpha) \sin(\alpha) + C_l(\alpha) \cos(\alpha) \\
 540 \quad &= (C_d^{\nu,p}(\alpha) \sin(\alpha) + C_l(\alpha) \cos(\alpha)) + C_d^{\nu,f}(\alpha) \sin(\alpha). \quad (73)
 \end{aligned}$$

We have intentionally split each term into pressure and friction contributions. The net forces f_t and f_n (each split in pressure
 and friction contributions) and the moments, $m_{c/2}$ and $m_{c/4}$, are then finally obtained using V^{corr} , c and ρ . They can be
 compared to the values measured in CFD.

As we here do not explicitly impose $m_{c/4}$ in the ALM, we also do not impose f_t^i . The tangential force that we impose in the
 545 ALM is hence simply the tangential projection of the lift and parasitic drag forces.



6.1.2 Case with explicitly enforcing $m_{c/4}$ in the ALM

We next consider the case where we explicitly impose $m_{c/4}$ in the ALM (using the procedure developed in Section 5); hence we also include the contribution of f_t^i in the imposition of the tangential force.

The values of C_l , C_d , $C_{m_{c/4}}$ and C_t^i are the same as those obtained in the above section.

550 Here, however, the velocity vector $\mathbf{V}$ must be corrected to compensate for the velocity induced by the shed vortex wake produced by the total net force imposed in the direction of $\mathbf{V}$; thus including the projection of f_t^i in that direction (as it will also shed some vorticity into the wake; even though this vorticity is not present in the real flow). The correction is thus:

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} (C_d^\nu + C_t^i \cos(\alpha))\right)}. \quad (74)$$

6.1.3 Results and comparison

555 We present the results of both types of simulations using ALM. The normalization used here is different from that used for the airfoil in rotation: here, the forces f_n and f_t are normalized using the wind velocity U_w and the turbine diameter $D = 2R$; thus, using $\frac{1}{2}\rho U_w^2 D$. To normalize the moments $m_{c/4}$ and $m_{c/4}$, we use $\frac{1}{2}\rho U_w^2 D c$. As for the power coefficient C_p of the VAT, it is obtained using

$$C_p = - \frac{(f_t R + m_{c/4}) \Omega}{\frac{1}{2} \rho U_w^3 D}. \quad (75)$$

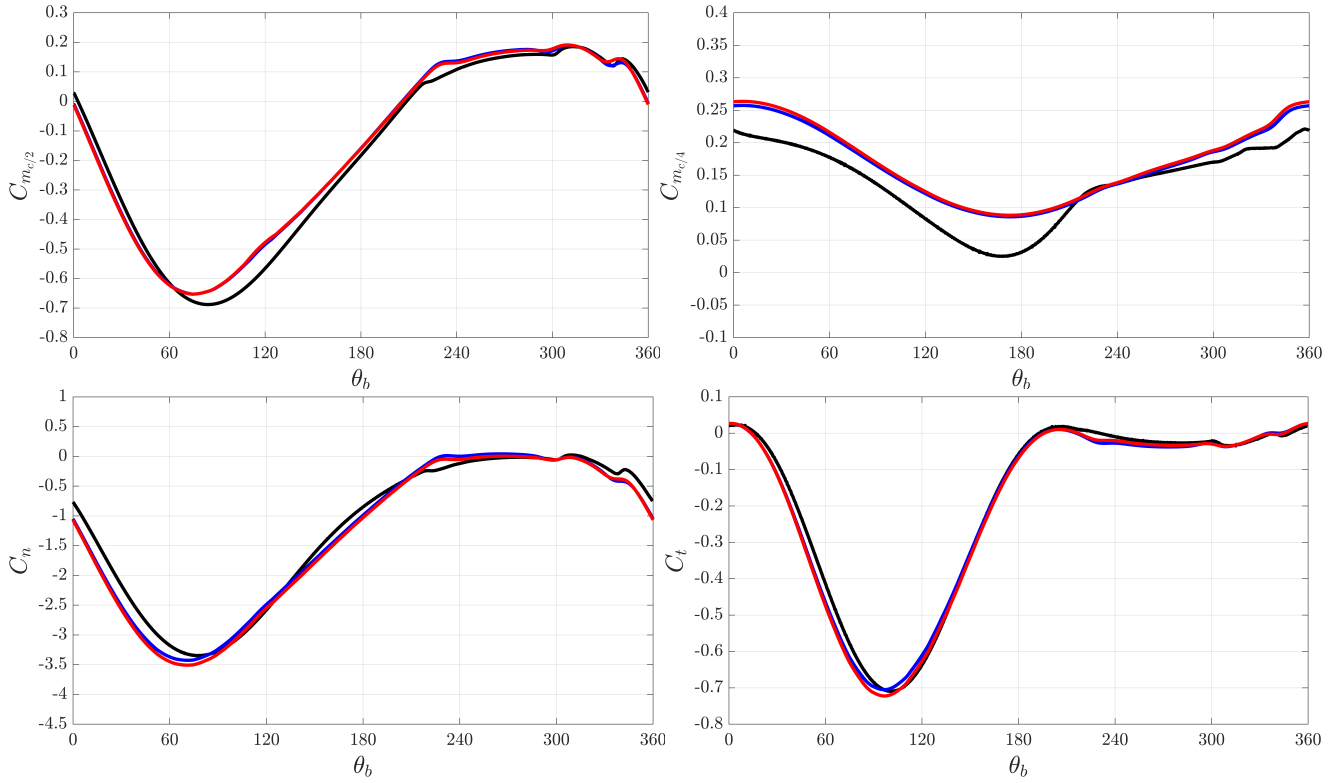


Figure 10. VAT with NACA0015 airfoil attached at $x_p = c/4$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/4}$ enforced (red), the ALM without $m_{c/4}$ enforced (blue) and the blade-resolved reference simulation (black) of $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t .

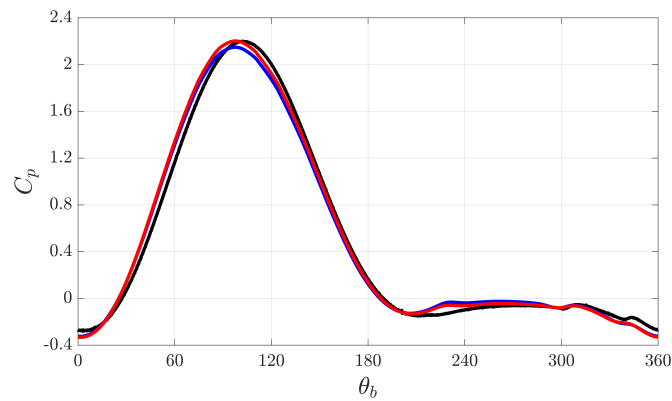


Figure 11. VAT with NACA0015 airfoil attached at $x_p = c/4$ and operating at $TSR = 3.25$: comparison between the results of the ALM with $m_{c/4}$ enforced (red), the ALM without $m_{c/4}$ enforced (blue) and the blade-resolved reference simulation (black) of C_p .



Coefficients	$\overline{C_{m_{c/2}}}$	$\overline{C_{m_{c/4}}}$	$\overline{C_n}$	$\overline{C_t}$	$\overline{C_p}$
Blade-resolved	-0.177	0.136	-1.255	-0.187	0.482
ALM without $m_{c/4}$	-0.161	0.166	-1.325	-0.196	0.485
ALM with $m_{c/4}$	-0.162	0.164	-1.320	-0.199	0.494

Table 3. Mean $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t and C_p over a blade cycle for a VAT with airfoil attached at $x_p = c/4$ and operating at $TSR = 3.25$.

560 It is also interesting to compare the ALM-predicted contributions of pressure and friction to the tangential force f_t with those from the blade-resolved simulation. Given that the two ALM cases should be similar, we chose to perform the comparison only for the ALM with $m_{c/4}$ enforced.

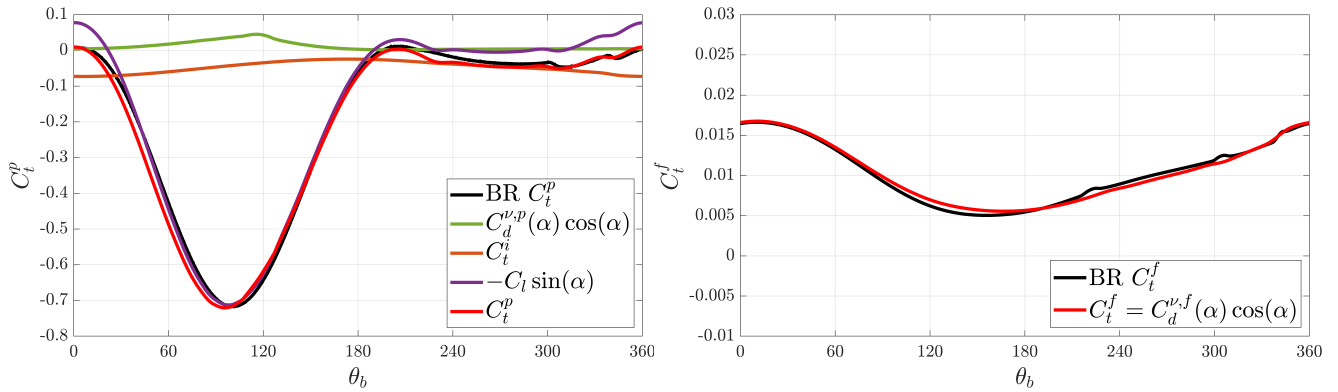


Figure 12. VAT with NACA0015 airfoil attached at $x_p = c/4$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/4}$ enforced (red) and the reference blade-resolved simulation (black) of the pressure contribution C_t^p (with its components) and of the friction contribution C_t^f to the tangential force coefficient.

Finally, the vorticity fields from the ALM methods are also compared to that of the blade-resolved simulation in Figure 13.

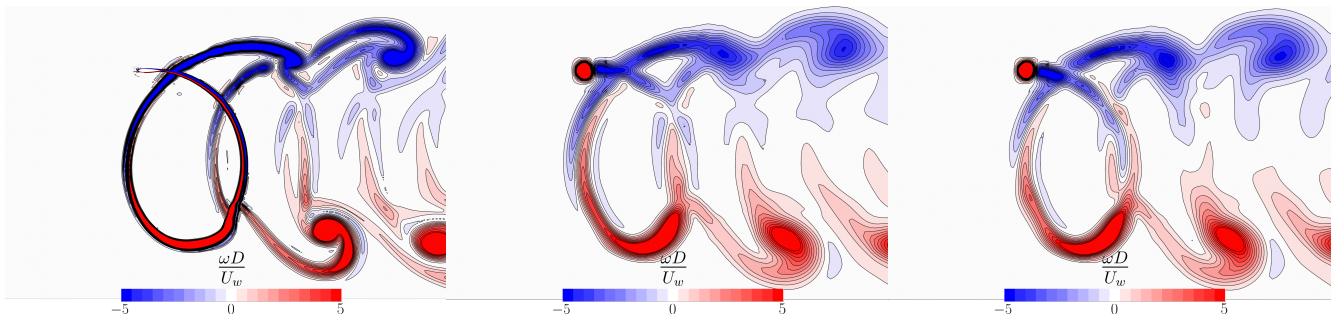


Figure 13. Vorticity field for the blade-resolved simulation (left), the ALM simulation without $m_{c/4}$ enforced (center) and the ALM simulation with $m_{c/4}$ enforced (right).



6.2 Case of airfoil attached at $x_p = c/2$

565 We consider next the case where the blade is attached to the arm at $x_p = c/2$, again without pitch.

6.2.1 Case with ALM centered at $c/4$

In the usual case where the ALM is strictly applied at the aerodynamic center, the airfoil attached at $x_p = c/2$ and without pitch must be considered “equivalent to the same airfoil attached at $x_p = c/4$ and with a pitch angle of α_c ”.

570 The components of the flow velocity $\mathbf{v}$ measured at $c/4$ are (v_t, v_n) . The components of the apparent velocity vector $\mathbf{V}$ are thus

$$\begin{aligned} V_t &= \Omega \frac{R}{\cos(\alpha_c)} \cos(\alpha_c) + v_t = \Omega R + v_t, \\ V_n &= \Omega \frac{R}{\cos(\alpha_c)} \sin(\alpha_c) + v_n = \Omega R \tan(\alpha_c) + v_n \end{aligned} \quad (76)$$

where $R/\cos(\alpha_c)$ corresponds to the slightly extended effective arm length when considering the circle passing through that point.

575 The angle of the velocity vector $\mathbf{V}$ relative to the airfoil chord is noted α . The airfoil chord makes an angle α_c relative to the curved flow streamline considered at $c/4$. The angle of the velocity vector relative to the curved flow streamline at $c/4$ is thus $\alpha_e = \alpha - \alpha_c$.

Again, there are then two ways of applying the ALM, and those are investigated below.

580 We first consider the case where we do not explicitly enforce the moment $m_{c/4}$ in the ALM; hence we also cannot enforce the associated inviscid force in the ALM. Only the lift and the parasitic drag contributions will be enforced in the ALM.

The amplitude of the velocity must be corrected to compensate for the velocity induced by the shed vortex wake produced by the parasitic drag:

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} C_d^\nu\right)}. \quad (77)$$

The angle of this corrected velocity vector relative to the curved flow streamline at $c/4$ is therefore still $\alpha - \alpha_c$.

585

In terms of modeling, the lift force (i.e., force in the direction orthogonal to the apparent velocity) exerted by the velocity $\mathbf{V}^{\text{corr}}$ on the airfoil attached at $c/4$ and with pitch α_c is equivalent to the normal force (= also lift force) exerted by a horizontal flow of velocity V^{corr} on the airfoil attached at $c/4$ and pitched by an angle $\alpha_p = \alpha_e + \alpha_c$. The lift coefficient is thus modeled using

$$590 \quad C_l(\alpha) \simeq C_l^u((\alpha_e + \alpha_c) - \alpha_c) - C_l^u(\alpha_c) = C_l^u(\alpha - \alpha_c) - C_l^u(\alpha_c). \quad (78)$$

Likewise, the parasitic drag (i.e., force in the direction of the apparent velocity) is modeled using

$$C_d^\nu(\alpha) = C_d^{\nu,p}(\alpha) + C_d^{\nu,f}(\alpha) \simeq C_d^{\nu,p}(\alpha - \alpha_c) + C_d^{\nu,f}(\alpha). \quad (79)$$



Those expressions are the same as those obtained for the case where the airfoil was attached at $x_p = c/4$. This makes sense since the apparent velocity vector is that considered at $c/4$ in both cases, and since the angle α of that vector is defined relative to the airfoil chord in both cases. The only difference lies in the apparent velocity vector, see Eq. (76).

The models for the moment coefficients are also identical:

$$C_{m_{c/2}}(\alpha) = C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha - \alpha_c) \cos(\alpha - \alpha_c) \quad (80)$$

and

$$C_{m_{c/4}}(\alpha) = C_{m_{c/2}}(\alpha) - \frac{1}{4} C_l(\alpha) \cos(\alpha - \alpha_c) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha - \alpha_c) \quad (81)$$

The coefficient for the inviscid force on the airfoil “as if it were attached at $c/4$ ” still counterbalances the moment $m_{c/4}$. That inviscid force now acts in the direction of α_c , and thus we obtain $C_t^i = -\frac{c}{R} C_{m_{c/4}} \cos(\alpha_c)$ and $C_n^i = -\frac{c}{R} C_{m_{c/4}} \sin(\alpha_c)$. We finally obtain:

$$\begin{aligned}
 C_t(\alpha) &= (C_d^\nu(\alpha) \cos(\alpha) - C_l(\alpha) \sin(\alpha)) - \frac{c}{R} C_{m_{c/4}}(\alpha) \cos(\alpha_c) \\
 &= \left(C_d^{\nu,p}(\alpha) \cos(\alpha) - C_l(\alpha) \sin(\alpha) - \frac{c}{R} C_{m_{c/4}} \cos(\alpha_c) \right) + C_d^{\nu,f}(\alpha) \cos(\alpha), \\
 C_n(\alpha) &= (C_d^\nu(\alpha) \sin(\alpha) + C_l(\alpha) \cos(\alpha)) - \frac{c}{R} C_{m_{c/4}}(\alpha) \sin(\alpha_c) \\
 &= \left(C_d^{\nu,p}(\alpha) \sin(\alpha) + C_l(\alpha) \cos(\alpha) - \frac{c}{R} C_{m_{c/4}}(\alpha) \sin(\alpha_c) \right) + C_d^{\nu,f}(\alpha) \sin(\alpha).
 \end{aligned} \quad (82)$$

When we do not explicitly impose $m_{c/4}$ in the ALM, we also do not impose f_t^i and f_n^i .

When we explicitly enforce $m_{c/4}$ in the ALM, we must also explicitly enforce f_t^i and f_n^i . The correction of the measured velocity $\mathbf{V}$ must now also account for the component of the inviscid force in that direction:

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} (C_d^\nu + C_t^i \cos(\alpha - \alpha_c)) \right)}. \quad (83)$$

We present the results of both types of simulations using the ALM centered at $c/4$ for an airfoil attached at $x_p = c/2$.

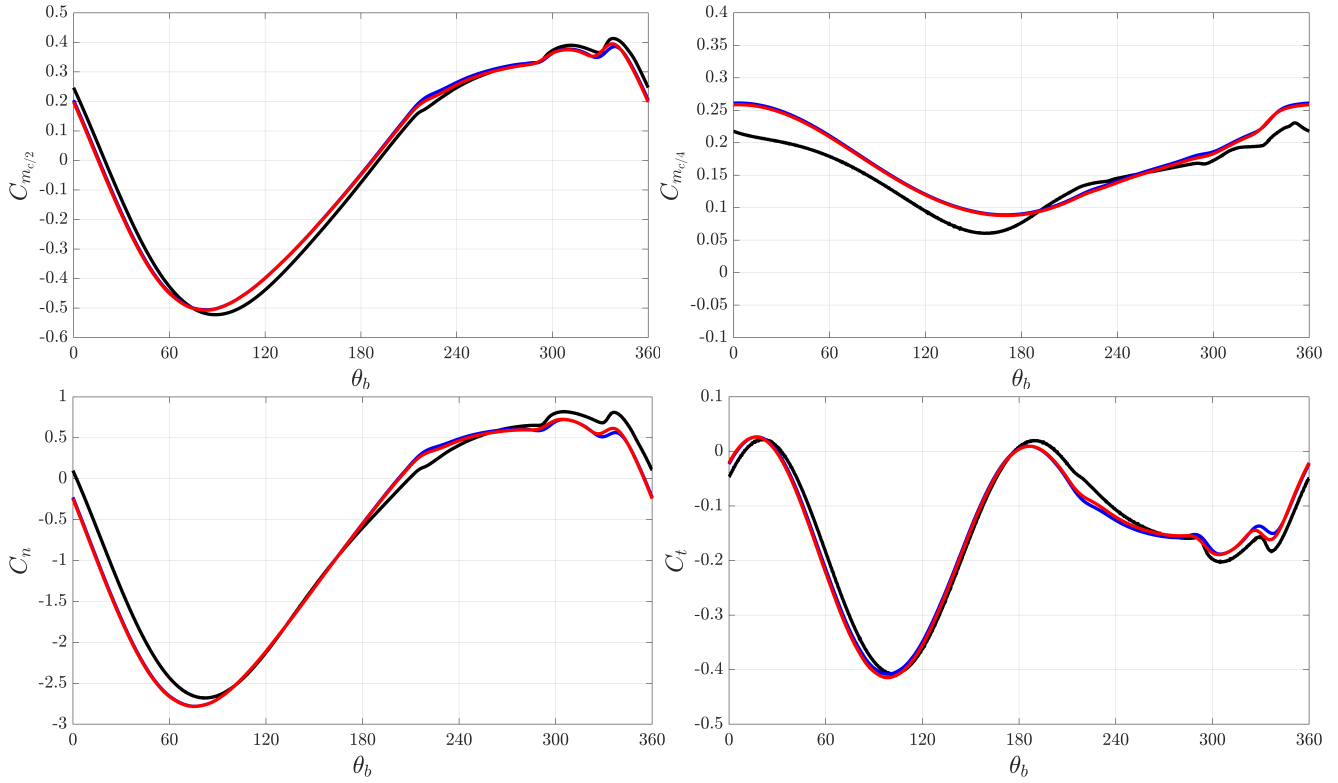


Figure 14. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/4}$ enforced (red), the ALM without $m_{c/4}$ enforced (blue) and the reference blade-resolved simulation (black) of $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t . Both ALM cases are centered at $c/4$.

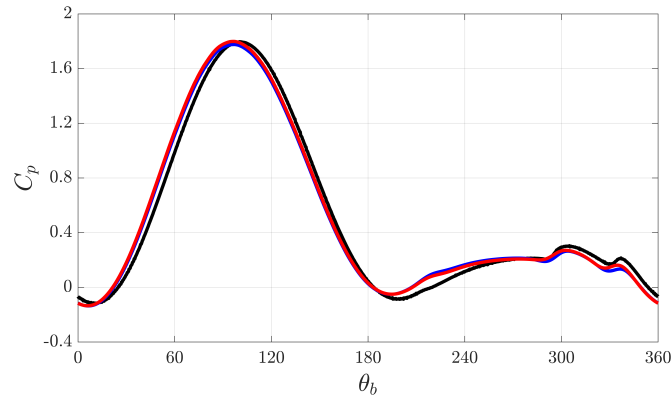


Figure 15. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the results of the ALM with $m_{c/4}$ enforced (red), the ALM without $m_{c/4}$ enforced (blue) and the reference blade-resolved simulation (black) of C_p . Both ALM cases are centered at $c/4$.



Coefficients	$\overline{C_{m_{c/2}}}$	$\overline{C_{m_{c/4}}}$	$\overline{C_n}$	$\overline{C_t}$	$\overline{C_p}$
Blade-resolved	-0.014	0.149	-0.650	-0.147	0.490
ALM without $m_{c/4}$	-0.011	0.169	-0.729	-0.149	0.493
ALM with $m_{c/4}$	-0.014	0.167	-0.733	-0.150	0.500

Table 4. Mean $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t and C_p over a blade cycle for a VAT with airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$. Both ALM cases are centered at $c/4$.

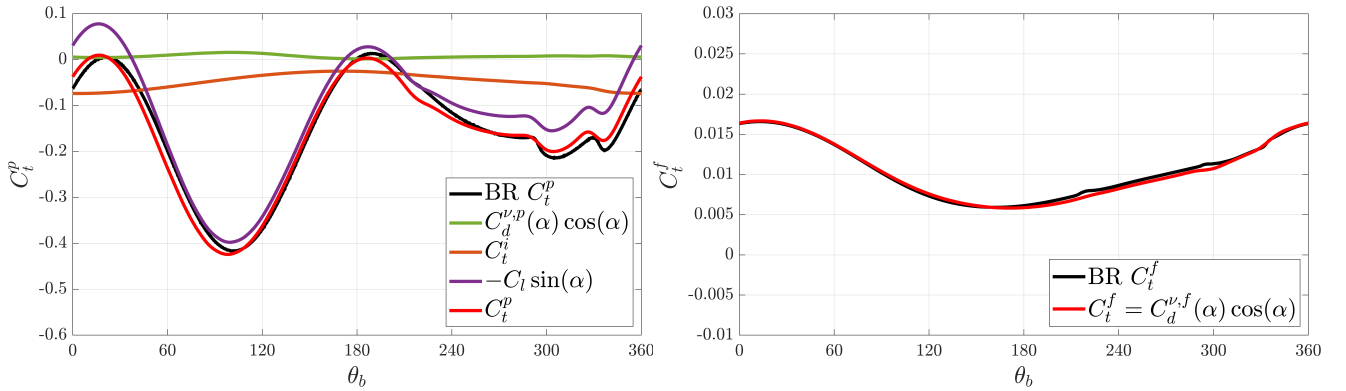


Figure 16. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/4}$ enforced (red) and the reference blade-resolved simulation (black) of the pressure contribution C_t^p (with its components) and of the friction contribution C_t^f to the tangential force coefficient. The ALM is centered at $c/4$.

6.2.2 Case with ALM centered at $c/2$

However, one could also consider that an ALM can be used as a model to represent the airfoil at its attachment point; thus, centering the ALM at $x_p = c/2$. In that case, we also measure the flow velocity vector $\mathbf{v}$ at $x_p = c/2$: its components are
 615 (v_t, v_n) . The components of the apparent velocity vector $\mathbf{V}$ are thus $(V_t = \Omega R + v_t, V_n = v_n)$. The angle it makes relative to the airfoil chord is α .

In terms of modeling, the force exerted in the direction orthogonal to the apparent velocity $\mathbf{V}^{\text{corr}}$ on the airfoil attached at $x_p = c/2$ and without pitch is equivalent to the normal force exerted by a horizontal flow of velocity V^{corr} on the airfoil attached at $x_p = c/2$ and pitched by an angle $\alpha_p = \alpha$; and the parasitic force exerted in the direction parallel to the apparent
 620 velocity is equivalent to the parasitic tangential force exerted by a horizontal flow of velocity V^{corr} on the airfoil pitched by an angle $\alpha_p = \alpha$. The related force coefficients are thus modeled using

$$\begin{aligned}
 C_{\perp}(\alpha) &= C_l^u(\alpha) - C_l^u(\alpha_c), \\
 C_{\parallel}(\alpha) &= C_d^{u,p}(\alpha) + C_d^{u,f}(\alpha + \alpha_c).
 \end{aligned} \tag{84}$$



The normal and tangential components are hence obtained as

$$\begin{aligned}
 625 \quad C_n(\alpha) &= C_\perp(\alpha) \cos(\alpha) + C_\parallel(\alpha) \sin(\alpha), \\
 C_t(\alpha) &= C_\parallel(\alpha) \cos(\alpha) - C_\perp(\alpha) \sin(\alpha).
 \end{aligned} \tag{85}$$

The corrected velocity vector is

$$\mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} C_\parallel\right)}. \tag{86}$$

The angle that $\mathbf{V}^{\text{corr}}$ makes relative to the curved flow streamline at the location of the aerodynamic center is $\alpha_e = \alpha - \alpha_c$.

630 The moment $m_{c/4}$ is hence obtained as

$$C_{m_{c/4}}(\alpha) \simeq C_{m_{c/2}}^0 + \frac{1}{4} C_l^u(\alpha_c) \cos(\alpha - \alpha_c). \tag{87}$$

The moment coefficient $C_{m_{c/2}}$ is thus obtained as

$$C_{m_{c/2}}(\alpha) \simeq C_{m_{c/4}} + \frac{1}{4} C_n(\alpha). \tag{88}$$

The associated inviscid tangential force is then $f_t^i = -m_{c/2}/R$, thus with coefficient

$$635 \quad C_t^i(\alpha) = -\frac{c}{R} C_{m_{c/2}}(\alpha). \tag{89}$$

If we do not explicitly enforce the moment $m_{c/2}$ in the ALM, we also do not enforce f_t^i . We then only enforce the normal and tangential forces associated to C_n and C_t of Eq. (85).

If we explicitly enforce the moment $m_{c/2}$ in the ALM, We must also enforce f_t^i ; hence the component $C_t^i(\alpha)$ must be added to the formula for C_t in Eq. (85). The corrected velocity vector must also be modified:

$$640 \quad \mathbf{V}^{\text{corr}} = \frac{\mathbf{V}}{\left(1 - \frac{1}{4\sqrt{\pi}} \frac{c}{\sigma} (C_\parallel + C_t^i \cos(\alpha))\right)}. \tag{90}$$

We present the results of both type of simulations using the ALM centered at $x_p = c/2$.

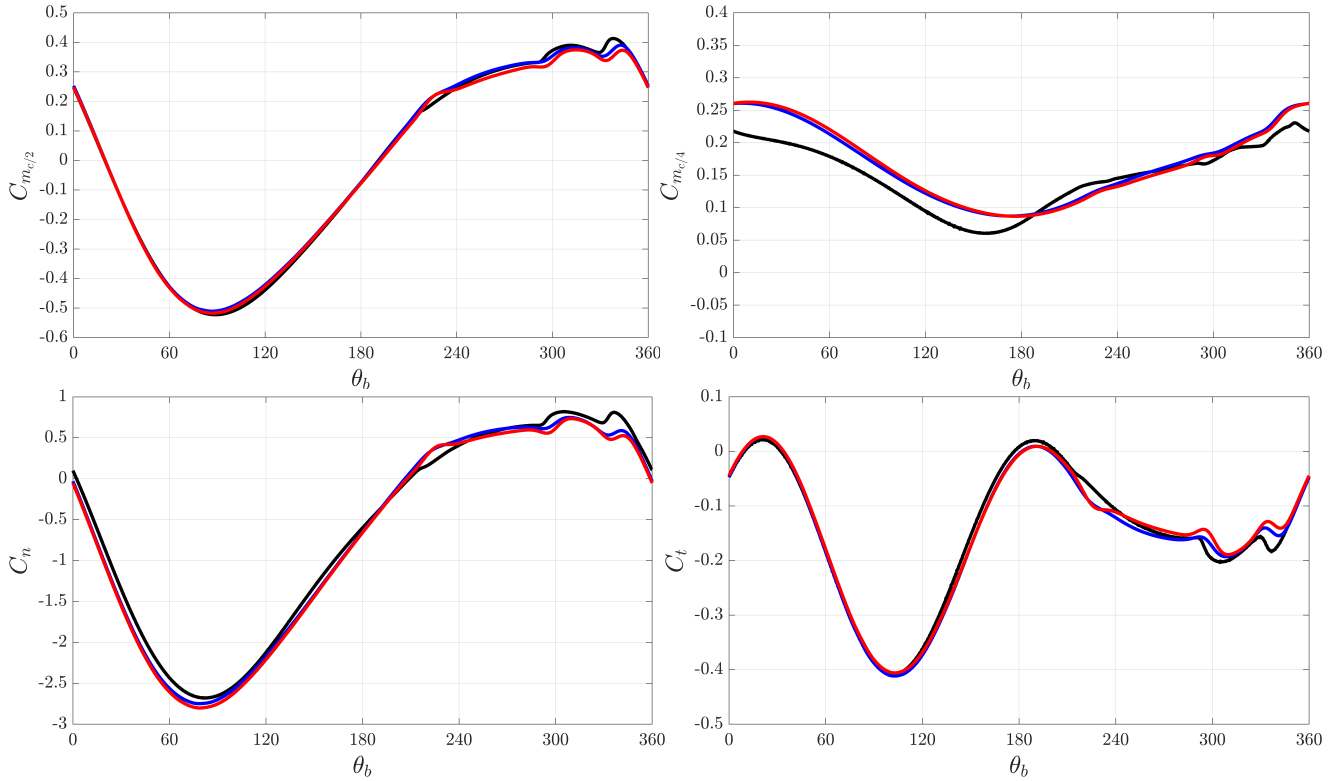


Figure 17. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/2}$ enforced (red), the ALM without $m_{c/2}$ enforced (blue) and the reference blade-resolved simulation (black) of $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t . Both ALM cases are centered at $x_p = c/2$.

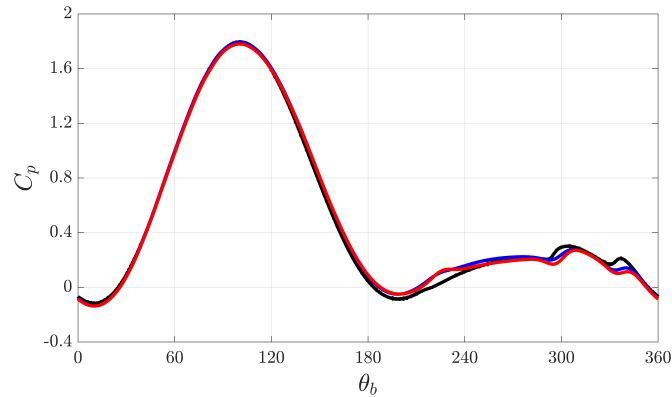


Figure 18. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the results of the ALM with $m_{c/2}$ enforced (red), the ALM without $m_{c/2}$ enforced (blue) and the reference blade-resolved simulation (black) of C_p . Both ALM cases are centered at $x_p = c/2$.



In this configuration (ALM centered at $x_p = c/2$), it can be observed that the upstream prediction (0° to 180°) is excellent. In fact, when the ALM is centered at $c/4$, a small shift in the peak power coefficient is noticeable in the upstream part of the cycle. This shift is eliminated when using the ALM centered at $c/2$. For the downstream region (180° to 360°), small discrepancies persist, as the flow is disturbed by the wake of the upstream blade, making the effective angle of attack prediction more challenging.

Coefficients	$\overline{C_{m_{c/2}}}$	$\overline{C_{m_{c/4}}}$	$\overline{C_n}$	$\overline{C_t}$	$\overline{C_p}$
Blade-resolved	-0.014	0.149	-0.650	-0.147	0.490
ALM without $m_{c/2}$	-0.011	0.167	-0.712	-0.151	0.502
ALM with $m_{c/2}$	-0.019	0.167	-0.744	-0.146	0.492

Table 5. Mean $C_{m_{c/2}}$, $C_{m_{c/4}}$, C_n , C_t and C_p over a blade cycle for the three cases studied of a VAT at $TSR = 3.25$ with the blade attached at $x_p = c/2$. Both ALM cases are centered at $x_p = c/2$.

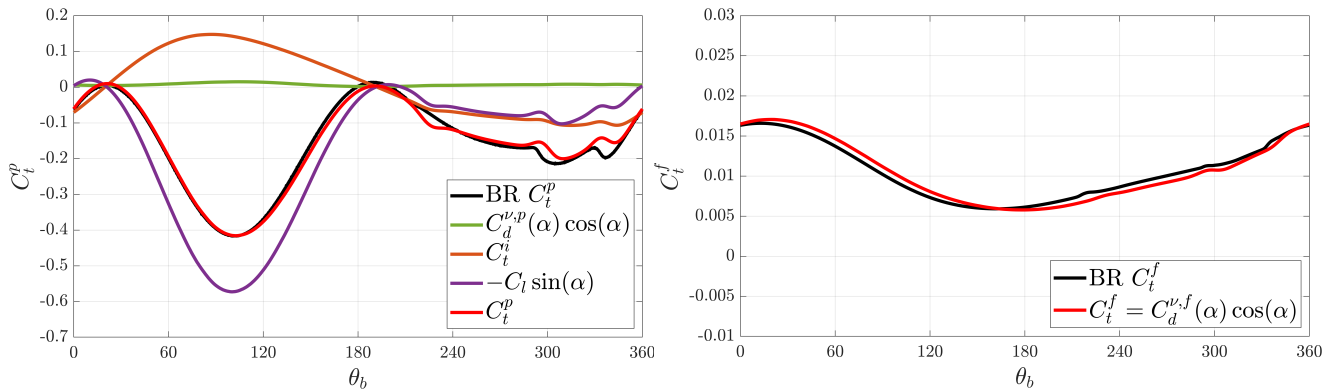


Figure 19. VAT with NACA0015 airfoil attached at $x_p = c/2$ and operating at $TSR = 3.25$: comparison between the ALM with $m_{c/4}$ enforced (red) and the reference blade-resolved simulation (black) of the pressure contribution C_t^p (with its components) and of the friction contribution C_t^f to the tangential force coefficient. The ALM is centered at $x_p = c/2$.

7 Conclusions

The modeling of flow-curvature effects on airfoils was further improved and implemented in an actuator line method (ALM); the rule being that such modeling can only use, as input, the aerodynamic coefficients of the airfoil in a uniform flow as a function of its angle of attack.

The basic models for the normal force and moment coefficients are not new and are obtained using the usual analogy with potential flow past an airfoil with curved streamlines. The model for the normal force was further improved here, to remain valid up to large pitch angles; even close to stall. The effect of the aerodynamic moment induced by the flow curvature is also



found to be important and cannot be ignored. The curvature effects depend on both the attachment point of the airfoil to the
655 arm and the pitch angle, and we have developed models that cover all possibilities.

The model for the tangential force is new (here, the analogy with a potential flow is flawed). The model is based on the fact
that, for an airfoil attached to a rotating arm, the moment at the root of the arm must be zero if the flow is assumed inviscid.
This was verified numerically by performing inviscid CFD simulations at various airfoil pitch angles. The inviscid tangential
force that is compatible with the aerodynamic moment can, hence, be computed. For the modeling viscous flows, we then add
660 the parasitic drag to that inviscid tangential force.

The improved models were first validated against reference data corresponding to wall-resolved CFD of a rotating NACA0015
airfoil with $cR = \frac{2}{7}$ at various pitch angles and for angle up to stall; also for two attachment points of the airfoil to the arm (at
mid-chord and quarter-chord).

The ALM with the improved models was then implemented in the CFD framework, and used to simulate the unsteady flow
665 corresponding to a VAT configuration: a rotating airfoil without pitch placed in a free stream at $TSR = \frac{\Omega R}{U_w} = 3.25$. For that,
a novel method was also developed to enforce the aerodynamic moment in the ALM. The various components of the ALM
results were compared with those of the reference CFD data. They were found to be in good agreement throughout the rotation
cycle.

It was also shown that the ALM can here be used in various ways: if one ignores the contribution of the aerodynamic moment
670 in the ALM, then one must also ignore the inviscid part of the tangential force in the ALM; if one enforces the contribution of
the moment in the ALM, then one must also enforce the inviscid part of the tangential force.

The above validations give us confidence that the proposed models, together with their implementation in an ALM simulation
framework (including the new method for enforcing a moment), constitute useful simplified tools for modeling complex un-
steady flows with good accuracy, such as VAT configurations where the flow curvature modifies both the normal and tangential
675 forces, and also produces an aerodynamic moment.

We also note that the method proposed here for enforcing the aerodynamic moment in an ALM, in addition to enforcing the
forces, can also be used to model unsteady flows involving cambered airfoils (thus with intrinsic aerodynamic moment); hence
also VAT using cambered blades.

Finally, it is recognized that the proposed models are, so far, limited to flows without fast unsteady effects or dynamic stall.
680 A next step in their development should focus on modeling such effects.

Appendix A: NACA0015 airfoil in uniform flow and associated aerodynamic coefficients

The aerodynamic coefficients of the NACA0015 airfoil in uniform flow were here obtained using wall-resolved CFD in RANS
at $Re_c = 6.0 \times 10^6$: a high Reynolds number case where the boundary layers are turbulent. The coefficients are noted using
a superscript “u” (that stands for “uniform” flow), so as to avoid any confusion with the coefficients measured in CFD of the
685 rotating NACA0015 airfoil.



The variation of the coefficients as a function of the angle of attack (noted β to avoid any confusion with the notation α used for the angle of attack of a rotating airfoil) is shown in Figure A1.

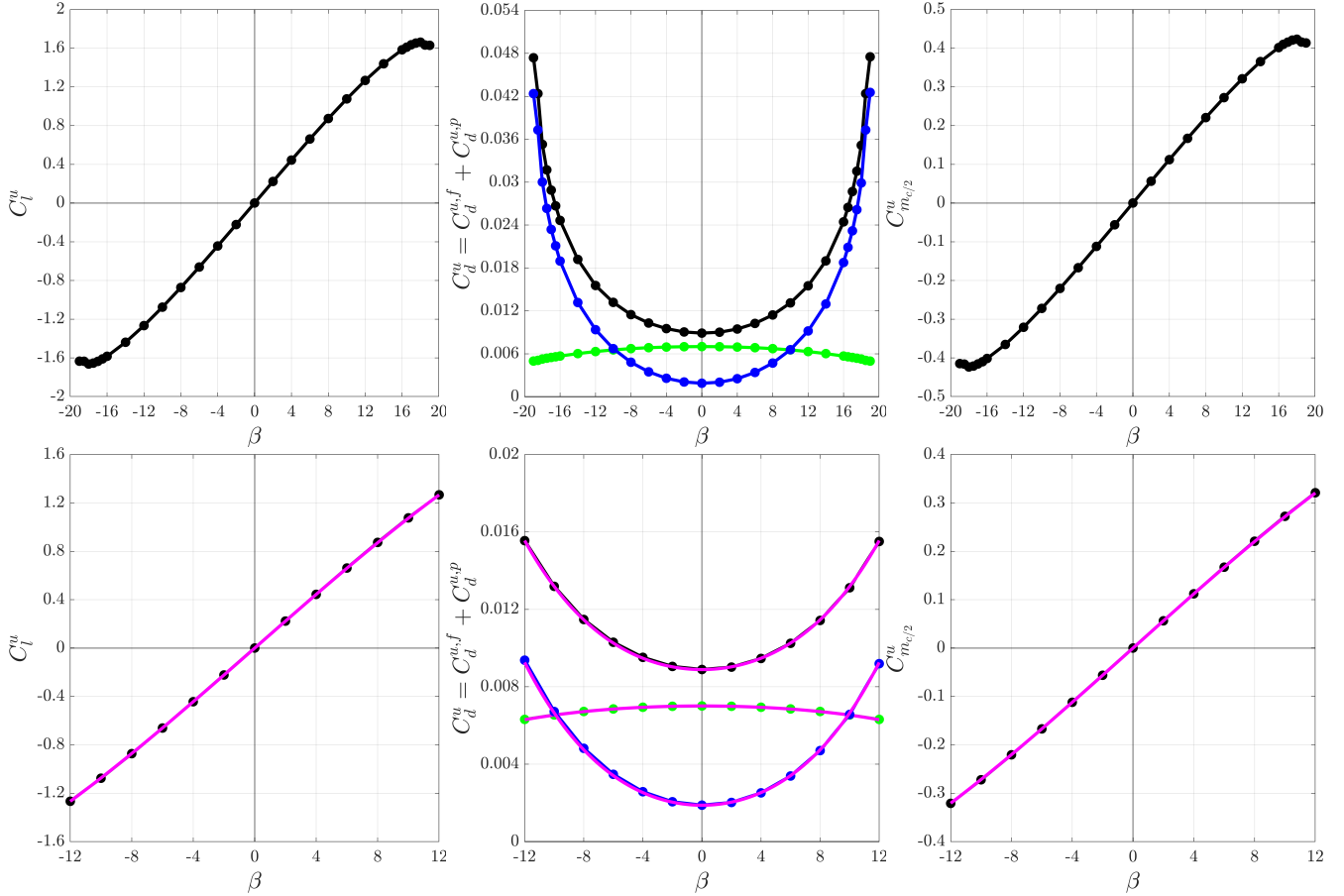


Figure A1. Coefficients C_l^u , C_d^u (with its $C_d^{u,f}$ (green) and $C_d^{u,p}$ (blue) contributions) and $C_{m_{c/2}}^u$ as a function of the angle of attack β for the NACA0015 airfoil in uniform flow: data for the complete range (top); data for the range $|\beta| \leq 12$ deg (bottom) and with the mathematical fit (magenta).

For the range with $|\beta| \leq 12$ deg, the lift coefficient can be fitted well using

$$C_l^u(\beta) \simeq 6.390\beta - 7.8\beta^3 = 6.390\beta(1 - 1.22\beta^2) \quad (\text{A1})$$

690 with β in rad. The slope of 6.390 at $\beta = 0$ was fitted using the CFD data at small angle ($|\beta| \leq 4$ deg): it is a bit larger than 2π due to the beneficial effect of thickness (also predicted by potential flow theory). The degradation when β increases (due to parasitic viscous effects on the pressure field), is however significantly faster than that of a potential flow (where it would go as $\sin\beta \simeq \beta(1 - \beta^2/6)$ for small β).



Using the same procedure, the moment coefficient around the mid-chord point, $x = c/2$, is fitted well using

$$C_{m_{c/2}}^u(\beta) \simeq 1.614\beta - 1.9\beta^3. \quad (A2)$$

The parasitic drag coefficient can also be fitted using

$$C_d^u(\beta) \simeq 0.00889 + 0.116\beta^2 + 0.82\beta^4. \quad (A3)$$

Contrary to the lift and moment coefficients (where it is mostly the pressure field that contributes, the contribution due to wall friction being negligible), there are two important contributions to the parasitic drag: the friction drag and the pressure drag.

Each can be fitted independently. We obtain:

$$\begin{aligned} C_d^{u,f}(\beta) &\simeq 0.00701 - 0.014\beta^2 - 0.05\beta^4, \\ C_d^{u,p}(\beta) &\simeq 0.00188 + 0.130\beta^2 + 0.87\beta^4. \end{aligned} \quad (A4)$$

The friction contribution is dominant at small angles (it represents 79% of the drag when $\beta = 0$), and it decreases slowly as the angle increases. The pressure contribution is moderate at small angles, and it increases rapidly as the angle increases; it even becomes dominant beyond about 10 deg.

The precise position of the aerodynamic center is then further obtained using the CFD data for $C_l^u(\beta)$ and $C_{m_{c/2}}^u(\beta)$ with $|\beta| \leq 4$ deg, and the result is $x_{ac} \simeq 0.2476c$. The distance between the aerodynamic center and the mid-chord point is thus $= (0.5 - 0.2476)c = 0.2524c$. The aerodynamic moment coefficient is then obtained using

$$C_{m_{ac}}^u(\beta) = C_{m_{c/2}}^u(\beta) - 0.2524 \cos(\beta) C_l^u(\beta). \quad (A5)$$

It is shown in Figure A2. It is indeed constant for small angles (i.e., no linear term, as it should) and it is well fitted for moderate angles using

$$C_{m_{ac}}^u(\beta) \simeq 0.710\beta^3 + 2.4\beta^5. \quad (A6)$$

The moment coefficient around $x = c/4$ (i.e., the approximate location of the aerodynamic center) is also shown: as expected, it is not totally flat for small angles. It can be fitted using

$$C_{m_{c/4}}^u(\beta) \simeq 0.0144\beta + 0.81\beta^3. \quad (A7)$$

The coefficient of the linear term is non-zero yet very small.

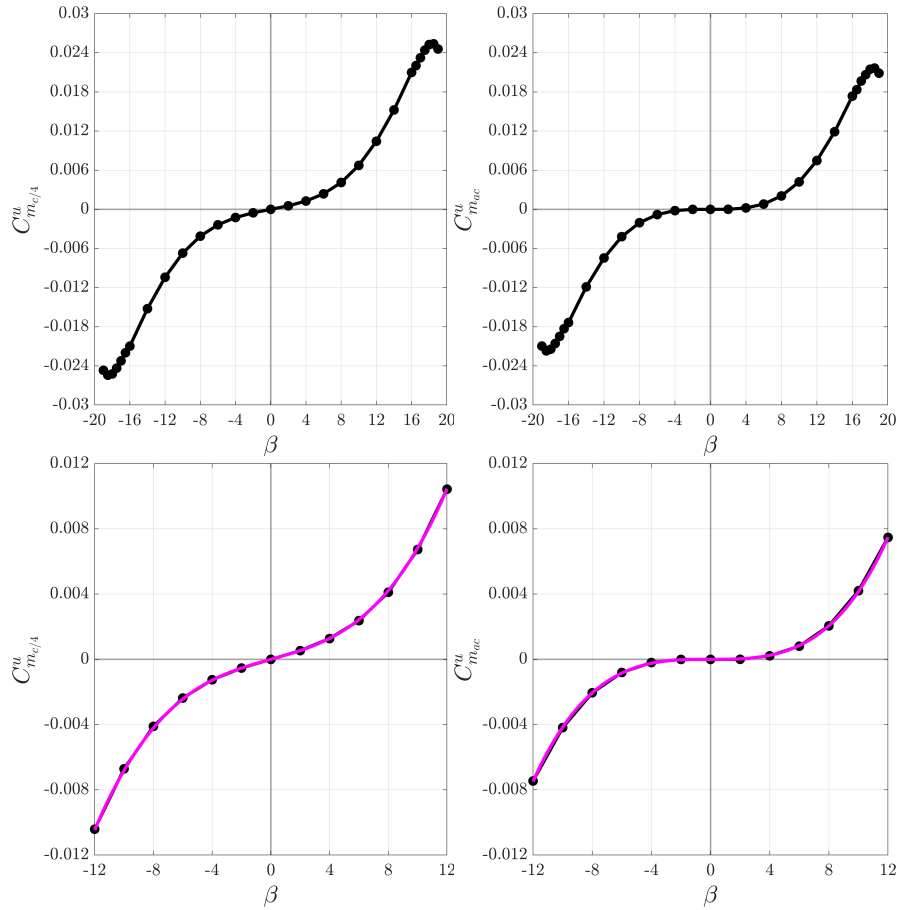


Figure A2. Coefficients $C_{\mu_{ac}}^u$ and $C_{\mu_{c/4}}^u$ as a function of the angle of attack β : data for the complete range (top); data for the range $|\beta| \leq 12$ deg (bottom) and with the mathematical fit (magenta).

Appendix B: NACA0015 airfoil in rotation and associated aerodynamic coefficients

The NACA0015 airfoil in rotation was also simulated using wall-resolved CFD at $Re_c = 6.0 \times 10^6$. Two cases were simulated: case with attachment point at mid-chord ($x_p = c/2$) and case with attachment point at quarter-chord ($x_p = c/4$).

720 B1 Simulation setup

The simulation setup, with the key-hole mesh, is shown in Figure B1. A zoom of the body-fitted mesh is also provided. The vorticity field associated with the boundary layers and the wake is also shown.

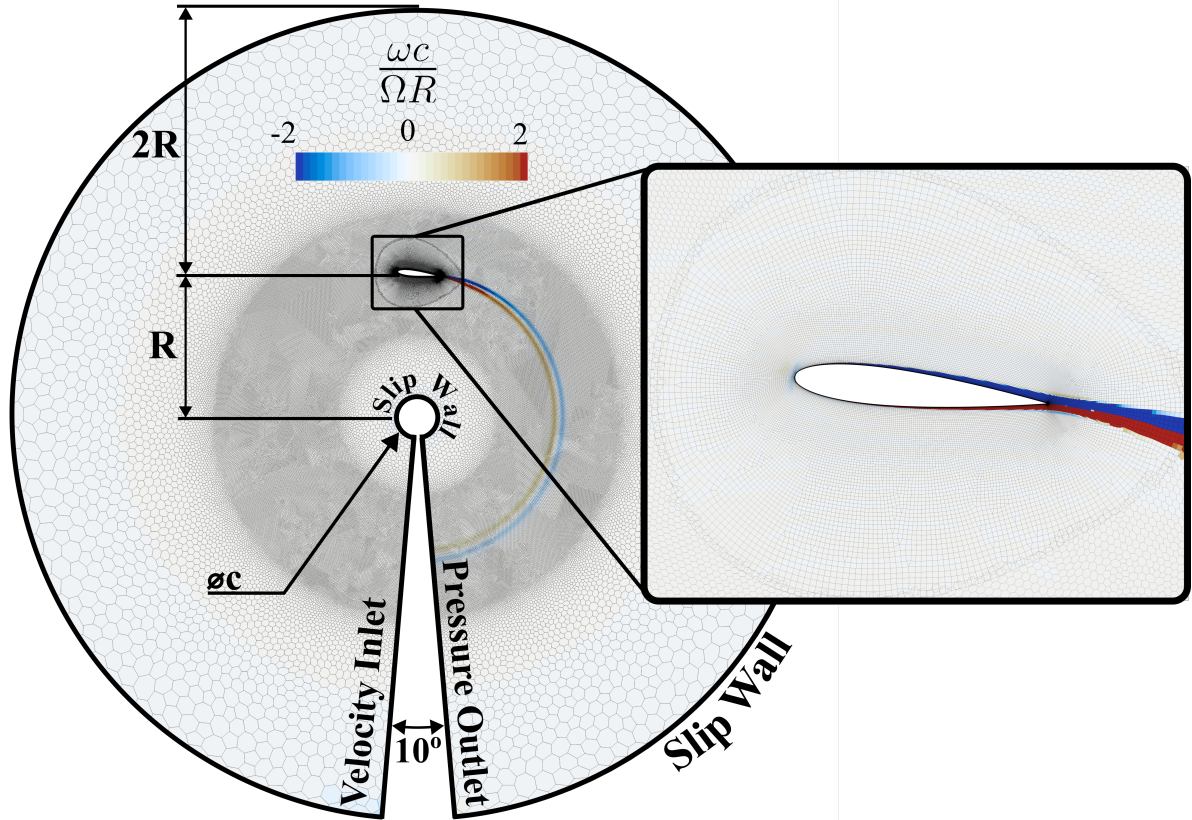


Figure B1. Simulation setup and key-hole mesh used for the simulation of the NACA0015 airfoil in rotation with $c/R = 2/7$. Case with airfoil attached at mid-chord ($x_p = c/2$) and tilting angle $\alpha_p = 6$ deg. A zoom on the body-fitted mesh around the airfoil is also shown.

B2 Airfoil attached at mid-chord ($x_p = c/2$)

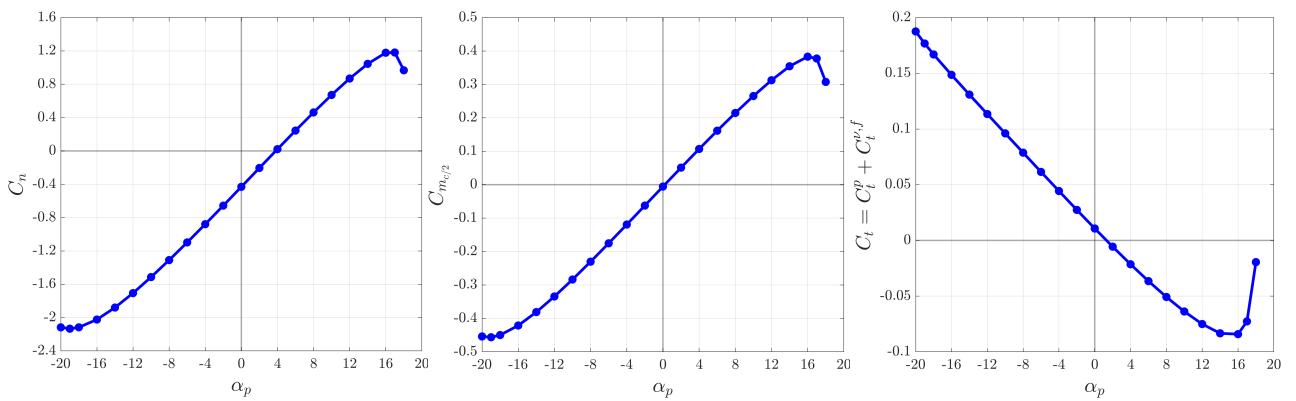


Figure B2. C_n , $C_{m_{c/2}}$ and C_t as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$.

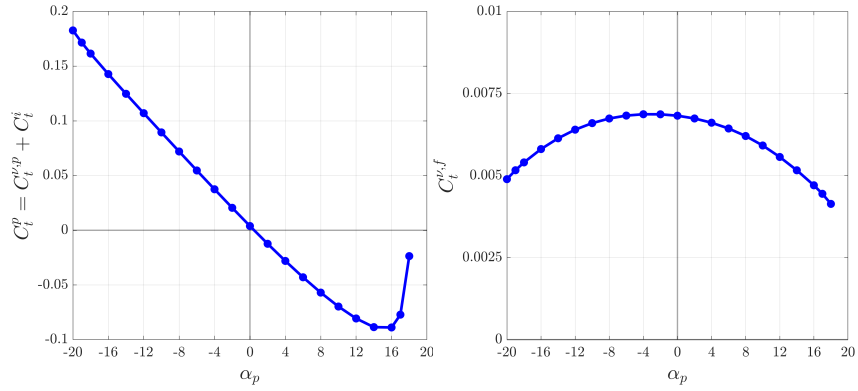


Figure B3. C_t^p and $C_t^{p,f}$ as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$.

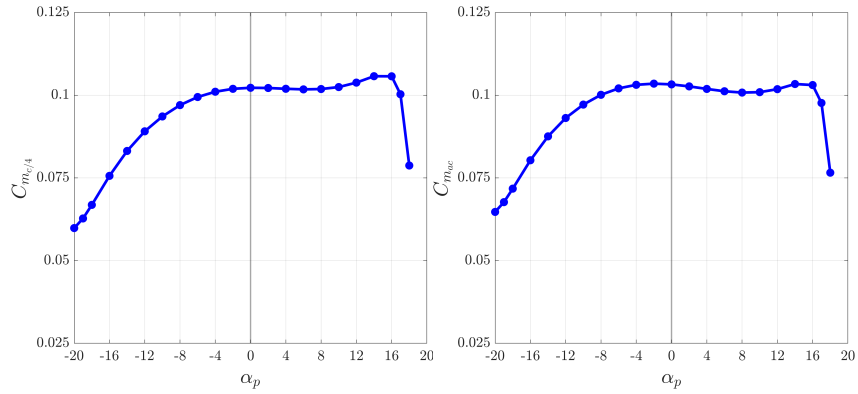


Figure B4. $C_{m_{c/4}}$ and $C_{m_{ac}}$ as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$.

B3 Airfoil attached at quarter-chord ($x_p = c/4$)

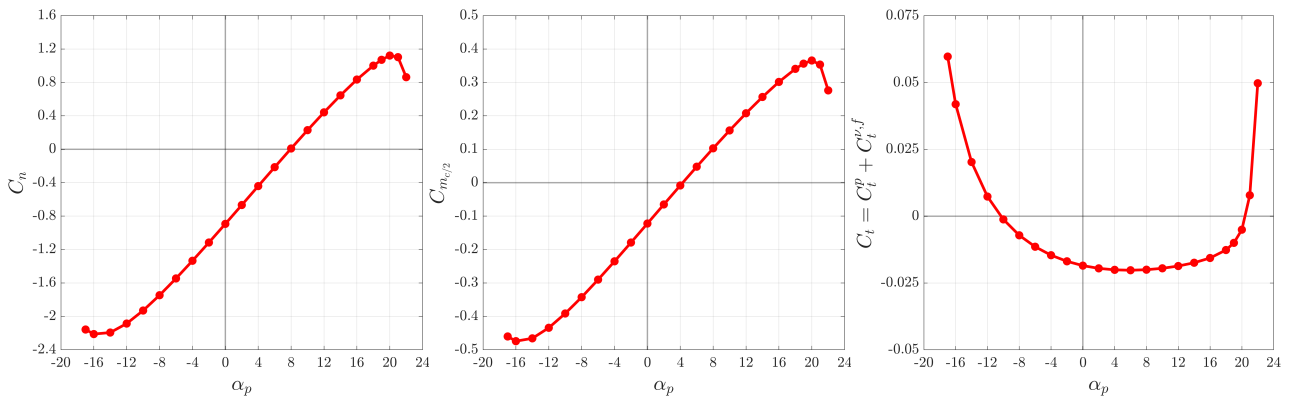


Figure B5. C_n , $C_{m_{c/2}}$ and C_t as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/4$.

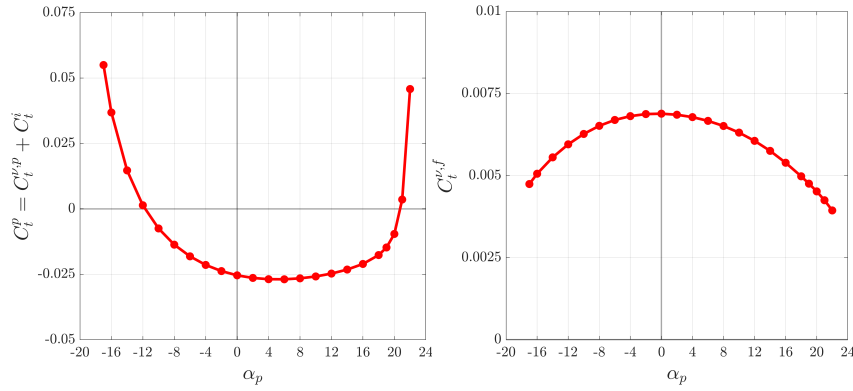


Figure B6. C_l^p and $C_l^{u,f}$ as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/4$.

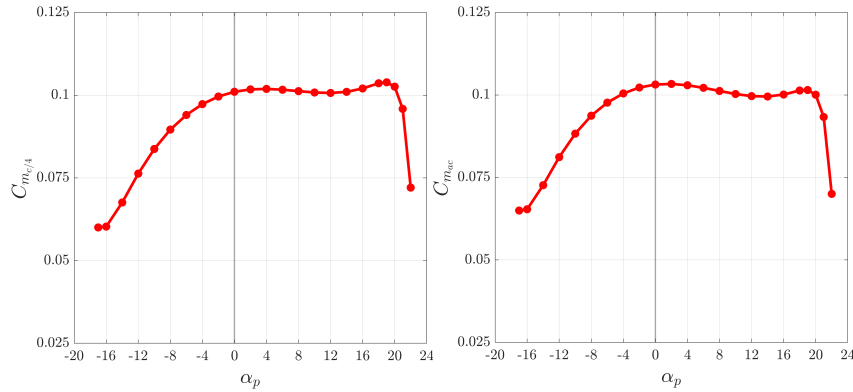


Figure B7. $C_{m_{c/4}}$ and $C_{m_{ac}}$ as a function of the pitching angle α_p for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/4$.

725 B4 Comparison

The coefficients C_l , $C_{m_{c/2}}$, C_d , C_d^p , $C_d^{u,f}$, $C_{m_{c/4}}$ and $C_{m_{ac}}$ expressed as a function of the angle of attack α at $c/4$ are compared in Figure B8, B9 and B10. For the case where $x_p = c/2$, $\alpha = \alpha_p + \alpha_c$ and for the case where $x_p = c/4$, $\alpha = \alpha_p$.

It is seen that the overlap is quite good. This supports the discussion made in the core of the paper that the “effective angle of attack” to consider in all cases is the angle that the airfoil chord makes relatively to the streamline of the curved flow at the

730 location of the aerodynamic center, in absence of that profile.

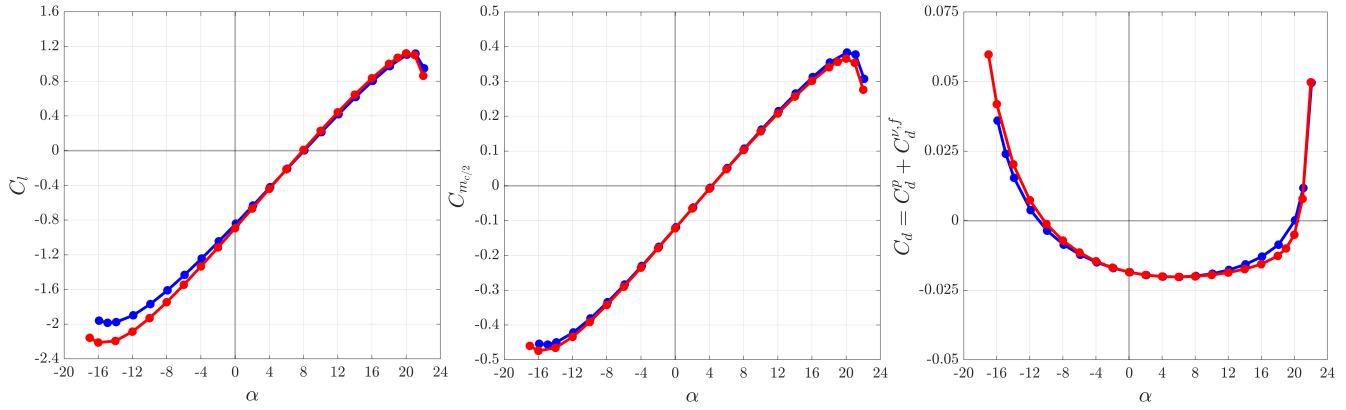


Figure B8. Comparison of the coefficients C_l , $C_{m_{c/2}}$ and C_d as a function of the angle of attack α for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$ (blue) or $x_p = c/4$ (red).

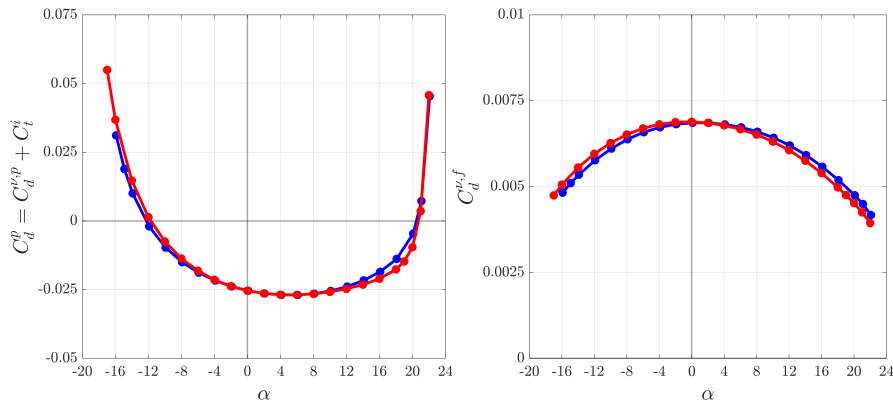


Figure B9. Comparison of the coefficients C_d^p and $C_d^{\nu,f}$ as a function of the angle of attack α for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$ (blue) or $x_p = c/4$ (red).

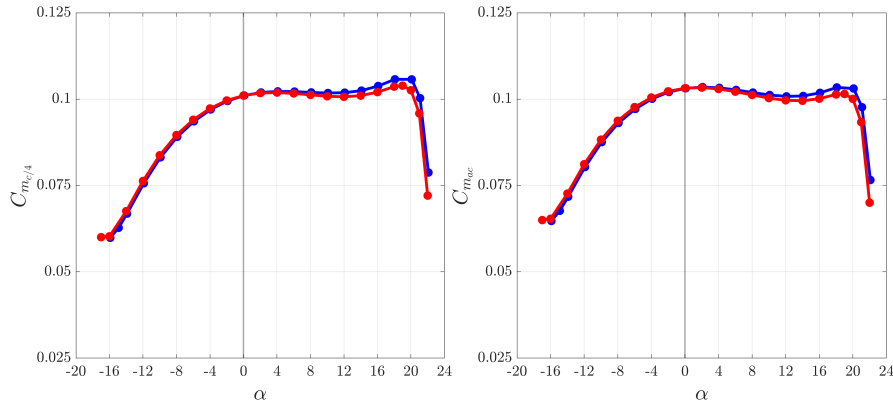


Figure B10. Comparison of the coefficients $C_{m_{c/4}}$ and $C_{m_{ac}}$ as a function of the angle of attack α for the NACA0015 airfoil in rotation with $c/R = 2/7$ and $x_p = c/2$ (blue) or $x_p = c/4$ (red).

Appendix C: Deformed NACA0015 airfoil in uniform flow and associated aerodynamic coefficients

The flow past the NACA0015 airfoil deformed using the transformation $Z(z) = z + \frac{i}{2} \frac{z^2}{R}$ was also obtained in uniform flow, using wall-resolved CFD at $Re_c = 6.0 \times 10^6$. This allows to measure the moment coefficient at mid-chord $C_{m_{c/2}}^0$ and then also account for it in the corrections for flow curvature effects. Table C1 provides the results for the deformed NACA0015 airfoil in uniform flow and the NACA0015 airfoil in curved flow, and also an ALM simulation of the deformed NACA0015 airfoil in uniform flow with moment distribution.

As predicted using the analogy presented in Section 3, the coefficients of the deformed airfoil in uniform flow match those of the airfoil in curved flow. We also see that the ALM with distribution of the aerodynamic moment is able to successfully reproduce the results of the deformed airfoil in uniform flow. Figure C1 presents the vorticity field.

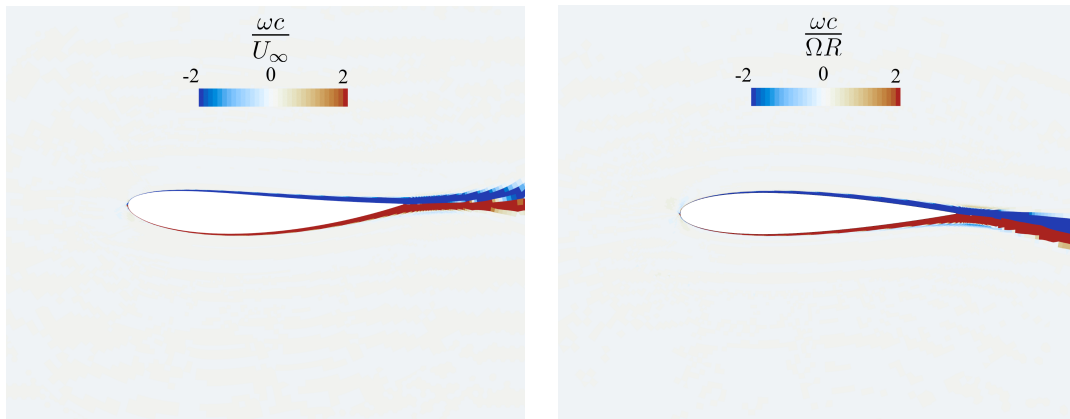


Figure C1. Vorticity field near the airfoil for the deformed airfoil in uniform flow (left) and the airfoil in curved flow (right).



Case	C_n	C_t	C_t^p	C_t^f	$C_{m_{c/2}}$	$C_{m_{c/4}}$
Deformed NACA0015 in uniform flow	-0.423	0.00926	0.00248	0.00678	-0.00746	0.0983
NACA0015 in curved flow	-0.431	0.0107	0.00387	0.00682	-0.00551	0.102
ALM of the deformed NACA0015	-0.442	0.00961	0.00268	0.00693	-0.00450	0.106

Table C1. Coefficients for the deformed NACA0015 airfoil in uniform flow, the NACA0015 airfoil in curved flow and the ALM results of the deformed NACA0015 airfoil in uniform flow.

740 Appendix D: Inviscid flow simulations

The purpose of the inviscid flow simulations was to verify that the moment m_0 around the center of rotation of the arm is indeed zero when the flow is inviscid. We employ the same setup as that presented in Figure B1. Additionally, a second-order implicit coupled flow solver is used to obtain the solution. The case studied is the NACA0015 airfoil attached at $x_p = c/4$. Each term of $C_{m_0}(\alpha_p) = C_{m_{c/4}}(\alpha_p) + \frac{R}{c} C_t^i(\alpha_p)$ is provided in Table D1. We see that C_{m_0} is indeed very close to zero; thereby confirming the argumentation that was used in developing the model for the inviscid contribution to the tangential force. Simulations were also attempted at higher pitch angles, but convergence became severely challenging due to the inviscid flow assumption.

α_p [°]	C_t^i	$C_{m_{c/4}}$	C_{m_0}
-5	-0.0388	0.1380	0.00064
0	-0.0361	0.1281	0.00045
5	-0.0335	0.1172	0.00003

Table D1. C_t , $C_{m_{c/4}}$ and C_{m_0} as functions of the pitch angle α_p for the NACA0015 airfoil in inviscid curved flow with $c/R = 2/7$. These values are obtained directly from the simulation, which explains why $C_{m_{c/4}}(\alpha_p) + \frac{R}{c} C_t(\alpha_p)$ does not exactly equal C_{m_0} .

Appendix E: Solution of curved potential flow past a plate with pitch

We develop the exact solution for curved potential flow past a flat plate at various pitch angles. For that, we first consider the circle of radius a centered at the origin in a Z -plane with (X, Y) coordinate system. We add a point vortex of circulation $\Gamma_0 < 0$ below the circle, at $Z_0 = -i R_0 e^{i\alpha_0} = R_0 \sin \alpha_0 - i R_0 \cos \alpha_0$. For the circle to be a streamline, we must add an image vortex of circulation $-\Gamma_0 > 0$ inside the circle, at $Z_1 = -i(a^2/R_0)e^{i\alpha_0}$. The complex potential is thus

$$\begin{aligned}
 F(Z) = \Phi(Z) + i\Psi(Z) &= -i\frac{\Gamma_0}{2\pi} \left[\log\left(\frac{Z-Z_0}{a}\right) - \log\left(\frac{Z-Z_1}{a}\right) \right] \\
 &= -i\frac{\Gamma_0}{2\pi} \left[\log\left(\frac{Z+iR_0e^{i\alpha_0}}{a}\right) - \log\left(\frac{Z+i(a^2/R_0)e^{i\alpha_0}}{a}\right) \right].
 \end{aligned} \tag{E1}$$



with Φ the potential and Ψ the streamfunction. The velocity field is obtained as

$$\begin{aligned}
 755 \quad \frac{dF}{dZ}(Z) = U(Z) - iV(Z) &= -i \frac{\Gamma_0}{2\pi} \left[\frac{1}{(Z - Z_0)} - \frac{1}{(Z - Z_1)} \right] \\
 &= -\frac{\Gamma_0}{2\pi} \frac{(R_0 - a^2/R_0) e^{i\alpha_0}}{(Z^2 + i(R_0 + a^2/R_0)Z e^{i\alpha_0} - a^2 e^{i2\alpha_0})}.
 \end{aligned} \quad (E2)$$

The circle in the Z -plane can be transformed into a horizontal flat plate in the z -plane, with (x, y) coordinate system, using the conformal transformation

$$z(Z) = Z + \frac{a^2}{Z}. \quad (E3)$$

760 The point $Z_l = -a$ is transformed into the plate leading edge $z_l = -2a$ and the point $Z_T = a$ is transformed into the plate trailing edge $z_T = 2a$. The chord of the plate is thus $c = 4a$. This transformation is also invertible, and we obtain

$$Z(z) = \frac{1}{2} \left(z + \sqrt{z^2 - 4a^2} \right). \quad (E4)$$

The vortex located at $Z_0 = -i R_0 e^{i\alpha_0}$ in the Z -plane is now located at $z_0 = Z_0 + a^2/Z_0 = -i R e^{i\alpha_p}$ in the z -plane, with

$$\left(R_0 + \frac{a^2}{R_0} \right) \sin \alpha_0 = R \sin \alpha_p \quad \text{and} \quad \left(R_0 - \frac{a^2}{R_0} \right) \sin \alpha_0 = R \sin \alpha_p. \quad (E5)$$

765 To each choice of distance R and tilt angle α_p in the z -plane, there corresponds a distance R_0 and angle α_0 in the Z -plane. For that, we write

$$z_0^2 - 4a^2 = - \left((R^2 \cos(2\alpha_p) + 4a^2) + i R^2 \sin(2\alpha_p) \right) = -\tilde{R}^2 e^{i2\beta}. \quad (E6)$$

The expressions for $\tilde{R}$ and β are obtained by identification:

$$\begin{aligned}
 \tilde{R} &= \left[(R^2 \cos(2\alpha_p) + 4a^2)^2 + (R^2 \sin(2\alpha_p))^2 \right]^{1/4}, \\
 770 \quad \beta &= \frac{1}{2} \arctan \left(\frac{R^2 \sin(2\alpha_p)}{R^2 \cos(2\alpha_p) + 4a^2} \right).
 \end{aligned} \quad (E7)$$

We hence finally obtain the position of Z_0

$$Z_0 = \frac{1}{2} \left(z_0 + \sqrt{z_0^2 - 4a^2} \right) = -\frac{i}{2} \left(R e^{i\alpha_p} + \tilde{R} e^{i\beta} \right). \quad (E8)$$

Once Z_0 is known, its modulus R_0 and its angle α_0 can be easily calculated.

775 The baseline case $\alpha_0 = 0$ is immediate as it corresponds to $\alpha_p = 0$, and thus also $R = R_0 \left(1 - \frac{a^2}{R_0^2} \right)$. Inverting this relation gives

$$R_0 = \frac{1}{2} \left(R + \sqrt{R^2 - 4a^2} \right) R_0 = \frac{R}{2} \left(1 + \sqrt{1 + \left(\frac{2a}{R} \right)^2} \right) = \frac{R}{2} \left(1 + \sqrt{1 + \left(\frac{c}{2R} \right)^2} \right). \quad (E9)$$



We note that the values of R_0 and α_0 are obtained quite close to R and α_p when the vortex is far enough from the plate (i.e., when the ratio c/R is “small enough”). For the case with significant curvature effects considered in this paper, that ratio is $c/R = 2/7 \simeq 0.286$. Clearly, this not very small; nevertheless, we obtain, for the case without tilt angle,

$$\frac{R_0}{c} \simeq 3.518, \quad (\text{E10})$$

which is still quite close to 3.50.

The complex potential of the flow past the plate in the z -plane is

$$f(z) = \phi(z) + i\psi(z) \quad (\text{E11})$$

and it is obtained by writing that $f(z) = F(Z)$ for $z = z(Z)$. The velocity field is then obtained using

$$\frac{df}{dz}(z) = u(z) - iv(z) = \frac{\frac{dF}{dZ}(Z)}{\frac{dz}{dZ}(Z)} \quad \text{for } z = z(Z). \quad (\text{E12})$$

To enforce the condition that the flow leaves the plate smoothly at its trailing edge, with a finite velocity there (i.e., the “Kutta-Joukowski condition”), we must add another vortex of circulation Γ_c at the center of the circle, the circulation of which being computed so that $\frac{df}{dz}(z_t)$ is finite. As $\frac{dz}{dZ}(Z_T = a) = 0$ (since $Z_T = a$ is a singular point of the transformation $z(Z)$), we must impose that $\frac{dF}{dZ}(Z_T = a) = 0$. The complex potential with the added vortex is, hence,

$$F(Z) = -i \frac{\Gamma_0}{2\pi} \left[\log \left(\frac{Z - Z_0}{a} \right) - \log \left(\frac{Z - Z_1}{a} \right) + \frac{\Gamma_c}{\Gamma_0} \log \left(\frac{Z}{a} \right) \right], \quad (\text{E13})$$

and thus

$$\begin{aligned} \frac{dF}{dZ}(Z) &= -i \frac{\Gamma_0}{2\pi} \left[\frac{1}{(Z - Z_0)} - \frac{1}{(Z - Z_1)} + \frac{\Gamma_c}{\Gamma_0} \frac{1}{Z} \right] \\ &= -i \frac{\Gamma_0}{2\pi} \left[\frac{i(a^2/R_0 - R_0) e^{i\alpha_0}}{(Z^2 + i(R_0 + a^2/R_0) e^{i\alpha_0} Z - a^2 e^{i2\alpha_0})} + \frac{\Gamma_c}{\Gamma_0} \frac{1}{Z} \right]. \end{aligned} \quad (\text{E14})$$

Imposing that $\frac{dF}{dZ}(a) = 0$ determines the circulation of the added vortex:

$$\Gamma_c = \frac{(R_0 - a^2/R_0)}{((R_0 + a^2/R_0) - 2a \sin \alpha_0)} \Gamma_0. \quad (\text{E15})$$

The total circulation of the flow around the plate in the z -plane (and also around the circle in the Z -plane) is the sum of the circulations of the two vortices inside the circle:

$$\Gamma = \Gamma_c + (-\Gamma_0) = -2 \frac{(a^2/R_0 + a \sin \alpha_0)}{((R_0 + a^2/R_0) - 2a \sin \alpha_0)} \Gamma_0. \quad (\text{E16})$$

For the case $\alpha_0 = 0$, we obtain

$$\Gamma = -2 \frac{\frac{a^2}{R_0^2}}{\left(1 + \frac{a^2}{R_0^2}\right)} \Gamma_0 > 0. \quad (\text{E17})$$



We also note that the circulation Γ is zero when $\sin(\alpha_0) = -a/R_0$; this corresponds to the case without force exerted by the flow on the plate.

805 Finally, the forces exerted by the flow on the plate can be obtained using Blasius' formula:

$$f_x - i f_y = i \frac{\rho}{2} \oint_{C_z} \left(\frac{df}{dz}(z) \right)^2 dz = i \frac{\rho}{2} \oint_{C_Z} \frac{\left(\frac{dF}{dZ}(Z) \right)^2}{\frac{dz}{dZ}(Z)} dZ. \quad (\text{E18})$$

We have used the change of variable, $z(Z)$, as the integral must be performed in the Z -plane, using a contour C_Z that contains the circle but excludes the outer vortex. Since $\frac{dz}{dZ}(Z) = \frac{(Z^2 - a^2)}{Z^2}$, we obtain

$$f_x - i f_y = -i \frac{\rho}{2} \left(\frac{\Gamma_0}{2\pi} \right)^2 \oint_{C_Z} \left[\frac{1}{(Z - Z_0)} - \frac{1}{(Z - Z_1)} + \frac{\Gamma_c}{\Gamma_0} \frac{1}{Z} \right]^2 \frac{Z^2}{(Z^2 - a^2)} dZ = -i \frac{\rho}{2} \left(\frac{\Gamma_0}{2\pi} \right)^2 \oint_{C_Z} G(Z) dZ. \quad (\text{E19})$$

810 The contour integral must be performed using the residue theorem:

$$\oint_{C_Z} G(Z) dZ = 2\pi i \sum \text{residues of } G(Z) \text{ inside } C_Z. \quad (\text{E20})$$

We thus finally obtain

$$f_x - i f_y = \rho \frac{\Gamma_0^2}{4\pi} \sum \text{residues of } G(Z) \text{ inside } C_Z. \quad (\text{E21})$$

It turns out that the only non-zero residues of $G(Z)$ are those evaluated at $Z_1 = -i(a^2/R_0) e^{i\alpha_0}$.

815

For the case $\alpha_0 = 0$ with the vortex just below the plate, and thus $\alpha_p = 0$, the flow past the plate is left/right symmetric; hence $f_x(0) = 0$ and $f_y(0)$ is also the force normal to the plate, $f_n(0)$. The sum of the residues is then obtained as

$$i \frac{1}{R_0 \left(1 - \frac{a^2}{R_0^2} \right)} \frac{4 \frac{a^2}{R_0^2}}{\left(1 + \frac{a^2}{R_0^2} \right)^2}, \quad (\text{E22})$$

and we finally obtain

$$820 \quad f_y(0) = f_n(0) = -\rho \frac{\Gamma_0}{2\pi R_0 \left(1 - \frac{a^2}{R_0^2} \right)} \frac{2 \frac{a^2}{R_0^2}}{\left(1 + \frac{a^2}{R_0^2} \right)^2} \Gamma_0 = \rho \frac{\Gamma_0}{2\pi R} \frac{\Gamma}{\left(1 + \frac{a^2}{R_0^2} \right)} = -\rho U \frac{\Gamma}{\left(1 + \frac{a^2}{R_0^2} \right)} < 0, \quad (\text{E23})$$

where we recalled the horizontal velocity $U = -\frac{\Gamma_0}{2\pi R} > 0$ induced by the vortex and evaluated at the plate center. The lift of the curved flow past the plate with the vortex directly below it is thus almost equal to $-\rho U \Gamma$: the usual formula for a plate or airfoil in a free stream at velocity U , with circulation Γ of the flow around it; and without anything else nearby.

825 Using Eq. (E9), we obtain

$$\frac{a}{R_0} = \frac{\frac{c}{2R}}{\left(1 + \sqrt{1 + \left(\frac{c}{2R} \right)^2} \right)}, \quad (\text{E24})$$



and the normal force coefficient is finally obtained as

$$C_n(0) = \frac{f_n}{\frac{1}{2} \rho U^2 c} = -\pi \frac{\frac{c}{2R}}{\left(1 + \left(\frac{c}{2R}\right)^2\right)} = -\frac{2\pi}{\left(1 + \left(\frac{c}{2R}\right)^2\right)} \frac{c}{4R}. \quad (\text{E25})$$

Consider the straight line that goes from the vortex to the plate leading (or trailing) edge: it makes an angle ϕ_c with the line
 830 that goes from the vortex to the center of the plate, such that $\tan(\phi_c) = \frac{c}{2R}$. We thus obtain that

$$\frac{\frac{c}{2R}}{\left(1 + \left(\frac{c}{2R}\right)^2\right)} = \frac{\tan(\phi)}{\left(1 + \tan^2(\phi_c)\right)} = \frac{1}{2} \sin(\phi_c) \cos(\phi_c) = \frac{1}{4} \sin(2\phi_c), \quad (\text{E26})$$

and hence

$$C_n(0) = -2\pi \frac{\sin(2\phi_c)}{4}. \quad (\text{E27})$$

We also define the half angle (used in the curvature correction models), $\alpha_c = \phi_c/2$, and write this as

$$835 \quad C_n(0) = -2\pi \frac{\sin(4\alpha_c)}{4}. \quad (\text{E28})$$

We have that

$$\frac{c}{2R} = \tan(\phi_c) = \tan(2\alpha_c) = \frac{2 \tan(\alpha_c)}{1 - \tan^2(\alpha_c)}, \quad (\text{E29})$$

which constitutes a quadratic equation for $\tan(\alpha_c)$. The solution is

$$\tan(\alpha_c) = \frac{\left(\sqrt{1 + \left(\frac{c}{2R}\right)^2} - 1\right)}{\frac{c}{2R}} \quad (\text{E30})$$

840 The streamlines of the flows obtained are shown in Fig. 1 for the case $c/R = 2/7$ and for different values of the tilt angle α_p of the plate relative to the vortex. To clearly show the flow relative to the plate, we present the streamlines in the coordinate system used in the core of the paper: the vortex Γ_0 is located at $(0, -R)$ below the plate and the plate is tilted by an angle α_p relative to the horizontal. For the case with $\alpha_p = 0$, the flow is left/right symmetric. For the case with $\alpha_p = -4$ deg, the force exerted by the flow is close to zero (it is exactly zero when $\alpha_p \simeq 4.185$ deg).

845

All the results obtained above are exact within the framework of potential flow theory. Now: for cases with $\left(\frac{c}{2R}\right)^2 \ll 1$, as those considered in this paper, we can simply write

$$C_n(0) \simeq -2\pi \sin(\alpha_c) \quad \text{with} \quad \sin(\alpha_c) \simeq \alpha_c \simeq \frac{\phi_c}{2} \simeq \frac{1}{2} \left(\frac{c}{2R}\right). \quad (\text{E31})$$

Author contributions. GW has developed the curvature correction models for the normal and tangential forces acting on a rotating airfoil
 850 attached to an arm at an arbitrary position and for pitch angles up to stall, the ALM for distributing an aerodynamic moment, the potential



flow solution of curved flow past a plate, and the curvature correction models for the normal and tangential forces of VAT configurations. PR and TV have carried, using the software StarCCM+, the blade-resolved simulations to obtain all reference results, and the ALM simulations used for comparison and validation of the curvature correction models; and using an overset mesh for the VAT simulations. FT has performed ALM simulations using an in-house code where the AL moves through the flow solver grid, also supporting the validation of the models.

855 MD has supported the development of the curvature correction models; also a first version of the tangential force model. GD has supervised the work of PR and has participated in the validation of the ALM simulations as compared to the blade-resolved simulations. The figures have been mostly produced by PR and FT.

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