

A semi-empirical model for near sea surface wind speed deficits downstream of offshore wind parks in the German Bight fitted to satellite synthetic aperture radar measurements

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Abstract. A two-dimensional advection/diffusion model for the near sea surface wind speed deficit downstream of offshore windparks is fitted to satellite synthetic aperture radar (SAR) data. The Wake2Sea model enables the inclusion of offshore wind farm (OWF) wake effects in existing atmospheric model data at low computational costs and employs the standard Fitch parameterisation to describe the momentum sink associated with wind turbines. Model wind fields from the German weather centre are used as prior information about the unperturbed atmosphere without OWFs. Using 30 Sentinel-1A/B satellites SAR scenes acquired over the German Bight representing different stability and wind speed regimes, a 4DVAR scheme is applied to optimize the agreement between simulated and observed radar cross sections. The method adjusts 8 parameters in the wake model and also applies corrections to the background wind field on a spatial scale of 40 km. An L-curve analysis is applied to choose the weighting of prior knowledge and observations in the cost function. The method improves the match between observations and simulations significantly, if uncorrected model wind fields are used as a baseline. Furthermore, the inclusion of the empirical wake model leads to improvements when the background corrected wind field is used as a reference. Comparisons with data measured at the fixed platform FINO-1 adjacent to the first German offshore wind park Alpha Ventus, showed that the proposed inclusion of wakes in the atmospheric model data leads to a significantly improved match.

1 Introduction

The global installed offshore wind energy power has increased about tenfold over the last decade reaching 64 GW in 2023 (WFO, 2023). With a share of about 47% China is currently the largest offshore wind farm (OWF) operator worldwide. Some studies suggest that over 380 GW of new offshore wind capacity will be added over the next decade globally (Williams and Zhao, 2023). In Europe the United Kingdom is the country with most installations followed by Germany with 14 GW and 8 GW installed power by 2023 respectively. According to the European Union (EU) Strategy on Offshore Renewable Energy, the installed offshore wind power in Europe will grow from about 28 GW in 2022 to about 60 GW by 2030 (EU, 2021). In Germany the goal to achieve 70 GW offshore wind energy by 2045 is written in law (Deutscher Bundestag, 2024).

It is obvious that these rapid developments come with a large spectrum of challenges in the economic, political and research sector. A large number of studies already exists, which analyze the impact of offshore wind farms on the atmosphere, often with a focus on wakes in the atmospheric boundary layer (ABL) (Siedersleben et al., 2020; Akhtar et al., 2021; von Brandis et al.,

25 2023; Platis et al., 2018). One reason for this interest is the direct implications of these wakes for the optimisation of power yields considering shadowing effects, as well as the role of turbulent wakes for the fatigue loading on downstream turbines. The respective processes in the ABL have been studied with different types of numerical models including mesoscale models (Siedersleben et al., 2020), Large Eddy Simulation (LES) models (Vollmer et al., 2017) and engineering models (Cañadillas et al., 2020). Furthermore, different types of observation techniques were applied, e.g. Light Detection and Ranging (LIDAR) systems (Schneemann et al., 2020) and spaceborne synthetic aperture radar (SAR) sensors (Djath and Schulz-Stellenfleth, 2019). The existing studies show that OWF wakes can extend well above 100 km downstream in cases where the ABL is very stable. Typical wind speed deficits are in the range 10%-20% (Djath et al., 2018). There is ongoing research about atmospheric wakes, e.g. concerning the interaction of wakes, or the coupling with coastal effects (Djath et al., 2022; Schulz-Stellenfleth et al., 2022). Furthermore, there is still debate about optimal parameterisations of OWFs in numerical models (Fischereit et al., 2022; Ali et al., 2023).

In addition to the OWF effects in the ABL, potential impacts in the ocean have gained growing attention (Christensen et al., 2013; Broström, 2008; Christiansen et al., 2022; Daewel et al., 2022). There are basically two types of processes discussed in literature so far:

- Effects caused by the modified wind forcing at the sea surface (Christiansen et al., 2022; Daewel et al., 2022).
- 40 – Effects related to the interaction of the water with the OWF foundation structures (Christensen et al., 2013; Grashorn and Stanev, 2016; Carpenter et al., 2016; Carpenter and Guha, 2024).

The present study is connected to the modelling of the first type of processes, where accurate estimates of near surface wind speeds in the surroundings of OWFs are required. A standard parameter used both in the ocean circulation and ocean wave modelling community is the wind speed at 10 meter height. As mentioned above, most studies concerned with OWF wakes in the ABL have a focus on the impacts around the hub height, which are most relevant for power yields. Near surface wind speeds around OWFs modelled with mesoscale models have been used to drive ocean models (Christensen et al., 2013), but very little has been done concerning the validation of these data. On the other hand, there is a large amount of satellite SAR data available, which provide two-dimensional (2D) information on near ocean surface wind speeds with high spatial resolution (Lehner et al., 1998), but the condensation of respective information on OWF wakes in parameterised form is still at a very basic level (Djath et al., 2018; Christiansen et al., 2022). Against this backdrop, the main objectives of the present study are as follows:

- Condense the OWF wake information contained in SAR data in a 2D semi-empirical model, which captures the main characteristics, but has small computational demands compared to a 3D atmospheric model
- Design this model as a tool for ocean modellers, to generate wind forcing for OWF impact studies, allowing the consideration of a multitude of OWF scenarios not feasible with 3D atmospheric models.

The proposed semi-empirical model can be applied a posteriori to existing atmospheric model datasets to incorporate OWF wake effects. Many of these data sets, like ERA5 (Hersbach et al., 2020), are intensely used as references by the scientific

community and the proposed tool can massively enhance the applicability of these data for studying near-surface wake effects in the offshore wind farm context.

60 The proposed model, which will be referred to as *Semi-empirical model for atmospheric offshore wind farm wakes near the sea surface (Wake2Sea)* in the following, is based on the momentum conservation law and has a relatively simple functional form. It contains 8 parameters, which were estimated using 30 SAR scenes covering a variety of different ABL stability and wind speed situations. It is shown that the model is capable of capturing major characteristics of OWF wakes like deficit intensity and wake length. The commonly used wind turbine parameterisation (Fitch parametrisations (Fitch et al., 2012) is used to include OWF properties, e.g. the thrust coefficient curve (C_T -curve), in the model and a dependence on ABL stability is contained in the formulation as well.

We would like to emphasize that the development of this model was guided by the requirements of the ocean community, i.e. the provision of wind information near the ocean surface. The current version of the model is not designed for the estimation of wind power at hub height. As expected, the model shows significantly smaller absolute wind speed deficits within wind farms for the 10 m near surface wind compared to respective deficits at hub height computed with 3D atmospheric models.

70 The paper is structured as follows: In Section 2 the Wake2Sea model for wind speed deficits is introduced. Section 3 gives information about the atmospheric model and satellite data used in the study. The model inversion approach is described in Section 4. Results of the inversion including comparisons with independent data measured at the FINO-1 platform and applications of Wake2Sea for the derivation of a yearly deficit statistics are presented in Section 5. Some theoretical analysis of the inversion results are described in Section 6 and conclusions are drawn in Section 7.

2 2D advection/diffusion model Wake2Sea for wind speed deficits

The semi-empirical wake model used in this study is based on a simplified form of the Navier Stokes momentum conservation equation for a layer of thickness dZ of the atmosphere above the ocean surface (Frandsen et al., 2006). Neglecting the Coriolis force and assuming that the 2D divergence of the horizontal wind field $U = (u, v)$ is small, i.e. negligible vertical air motion, we have

$$\frac{\partial U}{\partial t} \approx -U \cdot \nabla U + \nu_V \frac{\partial^2 U}{\partial z^2} + \nu_H \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \nabla p, \quad (1)$$

where the horizontal diffusion coefficient ν_H is assumed constant, ν_V is the vertical diffusion coefficient and p is pressure. All quantities have to be interpreted as averages of the layer dZ . The vertical diffusion term can be approximated as

$$\nu_V \frac{\partial^2 U}{\partial z^2} \approx \nu_V \frac{U_- - 2U + U_+}{dZ^2} \approx (\alpha_\chi - 2) \frac{\nu_V}{dZ^2} U =: -\chi U. \quad (2)$$

85 Here, $U_- = 0$ is the wind speed below the layer and for the layer U_+ above we assume $U_+ \approx \alpha_\chi U$ with a stability dependent parameter α_χ . Including the additional components of the model described in the following, Wake2Sea contains eight uncertain parameters $\alpha_1, \alpha_2, \dots, \alpha_8$, which are estimated as part of the inversion process.

In the following it will be necessary to have a rough estimate of the ratio between the wind at 10 m height U_{10} and the mean wind speed between the sea surface and dZ . We are using a simple functional form for the wind profile

$$90 \quad U(z) = U_{10} \left(\frac{z}{10\text{m}} \right)^E \quad (3)$$

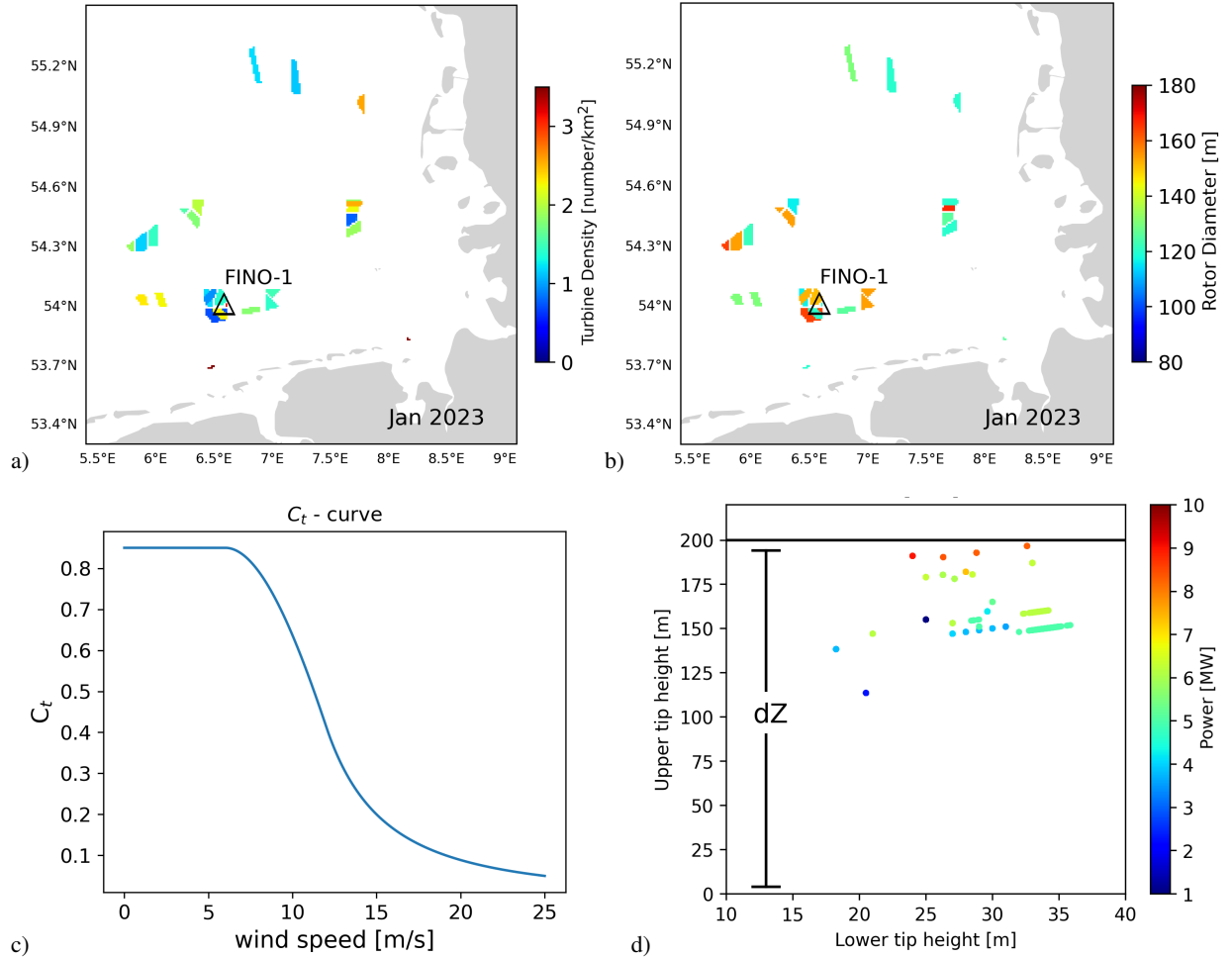


Figure 1. Number of OWF turbines per square kilometer (a) and average rotor diameter (b) for OWFs in the German Bight in January 2023. The black triangle indicates the position of the research measurement platform FINO-1. c) Smoothed version of the turbine thrust C_T -curve introduced in Siedersleben et al. (2018), shown as a function of hub-height wind speed. d) Scatter plot of lower vs. upper rotor tip height for offshore turbines in January 2023, colored by rated turbine power (in MW). The vertical bracket labeled dZ represents the vertical layer, which is used for the averaging of the prognostic variables in the two-dimensional wake model.

with an exponent E for this purpose (Jung and Schindler, 2021). A rough estimate of $E \approx 0.1$ for coastal environments can be obtained from the plots in Jung and Schindler (2021). By integration of the profile Eq. 3 we obtain

$$U = \frac{1}{dZ} \int_0^{dZ} U(z) dz = \frac{U_{10}}{E+1} \left(\frac{dZ}{10\text{m}} \right)^E \approx 1.22 U_{10} \quad (4)$$

for the layer averaged wind speed U . The wind fields obtained from the operational forecast centre refer to 10 m and are scaled according to Eq. 4 before being used as input for the Wake2Sea model. We are aware that the approximation used in Eq. 4 is very crude, however there is still overall uncertainty concerning model representations of the vertical ABL structure within wind farm wakes. Our decision to use this simplification was also driven by the necessity to keep the model simple to ensure a stable inversion of the satellite data.

Let's assume in a first step that the wind is going in the u direction. Adding the Fitch parameterisation (Fitch et al., 2012) to the momentum equation results in the following expression for the wind including atmospheric wakes

$$\frac{\partial U_{wake}}{\partial t} = \frac{\partial U}{\partial t} - 0.5 N C_T(|u_{wake}|) |u_{wake}| u_{wake} A/dZ, \quad (5)$$

where $\partial U/\partial t$ is defined in Eq. 1, N is the number of turbines per square meter, C_T is the thrust curve, and A is the rotor disc area. Eq. 5 corresponds to eq. 8 in Fitch et al. (2012). Plots of N and A representing the situation in the German Bight at the start of 2023 are shown in Figure 1a,b.

If we define the wind deficit as

$$D = \frac{|U| - |U_{wake}|}{|U|}, \quad (6)$$

a simplified version of an advection/diffusion equation for D is given by

$$\begin{aligned} \frac{\partial D}{\partial t} \approx & -U \cdot \nabla D + \frac{1}{2} N C_T[|U|(1-D)] |U|(1-D) A/dZ - \chi D \\ & + \nu_H \left(\frac{\partial^2 D}{\partial x^2} + \frac{\partial^2 D}{\partial y^2} \right). \end{aligned} \quad (7)$$

This approximation is based on the following simplifications:

- The velocity components vertical to the dominant flow are assumed to be small.
- Changes in the pressure field introduced by OWFs are not considered.
- The advection term for the deficit includes higher order terms, which were omitted to keep the numerical treatment simple.

The major impacts on the pressure field are very local with distances from the wind farm of the order of the wind farm size (Smith, 2023). As we have not observed pressure related phenomena on SAR images, e.g. blockage, and because an inclusion would lead to a much higher complexity of the the model we decided to not include this effect in the first version.. The

approximation of the advection term in particular means that the deficit is advected with the unperturbed background wind and not with the reduced wind speed, i.e. this can lead to errors in the advection of the order of 10%.

120 As Eq. 7 is invariant with respect to orthogonal transformation of the underlying grid, it is valid even if the dominant wind is not in u -direction.

The horizontal 2D wind velocity vector U required for the evaluation of the advection term in Eq. 7 is taken from existing atmospheric model data. As these wind vectors were computed with a 3D model, the vector U is not necessarily divergence free and the inclusion of a vertical advection term

$$125 \frac{\partial D}{\partial t} \approx \dots - w D/dZ \quad (8)$$

for D with w computed according to

$$w = -dZ \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (9)$$

makes sure that non-zero 2D divergence of U does not lead to meaningless production of D in Eq. 7.

The vertical diffusion parameter is known to depend on the stability of the boundary layer (Djath et al., 2018) and a respective
130 parameterisation was used in the model

$$\chi = \alpha_3^2 \Phi_+ [(1 + \alpha_4 D)(1 - \alpha_5 \Delta T)] \quad (10)$$

with the air/sea temperature difference ΔT (i.e., $\Delta T = T_{2m} \sim SST$) and a differentiable function Φ_+ defined as

$$\Phi_+[x] = \begin{cases} x^2 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

The quadratic formulation was used to ensure a differentiable dependency of the sink term on the control parameters and the
135 prognostic variable D , which is beneficial for gradient based inversion methods like applied in this study. Eq. 10 contains an additional dependence on the deficit which allows a wider range of different downstream profiles of D . The general idea is that the deficit has an impact on the vertical shear, which in turn is a key parameter for turbulence generation and hence deficit diffusion.

The C_T -curve required in the Fitch model depends on technical properties of the wind turbines under consideration and
140 respective information is notoriously hard to obtain. In this study a smoothed version of the \tilde{C}_T curve introduced in Siedersleben et al. (2018) was used as a baseline, which is shown in Figure 1c. To ensure a smooth dependence of the C_T curve on different parameters in the control vector, the following differentiable function was fitted to the "default" C_T curve shown in Figure 9 of Siedersleben et al. (2018).

$$\tilde{C}_t(u) = \begin{cases} 0.85 & \text{for } u \leq 6 \text{ m/s} \\ a_1 u^3 + b_1 u^2 + c_1 u + d_1 & \text{for } 6 \text{ m/s} < u < 12 \text{ m/s} \\ a_2 / (u^2 + b_2 u + c_2) & \text{for } 12 \text{ m/s} \leq u \leq 25 \text{ m/s} \\ 0.05 & \text{else.} \end{cases} \quad (12)$$

| a_1 [s^3/m^3] | b_1 [s^2/m^2] | c_1 [s/m] | d_1 | a_2 [m^2/s^2] | b_2 [m/s] | c_2 [m^2/s^2] |
|-----------------------------------|-----------------------------------|-------------------------------|----------|-----------------------------------|-------------------------------|-----------------------------------|
| 6.13E-04 | -2.68E-02 | 2.56E-01 | 1.50E-01 | 2.04E+01 | -9.4E+00 | 1.80E+01 |

Table 1. Coefficients used in the C_T curve Eq. 12 fitted to the "default" curve in Figure 9 of Siedersleben et al. (2018).

145 The respective coefficients are summarised in Table 1. To account for possible inaccuracies, a scaling of the C_T function according to

$$C_T^{\alpha_1, \alpha_2} = \alpha_1 \tilde{C}_T(\alpha_2 U) \quad (13)$$

is introduced as part of the inversion process described later on.

The deficit as defined in Eq. 6 refers to the mean flow in the bottom layer of thickness dZ . The deficit near the surface
150 observed by the satellite can deviate from that and hence an adjustment of the form

$$D_{10m} = D \Phi_+ [\alpha_7 + \alpha_8 D] \quad (14)$$

is applied with the function Φ_+ defined in Eq. 11. A height of 10 m has become a standard for near surface wind fields both in numerical modelling and in satellite retrieval schemes and we are therefore following this convention as well. The reasoning to include D in Eq. 14 is to allow possible dependencies of the ratio D/D_{10m} on the vertical shear and/or downstream distance.

155 The prognostic equation for the deficit D eq. 7, the expression for the vertical deficit diffusion eq.10, the correction function for the Fitch parameterisation eq. 13, and the simplified expression relating layer averaged wind deficits to surface deficits eq. 14 represent the semi-empirical wake model Wake2Sea that is used for the fitting procedure with control vector $\alpha_{\text{wake}} = (\alpha_1, \dots, \alpha_5, \nu_H, \alpha_7, \alpha_8)$ of dimension eight described in the next chapters.

For the definition of the layer thickness dZ required for the Fitch parameterisation it is important, that the major parts of
160 all turbine rotor discs are within this layer (Fitch et al., 2012). On the other hand dZ should be small enough, such that the mean deficit in this layer is still somewhat related to the conditions at the surface. The distribution of the upper tip heights for offshore wind turbines in the German Bight in January 2023 is depicted in Figure 1d. It can be seen that the layer thickness of $dZ = 200$ m used in this study includes all rotor discs. We are aware however that there is a trend towards larger wind turbines (Akhtar et al., 2024) and this must be taken into consideration in follow-up studies.

165 The Wake2Sea model was evaluated numerically using an explicit finite difference scheme on a grid with $\Delta x = \Delta y = 1$ km bin size in both dimensions, which corresponds to a typical spacing of wind turbines. This means that we are not trying to resolve wakes of individual turbines with the present setup of the model. As the typical cut-out wind speed is $u_{\text{cut}} = 25 \text{ ms}^{-1}$ and higher wind speeds do not need to be considered, the CFL criterion for the time step Δt gives

$$\Delta t \leq \left(\frac{u_{\text{cut}}}{\Delta x} + \frac{v_{\text{cut}}}{\Delta y} \right)^{-1} = 20 \text{ s} . \quad (15)$$

170 As the wind speeds of the analysed cases were well below the cut-out wind speed, a time step of 20 s was considered reasonable. The advection term was discretised using a total variation diminishing (TVD) scheme (Harten, 1997). For the simulation of a

particular SAR image, the model is started with $D = 0$ ten hours before the satellite acquisition time. This choice is based on the knowledge that wakes can extend well up to 100 km and in the extreme case of wind speeds just above the cut-in limit of 3.5 m/s the advection of deficits over such distances requires just below 10 hours.

175 3 Satellite observations and model data

3.1 SAR data

Satellite SAR is an active microwave radar, that provides information about sea surface roughness independent of daylight and weather conditions. Using the Doppler information obtained from the returned signals, SAR systems onboard European satellites such as Sentinel-1A and Sentinel-1B (Torres et al., 2012) achieve a high spatial resolution of the order of 10 m in
180 along and across flight direction. For C-band SAR systems the surface roughness length scale relevant for the radar cross section is of the order of a few centimeters and has a dependence on incidence angle. As this part of the surface spectrum is highly influenced by the surface wind, SAR has become an established tool for the derivation of 2D wind speed maps over the ocean (Lehner et al., 1998). SAR data have as well been used in the context of offshore windfarming in a larger number of studies (Ahsbahs et al., 2018; Djath et al., 2018; Djath and Schulz-Stellenfleth, 2019; Jacobsen et al., 2015; Li and Lehner,
185 2013; Hasager et al., 2011; Christiansen and Hasager, 2005). In this study Sentinel-1A/B data obtained in Interferometric Wide swath (IW) mode with VV polarisation are used. Sentinel-1A was launched on 3 April 2014 and is still active while Sentinel-1B launched on 25 April 2016 has become inoperative since December 2021. The two satellites together have an exact repeat cycle of 6 days, acquiring data with the same imaging geometry. Each acquired scene covers a swath approximately 250 km wide. For this study, SAR scenes with different imaging geometries, obtained during both ascending and descending passes,
190 were used. Sentinel-1A/B satellites operate in a sun-synchronous orbit, with overflights of the German Bight at around 6 AM UTC during descending passes and around 5 PM UTC during ascending passes, respectively.

The SAR data were radiometrically calibrated to obtain the Normalized Radar Cross Section (NRCS) using the SNAP (Zuhlke et al., 2015) software made available by the European Space Agency (ESA). This radiometric calibration ensures that the pixel values represent physically meaningful backscatter coefficients independent of acquisition geometry. After calibration,
195 the images were terrain-corrected. To reduce speckle noise (Kerbaol, 1997), the SAR images were smoothed down to 200 m grid resolution. The NRCS is a dimensionless quantity, which describes the intensity of the radar return and it is often expressed in decibels (dB) values. For assimilation into the inversion framework we used linear units of NRCS, while dB values are shown only for better visualisation in selected figures.

A couple of simple quality check were performed to exclude image points, which are likely related to perturbation due to
200 ships, wind turbines, or shallow water current features (Vachon et al., 1997; Alpers and Hennings, 1984):

1. points with $\text{NRCS} > 1$ are excluded,
2. NRSC values in areas with water depth < 10 m are excluded,
3. NRCS measurements within OWF areas are excluded, because of radar signals from turbines.

4. Finally, the total NRCS standard deviation is estimated and points with $\text{NRCS} > 2.576 \text{ STDV}$ are excluded, which corresponds to the 99% confidence limit in Gaussian distributions.

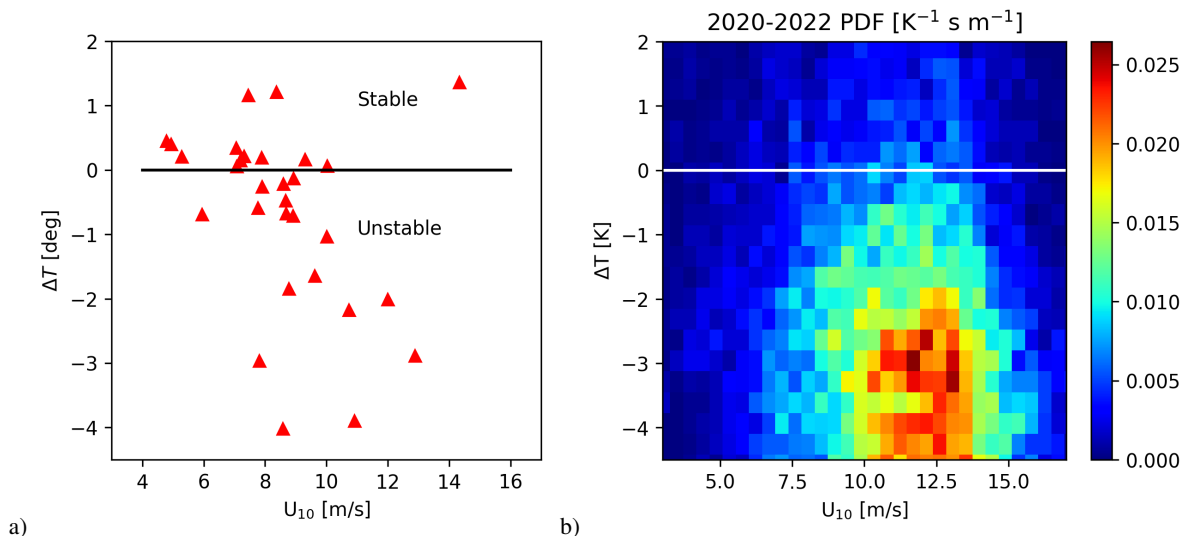


Figure 2. a) Overview of the 30 satellite radar scenes used in the study in terms of corresponding wind speed U_{10} and air sea temperature differences ΔT averaged over the OWF areas in the German Bight. b) 2D-histogram of U_{10} and ΔT estimated from DWD model data for the years 2020-2022 at the FINO-1 location.

Information about the 30 SAR scenes used in this study are summarised in Table A1. An overview of the corresponding environmental conditions in terms of the average 10 m wind speeds and the air/sea temperature differences in the OWF areas for the SAR acquisition times is shown in Figure 2a. We emphasize that we did not use the Sentinel-1 Level-2 Ocean (OCN) products, even though they provide wind speed, NRCS, and quality flags. Our assimilation framework requires continuous calibrated NRCS values as direct input, not preprocessed wind vectors. Moreover, the OCN products are delivered at coarser resolution and do not allow detailed control of calibration and filtering (e.g. removal of ship signatures, turbine returns, and shallow-water artefacts), which is essential for reliable wake inversion.

3.2 Atmospheric model data

Data from the icosahedral non-hydrostatic (ICON) atmosphere model run at the German Weather Service (DWD) were used as first guess information (Reinert et al., 2020). This model has a grid with about 7 km resolution and is routinely used for numerical weather prediction (NWP) at DWD. The model data contain hourly wind vectors at 10 m height as well as information about sea surface temperatures (SST) and temperatures at 2 m height (T_{2m}).

Figure 2b shows a 2D histogram of wind speeds and air/sea temperature differences $\Delta T = T_{2m} - \text{SST}$ computed inside the German Bight OWF areas for the years 2020-2022. One can see that there is dominance of unstable situations and that the selected SAR data provide a reasonable representations of typical conditions.

4 Inverse Modelling

The general objective of inverse modelling is to achieve a good agreement between simulations and observations by adjustment of uncertain model parameters. We have already described the wake model Wake2Sea that is applied to add wakes to existing 2D horizontal wind fields in Section 2. To simulate the NRCS measured by a SAR from these wind data a so called geophysical model function (GMF) is used. These empirical functions describe the dependency of NRCS on wind speed, wind direction and radar incidence angle and are derived by collocating SAR measurements with in-situ wind observations with a typical reference level of 10 m above the sea surface. In this study the GMF CMOD5.N is used (Portabella et al., 2002; Verhoef et al., 2008; Hersbach, 2008), which was tuned to neutral atmospheric conditions. The choice of GMF is not very critical for the present study, because the inversion is based on relative changes of NRCS due to OWF wakes and the absolute NRCS levels, which can vary slightly among different CMOD versions, are of less importance.

4.1 Implementation of the inverse modelling scheme

As the wind deficits caused by OWFs have an order of magnitude, which can be comparable to errors in the first guess model wind field, respective corrections have to be applied in addition to the adjustments of the wake model parameters. Technically, this was done using a 2D spline basis with two example basis functions shown in Figure 3. The used splines are quadratic in both dimensions and are defined on a grid with 40 km resolution. One can see that the scale of the splines is larger than the OWF dimensions and thus it can be expected that the spline corrections are not able to replicate OWF wakes and thus to interfere with the wake model.

The quadratic Bsplines are overlapping and sum up to unity (Schumaker, 2007). Denoting the spline basis as $B_j, j = 1, \dots, N_{sp}$, the correction of the u and v component of the first guess wind field for SAR image number k is computed as

$$240 \quad u_{BG}^k(x, y) = u_{DWD}^k(x, y) + \sum_{j=1}^{N_{sp}} \beta_{j,k}^u B_j(x, y) \quad (16)$$

$$v_{BG}^k(x, y) = v_{DWD}^k(x, y) + \sum_{j=1}^{N_{sp}} \beta_{j,k}^v B_j(x, y) \quad (17)$$

with coefficients $(\beta_{1,k}^u, \dots, \beta_{N_{sp},k}^u, \beta_{1,k}^v, \dots, \beta_{N_{sp},k}^v)$. The total control vector α for the optimisation problem is then defined as

$$\alpha = (\alpha_{wake}, \beta_{:,1}^u, \dots, \beta_{:,N_{images}}^u, \beta_{:,1}^v, \dots, \beta_{:,N_{images}}^v) \in \mathbb{R}^{8+2N_{sp}N_{images}} \quad (18)$$

Actually, the control vector dimension is 5728 and thus smaller than $8 + 2N_{sp}N_{images}$, because Bsplines completely outside of the area covered by the SAR scene were not considered in the inversion. The number of Bsplines used for each of the SAR images is given in the last column of Table A1. We denote the wind field obtained by feeding the corrected wind field $\mathbf{U}_{BG} = (u_{BG}, v_{BG})$ into the empirical wake model by $U_{sim}(\alpha)$. The structure of the complete forward model including the dependencies on the control vector is depicted in Figure 4.

The model inversion is then equivalent to the minimisation of the following cost function:

$$250 \quad J(\alpha) = \chi_{obs}(\alpha) + \lambda \chi_{prior}(\alpha) \quad (19)$$

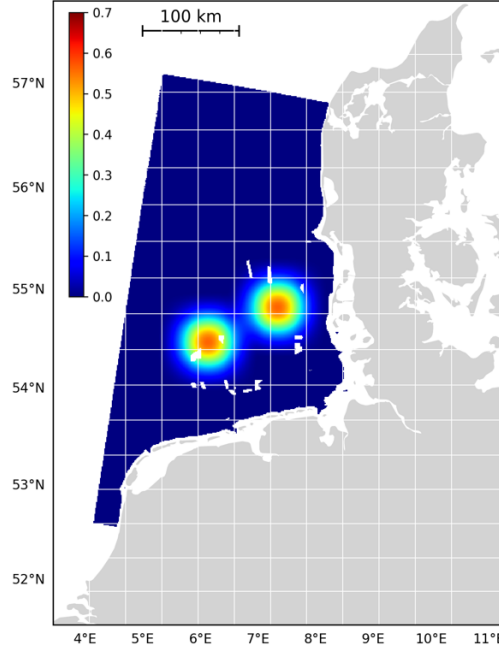


Figure 3. Computational domain used for the Wake2Sea wake simulations and inversions. The 40 km resolution grid for the quadratic Bsplines used for smooth corrections of the background wind field is superimposed. Two members of the basis are shown in color coding as examples. The color bar refers to the dimensionless spline values.

with a scalar weighting factor λ and the two cost function components associated with departures from prior information and differences between observations and simulations:

$$\chi_{prior}(\alpha) = \sum_{k=1}^8 \frac{(\alpha_k - \bar{\alpha}_k)^2}{(\sigma_k^\alpha)^2} + \sum_{k=1}^{N_{images}} \sum_{j=1}^{N_{sp}^j} \frac{(\beta_j^{u,k})^2 + (\beta_j^{v,k})^2}{(\sigma^\beta)^2} \quad (20)$$

$$\chi_{obs}(\alpha) = \sum_{k=1}^{N_{images}} \sum_{x \in SAR^k} \frac{(\text{NRCS}_{sim}(U_{sim}(\beta^u, \beta^v, \alpha_{wake})) - \text{NRCS}_{obs}^k)^2}{(\sigma_{NRCS}^k)^2}, \quad (21)$$

255 where the squared differences between the observed and simulated NRCS are summed over all SAR image points x , which fulfill the criteria described in Section 3.1. In total, about 40×10^6 SAR image pixels from $N_{images} = 30$ images were used in the inversion. For $\lambda = 1$ the parameters $\sigma^\alpha, \sigma^\beta, \sigma_{NRCS}$ can be interpreted as error standard deviations of the respective control and observation vector components. We will however not use this interpretation rigorously because the structure of the cost function implies some oversimplifying assumptions, particularly concerning the spatial independence of observation errors, and the σ -parameters are instead used to control the relative weighting of different components in the cost function. To avoid
260 the dominance of SAR images acquired at high wind speeds and large NRCS values, we chose

$$(\sigma_{NRCS}^k)^2 = \text{VAR}(\text{NRCS}^k) \times 10^5, \quad (22)$$

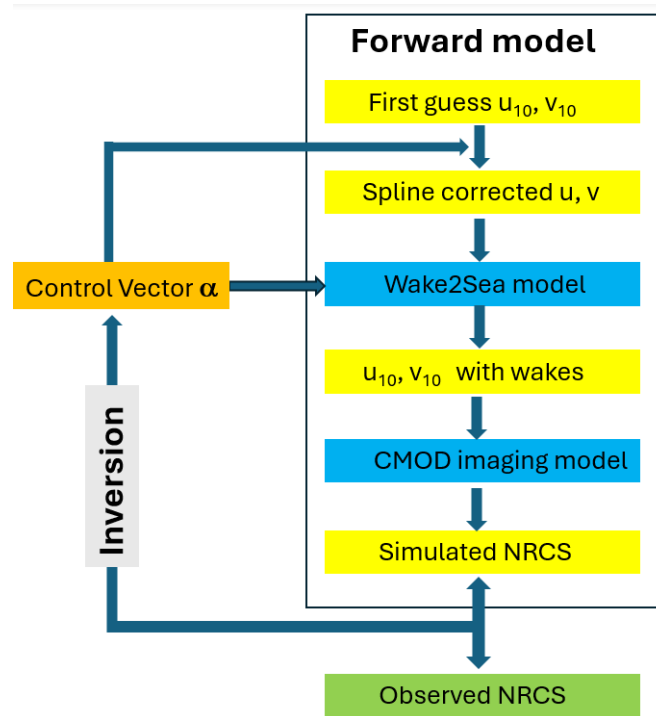


Figure 4. Structure of the forward model and the inversion procedure.

i.e., the deviations between simulations and observations are scaled with the standard deviation of each image with index k . For the weighting of the prior terms we chose

$$265 \quad \sigma^\beta = 1 \text{ m/s} \quad (23)$$

$$\sigma_1^\alpha = \sigma_2^\alpha = 144 \quad (24)$$

$$1/(\sigma_k^\alpha)^2 = 0 \text{ for } k = 3, \dots, 8 \quad (25)$$

i.e. the errors in the background wind field are assumed to be of the order 1 m/s. The error variance for the Fitch correction parameters is chosen such that a 50% correction leads to about the same increase of the cost function as a 2 m/s background
 270 wind field correction over a mesoscale patch, i.e. about 100 km. As a prior we are using $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 1$. Technically, α_1, α_2 were rescaled in the forward model, such that the cost function has the standard form commonly used for the L-curve analysis (Hansen and O'Leary, 1993). Very little is known about the remaining control vector components and they are not regularised at all. We will come back to the choice of the weighting factor λ later on in the section.

For each choice of λ the cost function Eq. 19 defines a nonlinear least-squares minimisation problem, which was solved
 275 numerically using a Gauss-Newton method (Press et al., 1992), which would probably be called incremental 4DVAR method in the context of data assimilation. In this iterative technique a linearised version of the problem is solved in each iteration step.

The equivalent linear system to be solved in each step for the correction $\Delta\alpha$ of the control vector has the form

$$0 = \frac{\partial J}{\partial \Delta\alpha} = [M_{AD}H^T G^{-1}(HM(\alpha_i) - y)]^T + M_{AD}H_{AD}G^{-1}H_{TL}M_{TL}\Delta\alpha . \quad (26)$$

Here, M_{AD} and M_{TL} denote the adjoint and tangent linear models of the advection diffusion model described in Section 2 respectively. The observation operator H and the respective tangent linear and adjoint operators H_{TL} , H_{AD} refer to the CMOD backscattering model. The tangent linear and adjoint models were hand-coded and tested using the dot-product test (Ji, 2009). Both the advection/diffusion and the CMOD imaging model are differentiable with the exception of the TVD advection scheme, which contains switches. The challenges with respect to adjoints of advection schemes have been discussed in previous studies (Liu and Sandu, 2008). The challenge of the linear problem Eq. 26 is that the matrices are not available in explicit form, but only as operators in the form of program subroutines, i.e. direct solvers are not practical. The standard approach in such situations is to use a conjugate gradient (CG) method, which is an iterative solver for symmetric systems (Press et al., 1992). With a solution $\Delta\alpha$ of the system Eq. 26 the next iteration of the control vector α is given by

$$\alpha_{i+1} = \alpha_i + \rho\Delta\alpha , \quad (27)$$

where $\rho = 1$ usually leads to a cost function reduction. If that is not the case smaller step sizes $\rho = 0.5, 0.25, \dots$ are tested until a decrease is observed.

All model components and the inversion scheme were implemented in FORTRAN90 and parallelized on a linux cluster computer. A 2D domain decomposition with 4 by 4 domains is used for the advection/diffusion model and the respective adjoint and tangent linear models. All 30 SAR images are inverted in parallel, i.e. the program requires 480 processors. The model trajectory required in the adjoint model is stored in memory with full temporal resolution, i.e. at 20 second time steps.

4.2 L-curve analysis

We now come back to the choice of the parameter λ in Eq. 19, which controls the weighting of the prior and the observation terms. A classical method to determine a reasonable value for λ is the L-curve analysis (Hansen and O'Leary, 1993), where the so called L-curve defined as

$$\Gamma(\lambda) = \left(\log(\chi_{obs}(\alpha(\lambda))), \log(\chi_{prior}(\alpha(\lambda))) \right) =: (\hat{\rho}(\lambda), \hat{\eta}(\lambda)) \quad (28)$$

is considered, with $\alpha(\lambda)$ denoting the solution of the minimisation problem for a given λ and χ_{obs}, χ_{prior} are defined in Eqs. 20,21. The L-shaped curve obtained for the inversion problem described in the previous section is shown in Figure 5a omitting the logarithm in Eq. 28. The triangle and square symbols correspond to the smallest and largest considered λ values respectively. The general idea is that it does not make sense to look at solutions which are very far on the left or very far on the right, because they are either replicating noisy observations or simply reproducing the prior knowledge. It can be justified that a choice of λ that maximises the curvature

$$\kappa(\lambda) = \frac{\hat{\rho}'\hat{\eta}'' - \hat{\rho}''\hat{\eta}'}{((\hat{\rho}')^2 + (\hat{\eta}')^2)^{3/2}} \quad (29)$$

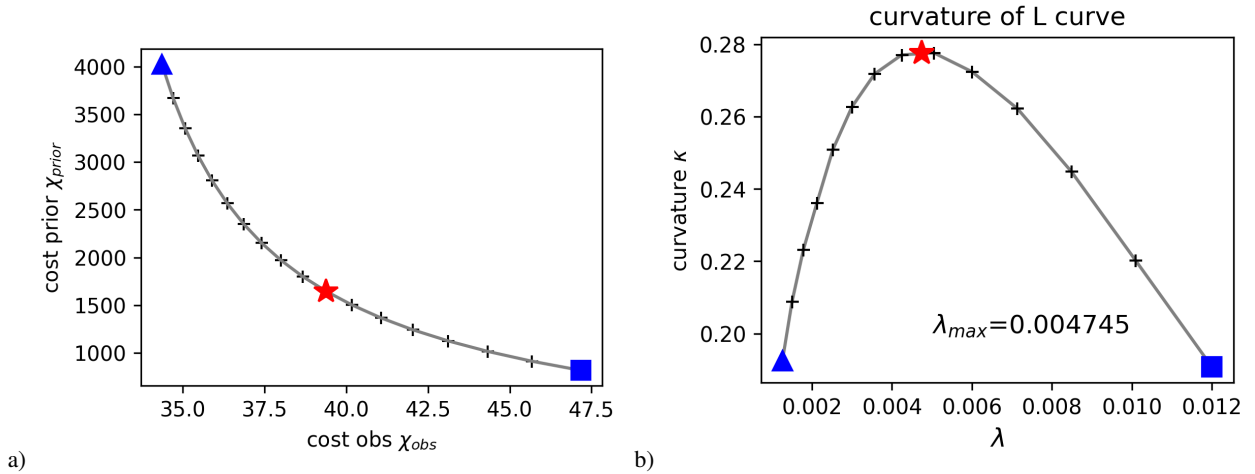


Figure 5. a) L-curve Γ without taking the logarithm (see Eq. 28). b) Curvature of Γ as a function of λ with maximum λ_{max} .

of Γ is a reasonable compromise between these two extremes (Hansen and O’Leary, 1993). The computed curvature for our case is shown in Figure 5b indicating a pronounced maximum at around $\lambda_{max} = 0.004791$. The wake model parameters estimated for different choices of λ are summarised in Figure 6b–d. The relatively smooth dependence of the solution vector
 310 on λ indicates a stable performance of the inversion procedure. The red asterisks indicate the solutions corresponding to the maximum curvature of Γ . A plot for α_1, α_2 is not shown because the departures from the first guess were negligible. Our general interpretation of the curves in Figure 6b–d is that the inversion scheme uses the wake model to compensate errors in the background wind field if the prior is weighted more strongly. As these background errors typically have a larger spatial scale than the OWF wakes, the inversion tends to diffuse the simulated wakes, e.g. by increasing the horizontal diffusion coefficient
 315 ν_H or by making the wakes longer by decrease of α_3 . Figure 6a shows the standard deviation for the magnitude of the wind speed corrections as well as the mean systematic correction of the u and v components as a function of λ . One can see that the magnitude of the systematic corrections is below 5 cm/s and that the non-systematic corrections start to grow quite rapidly below λ_{max} .

5 Inversion Results

320 The exact values for the wake model control vector components at the maximum curvature point are summarised in Table 2. The positive value for α_5 indicates that, as expected, the vertical diffusion is higher in unstable situations. The negative value for α_4 suggests that the sink term for the deficit is smaller for high deficits, which are found closer to the OWFs. This seems to be slightly counter-intuitive because higher deficits are associated with stronger vertical shear and turbulence production. We can only speculate that this might have to do with the time the turbulence requires to penetrate the layers above the rotor discs
 325 eventually increasing the downward momentum flux.

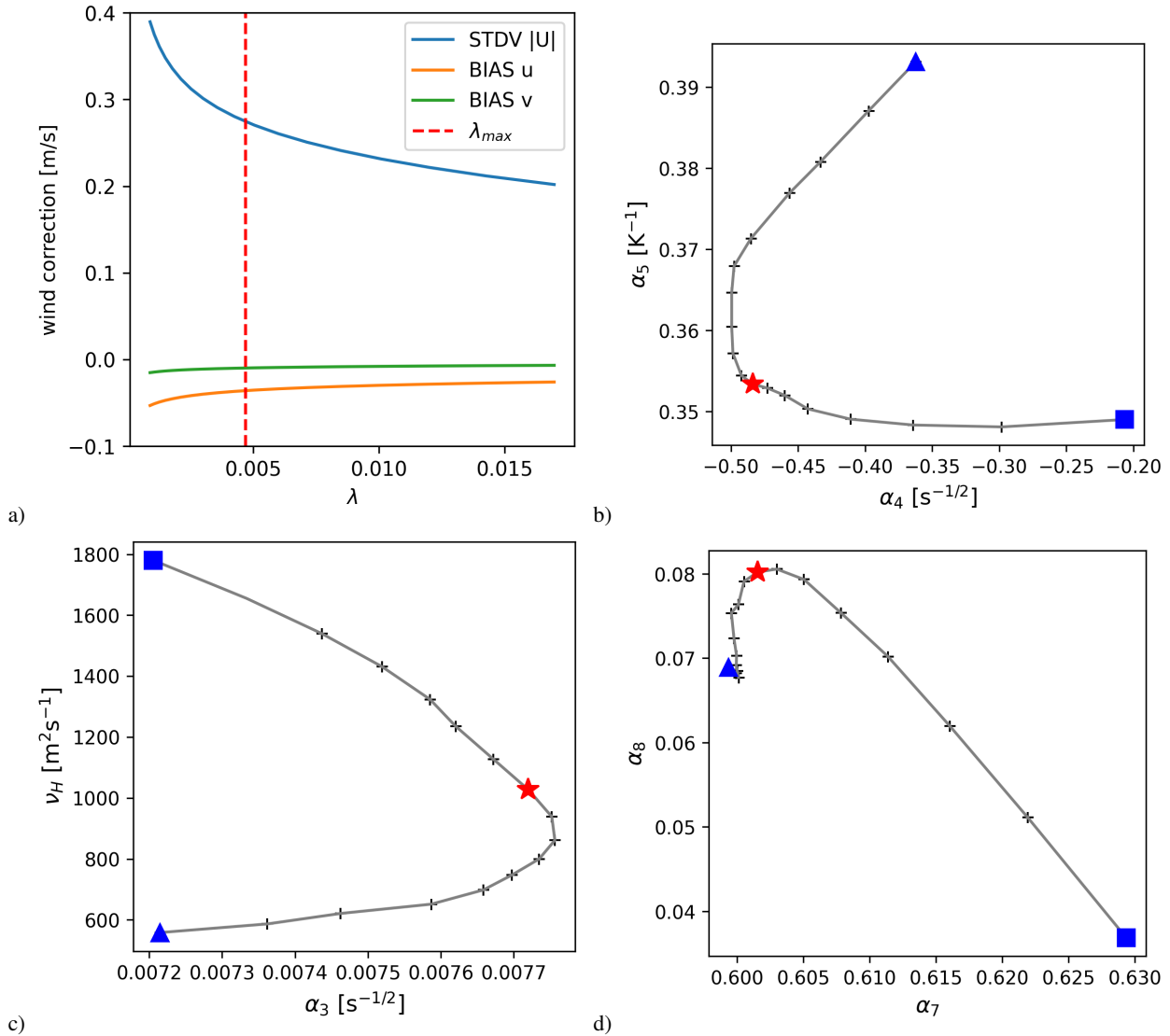


Figure 6. a) Systematic and non-systematic corrections of the background wind field as a function of λ . (b-d) Components of the estimated control vector α_{wake} as a function of λ . The red asterisk and blue triangle and square symbols correspond to the maximum curvature point λ_{max} as well as to the smallest and largest considered λ value.

The value of roughly $\nu_H=990\text{ m}^2/\text{s}$ for the horizontal diffusion emphasizes the importance of using higher order advection schemes in the simulation. The use of a first order upwind scheme in this study would have resulted in a numerical diffusion of the order of $0.5 U_{10}\Delta x \approx 5000\text{ m}^2/\text{s}$, i.e. significantly larger than the estimated physical diffusion.

As an example Figure 7 shows one of the 30 inversions for a SAR scene acquired on 15 April 2020 at 05:49 UTC. This
 330 was a slightly unstable situation with wind speeds around 9 m/s from westerly directions. The air/sea temperature map used

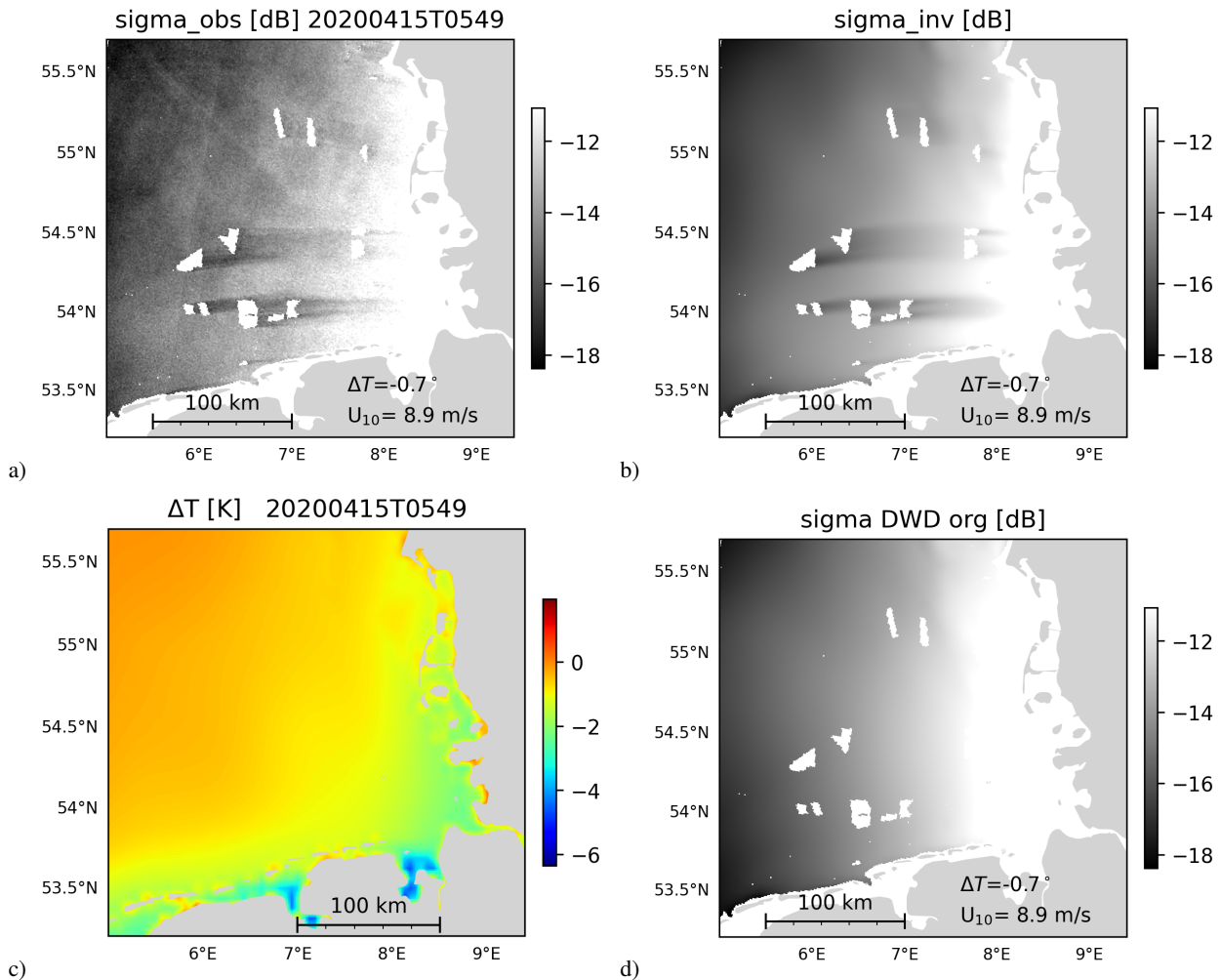


Figure 7. Example of an inversion for the SAR SENTINEL-1A SAR scene acquired on 15 April 2020 at 05:49 UTC (Copernicus Sentinel data [2020]) with measured NRCS (a), best simulated NRCS (b), sea surface temperature difference (c), and NRCS simulation based on the original DWD atmospheric model data without OWF parameterisation (d).

in the inversion is shown in Figure 7c exhibiting a typical increase towards the land caused by stronger warming of water in the shallow near coastal areas. Comparing the observed NRCS in Figure 7a with the best simulation in Figure 7b shows good agreement of the multiple wake structures in terms of intensity and size. The NRCS simulation without the wake model and background corrections of the wind field can be found in Figure 7d. It can be seen that the original DWD wind field is already quite consistent with the SAR measurements. Smaller corrections are mainly applied in the near coastal areas with stronger NRCS gradients.

A statistical overview of the inversions is presented in Figure 8. A map indicating the number of SAR acquisitions used at each location in the German Bight is shown in Figure 8a. One can see that for most of the central part close to 30 images are

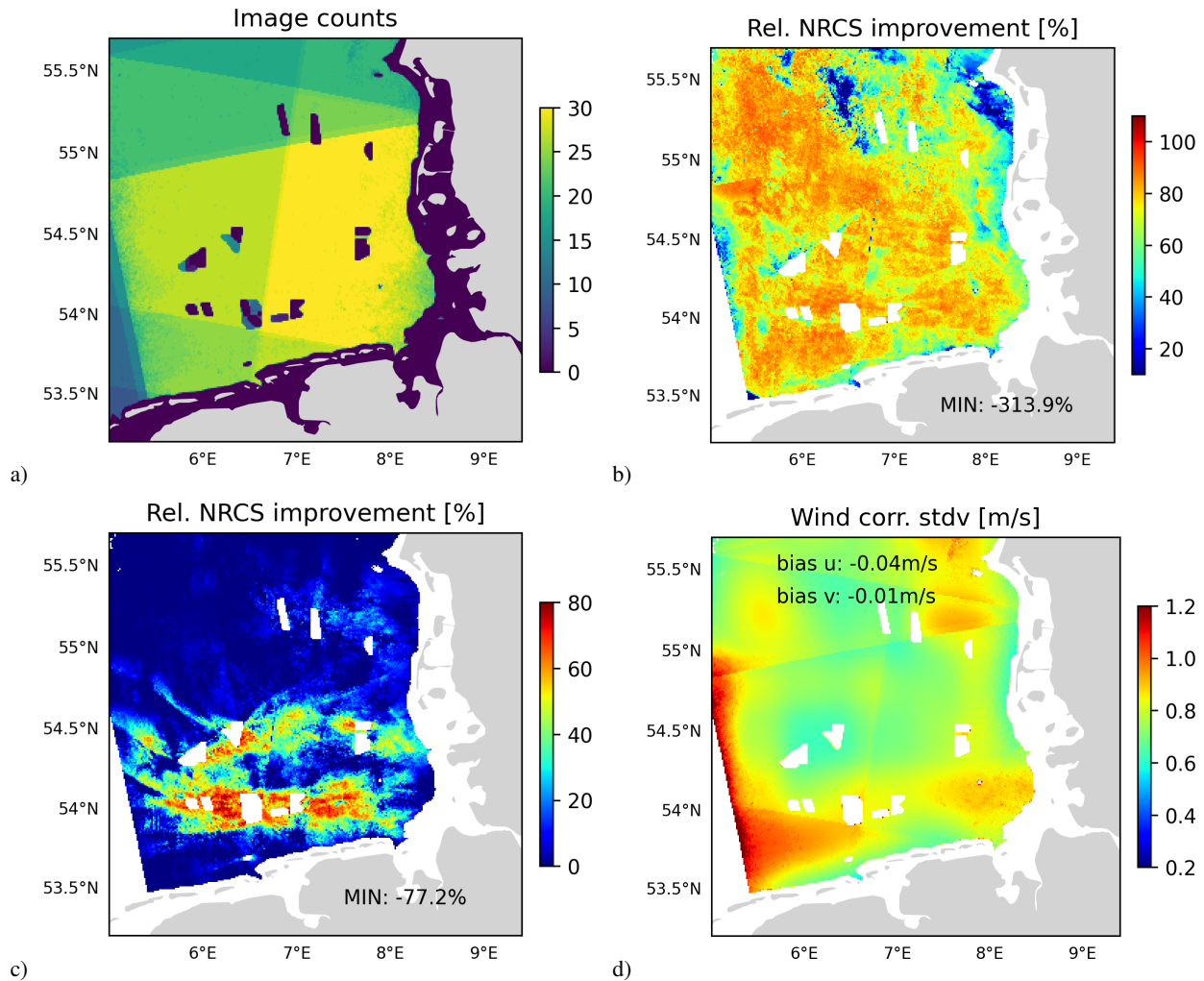


Figure 8. a) Number of images in the used Sentinel-1 data set of 30 images that cover different locations in the German Bight. b) Relative improvement of the agreement between observed and simulated radar cross section achieved by the fitting, if the original DWD wind data are used as reference. c) The same as b), but here the improvement due to the empirical wake model is shown using the large-scale corrected DWD wind data as a reference. d) Standard deviation of wind corrections applied on a scale of 40 km and above.

340 available. Only in the northern, western and near coastal margins the coverage drops to 25 images or below. Figure 8b shows the relative improvement of the RMSE comparing the NRCS simulations based on DWD data without wakes and background corrections with the best inversion results. One can see a relatively homogeneous improvement between 80% and 100%. The standard deviation of the respective background wind speed corrections is given in Figure 8d. For the most part the standard deviations are between 0.6 m/s and 0.8 m/s, which is consistent with the model error levels reported in previous studies (Rieger et al., 2021). Higher correction values are found in some areas with lower data availability in particular at the western margin

345 and these are likely dominated by some individual cases, which required stronger corrections. The overall biases for the zonal and meridional wind components are -0.04 m/s and -0.01 m/s respectively, i.e. the systematic inconsistencies between DWD data and SAR measurements are small.

In Figure 8c RMSE improvements are shown again, but this time the reference is the simulations including the background wind corrections, i.e. the plot isolates the improvements achieved by the empirical wake model alone. For that reason the main relative improvements between 40% and 80% are found in the neighborhood of the offshore wind farms with a particular focus on the eastern side corresponding to the dominant westerly wind directions.

| α_1 | α_2 | α_3 [$s^{-1/2}$] | α_4 [$s^{-1/2}$] | α_5 [K^{-1}] | α_6 [m^2/s] | α_7 | α_8 |
|------------|------------|---------------------------|---------------------------|-------------------------|------------------------|------------|------------|
| 9.9998e-01 | 1.0000e+00 | 7.7409e-03 | -4.8939e-01 | 3.5345e-01 | 9.8929e+02 | 6.0113e-01 | 7.9671e-02 |

Table 2. Estimates for the control vector α_{wake} used in the Wake2Sea model, which were obtained with the 4DVAR inversion procedure.

5.1 Simulations with Wake2Sea for a complete year

As a first application of the empirical model Wake2Sea OWF wakes were added to a complete year of DWD atmospheric model data. Figure 9 shows respective maps of estimated deficits at 10 m height for the year 2020. The wake model was applied the same way as in the inversion, i.e. hourly model data were used as input and a time step of 20 seconds was used in the explicit scheme. The only adjustment that was necessary is related to very high wind speeds above 30 m/s that occurred during some short periods in that year. Although we used a cut-out wind speed of 25 m/s above which the deficit production is switched off, spurious deficits generated at previous time steps can still exist and lead to instabilities. For this reason model wind vectors were scaled down to 30 m/s in those situation.

360 Figure 9a shows the mean percentage deficits averaged over the entire year. It can be seen that the maximum average densities with values around 8% are concentrated in a small area around the OWFs. Averages of the absolute wind speed reductions are shown in Figure 9b. One can see that the wind farm cluster in the south westerly part causes reduction of at least 0.2 m/s within an area of almost 100 km x 100 km. The deficit standard deviation displayed in Figure 9c illustrates that the area where wakes can occur from time to time is significantly larger than suggested by the mean deficit in Figure 9a. To get an impression about stronger shadowing effects Figure 9d shows the 90% deficit percentile, i.e. deficit values, which are exceeded in 10% of the cases. One can see that there are larger areas in particular between and inside the OWFs with deficit values of 15% and above.

5.2 Comparisons with FINO-1 measurements

As an independent validation of the inversion results comparisons were performed with measurements taken at the research platform FINO-1. The location of the platform is indicated by a black triangle in Figure 1a. There are two main challenges in this comparison: 1) The lowest FINO-1 measurement is at 34 m height, i.e. significantly above the 10 m reference level used in this study. 2) The FINO-1 platform is located in the centre of a large wind park cluster, i.e. the wakes measured by the platform are more inner park wakes rather than external wakes in nature. In this context it is important to remember that the interior of

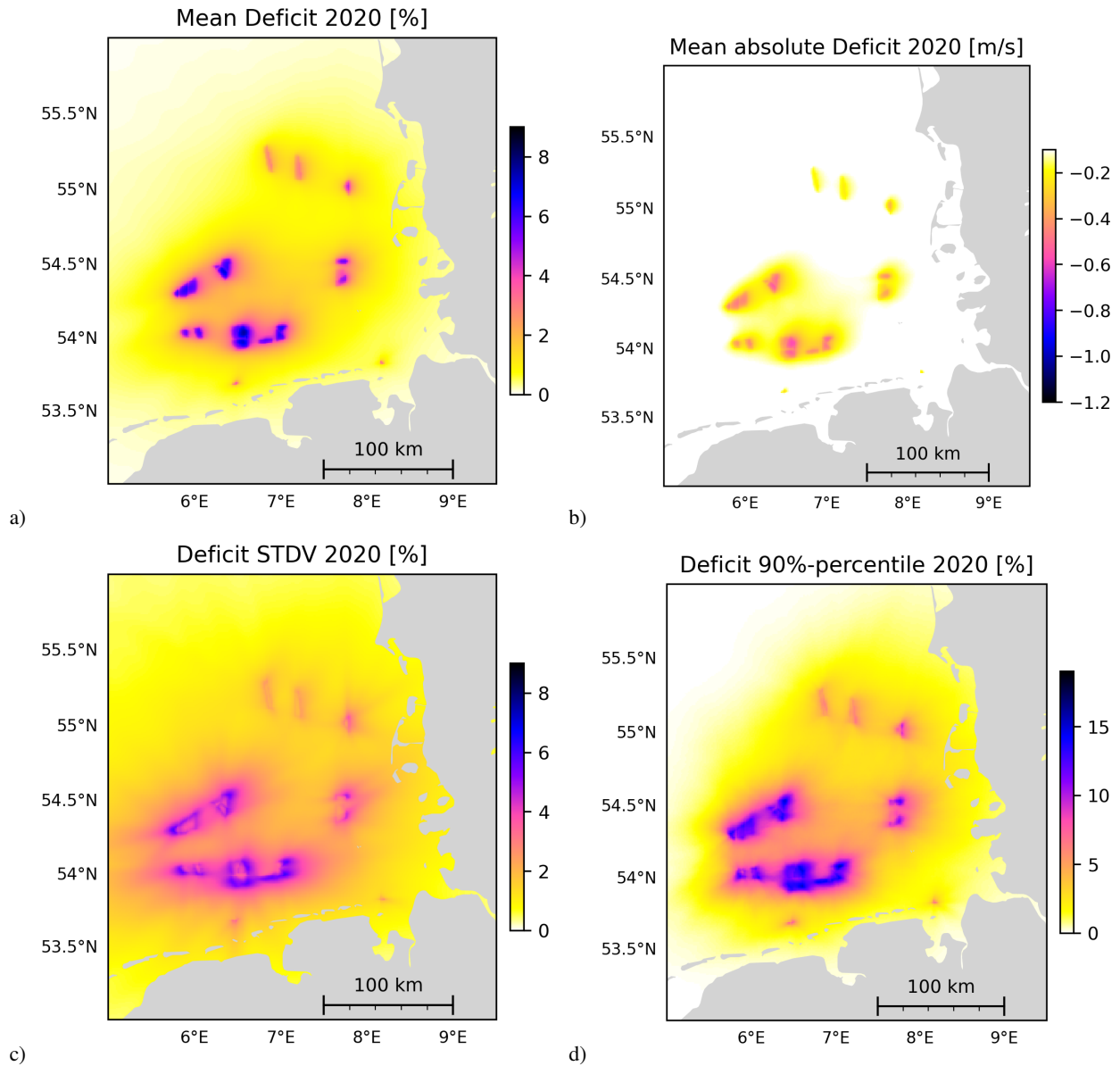


Figure 9. Mean relative (a) and absolute (b) wind speed deficits computed with the empirical model Wake2Sea for 2020. (c,d): Respective standard deviation (c) and 90%percentile of deficit.

wind parks was excluded in the inversion because of radar reflections from the turbine structures, i.e. the empirical model is not optimised for inner wind park wakes.

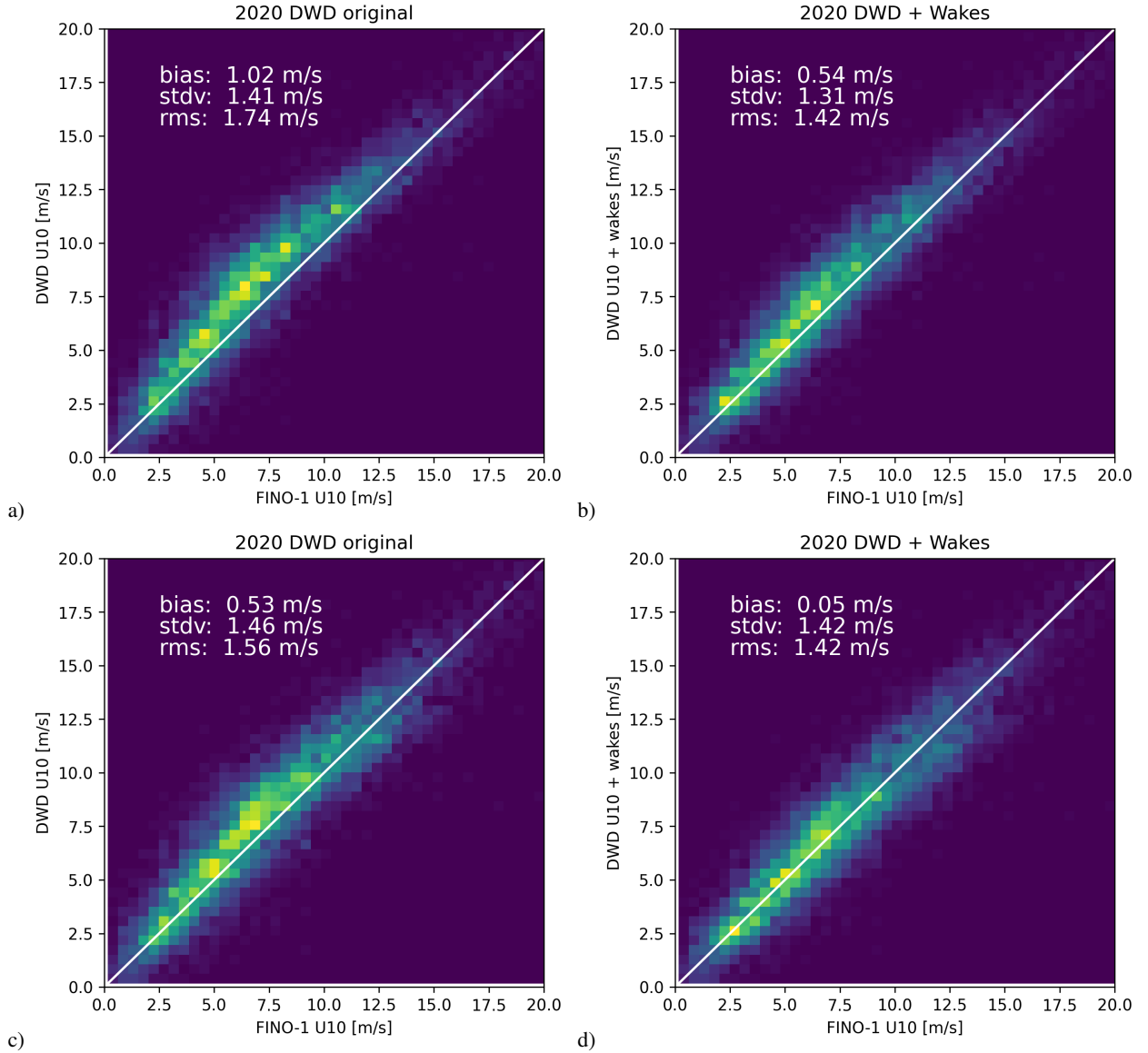


Figure 10. Comparison of FINO-1 measurements with DWD model 10 m wind speeds without wakes (a,c) and with Wake2Sea simulated wakes (b,d). Neutral conditions were assumed in the extrapolation from 34 m to 10 m in (a,b). Turbulence due to wakes was considered in the extrapolation in (c,d).

375 The 34 m FINO-1 measurements were extrapolated down to 10 m assuming a neutral wind profile $U(z) = u^* \log(z/z_0)/\kappa$ in combination with the Charnock equation

$$z_0 = 0.015(u^*)^2/g \quad (30)$$

with gravitational acceleration g , roughness length z_0 , friction velocity u^* and Karman constant κ .

The empirical model Wake2Sea was used to add OWF wakes to a complete year of DWD model data, resulting in a data set for wind speeds U_{10} with hourly sampling. The comparison of the original DWD data for the year 2020 with the extrapolated FINO-1 measurements is shown in Figure 10a. One can see a significant bias of about 1 m/s, which is consistent with the missing OWF wakes in the model data. The corresponding comparison with the model data including wakes can be found in Figure 10b. In this case the bias is reduced by a factor of two and the standard deviations and the RMSE are reduced significantly as well. Considering the remaining bias of 0.54 m/s one can now argue that the assumption of a neutral boundary layer is not realistic, particularly inside a wind park cluster. Inside the wakes the turbulence and vertical mixing is increased and one can expect that the wind speeds at 10 m and at 34 m are in closer agreement than suggested by the theoretical log profile. Following this argument a second comparison was performed in which the 10 m wind was assumed to be equal to the 34 m FINO-1 measurement for those cases where the wake model indicated a wind deficit of at least 3%. These estimates were then compared to model data with and without wakes as before. The results shown in Figures 10c,d indicate that the bias is now reduced by a factor of ten with a remaining value of 0.05 m/s and also the standard deviation is slightly improved.

In any case, the wake model improves the agreement of the model data with the FINO-1 observations, despite the fact that the model was never tuned to these interior wind cluster conditions. The order of magnitude of the respective deviations is now close to error levels associated with assumptions about the vertical profiles and this calls for more dedicated insitu measurement in offshore wind farm areas.

6 Theoretical considerations about Wake2Sea

6.0.1 Analytical downstream deficit profiles

A rough idea about the shape of the wakes produced with the Wake2Sea model can be obtained by assuming that the background wind field u is constant, neglecting lateral diffusion and considering the stationary case with $\frac{\partial D}{\partial t} = 0$. If we further assume $\alpha_4 = 0$, the solution for D is given by

$$D(x) \approx D_0 \exp(-\alpha_3^2 x/u) . \quad (31)$$

where D_0 is the initial deficit and x is the distance from the windfarm. If the deficit dependence in the sink term (see Eq. 10) is expanded to quadratic order, i.e.

$$\frac{\partial D}{\partial t} \approx \dots - D \alpha_3^2 (1 + 2\alpha_4 D)(1 - \alpha_5 \Delta T)^2 + \dots , \quad (32)$$

the stationary solution is given as

$$D(x) \approx \frac{\xi_1}{\exp(\bar{\alpha}_3 x/u) - 2\alpha_4 \xi_1} \quad (33)$$

with

$$\bar{\alpha}_3 = \alpha_3(1 - \alpha_5 \Delta T) \quad (34)$$

and

$$\xi_1 = \frac{D_0}{1 + 2\alpha_4 D_0} . \quad (35)$$

410 Figure 11a shows a comparison of the linear and quadratic approximation of the stationary deficit profiles as well the solution obtained by numeric integration of the complete sink expression for neutral conditions, i.e. $\Delta T = 0$. One can see that the quadratic approximation is in very good agreement with the full nonlinear solution and that the non-linearity leads to a slightly slower decay of the wake near the wind farm compared to the linear solution.

The distance at which the deficit has dropped to half of the initial value is then given by

$$415 \quad D_{1/2} = \frac{u}{\alpha_3^2} \log(2\xi_1(D_0^{-1} + \alpha_4)) . \quad (36)$$

Figure 11b shows this parameter as a function of air/sea temperature difference and initial deficit D_0 for a wind speed of 8 m/s. According to Eq. 36 a simple linear scaling can be applied to obtain $D_{1/2}$ for other wind speeds. One can see that there is a pronounced stability dependency and a relatively weak impact of the initial deficit D_0 .

6.0.2 Horizontal Diffusion of wake

420 The estimated lateral diffusion ν_H leads to an across wake expansion that can be analysed analytically based on some simplifying assumptions. If we assume that the across wake profile has a Gaussian shape at distance $x = 0$ km from the wind farm and furthermore the deficit is advected by a constant velocity u , then the evolution of the across track deficit profile as a function of x can be described as

$$D(y) = D(x) \exp\left(-\frac{1}{4} \frac{y^2}{\nu_H x/u}\right) , \quad (37)$$

425 where y denotes the across track coordinate with $y = 0$ corresponding to the centre and $D(x)$ is the along wake deficit profile. To separate the deficit decay due to the lateral diffusion from the effects caused by vertical diffusion, we simply assume $D(x) = \text{const}$. Figures 11c,d show examples of across wake profiles at $x=0$ km (solid black line) and at $x=50$ km (red dashed line) for $u=8$ m/s. The initial wake width is 10 km in Figure 11c and 30 km in Figure 11d. One can see that the lateral diffusion has less effect on the maximum deficit if the wake is wider, because it takes more time for the diffusive lateral fluxes to change
430 the shape of the profile.

6.0.3 Wake growth in large wind parks

Another interesting question is the maximum deficit that can be reached in very large wind parks. As the wind speed reduction caused by the wake is included in the source term in Eq. 7 the deficit cannot grow above 100% and there is a well defined saturation limit. If the linear approximation is used for the vertical diffusion, i.e. $\alpha_4 = 0$ and furthermore a constant C_T value
435 is assumed, the following solution is obtained inside the wind park

$$D(x) = D^\infty \left(1 - \exp(-x(\alpha_3^2/u + c_1))\right) , \quad (38)$$

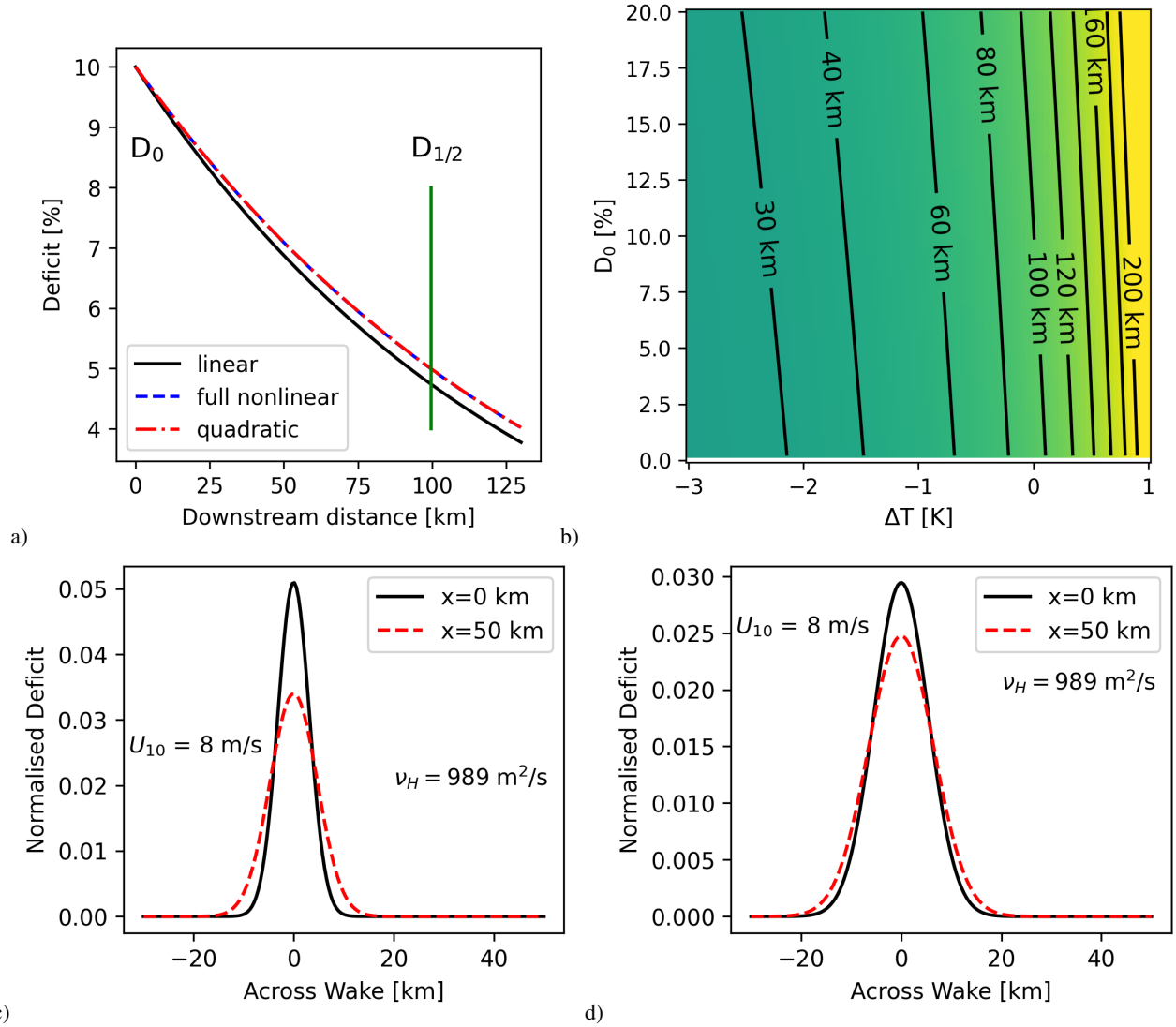


Figure 11. a) Theoretical deficit profiles computed using the linear approximation Eq. 31 (black curve), the quadratic approximation Eq. 33 (red dashed dotted) as well as numerical integration (dashed blue) assuming neutral conditions. b) Half-decay distance $D_{1/2}$ in the quadratic model as a function of the initial deficit D_0 and the air/sea temperature difference for a wind speed of 8 m/s. (c,d): Theoretical across wake profiles for $u = 8$ m/s at downstream distances of $x = 0$ km and $x = 50$ km using the lateral diffusion coefficient ν_H estimated based on SAR data. The initial wake width is 10 km (c) and 30 km (d).

where $x = 0$ defines the upstream boundary of the park and

$$c_1 = 0.5NC_T A/dZ \quad (39)$$

$$D^\infty = \frac{c_1 u}{\alpha_3^2 + c_1 u} \quad (40)$$

440 The distance after which half of the saturation deficit D^∞ is reached is then given by

$$x^{1/2} = \frac{\log(2)}{\alpha_3^2/u + c_1} . \quad (41)$$

Figures 12a,b show $x^{1/2}$ and $D_{10m}^\infty \approx \alpha_7^2 D^\infty$ as a function of wind speed U_{10} and the normalised rotor area $N \cdot A$ for $C_T = 0.8$. One can see that the internal wakes require longer distances to built up in high wind speed conditions and in OWFs with smaller turbine density. The model predicts that for slightly higher wind speeds the deficit can grow well beyond 30% in
 445 very dense wind parks. We have to emphasize again that the model was not tuned to internal wake measurements and it is implicitly assumed that the vertical deficit diffusion can be described with the same parameterisation inside and outside OWFs. Furthermore, this analysis does not include the horizontal diffusion discussed in the previous subsection ??, i.e. conceptually it refers to an infinitely wide wake. The comparison of the Wake2Sea model results with FINO-1 data in Section 5.2 suggest at least that there is no obvious overestimation of deficits inside of wind park clusters.

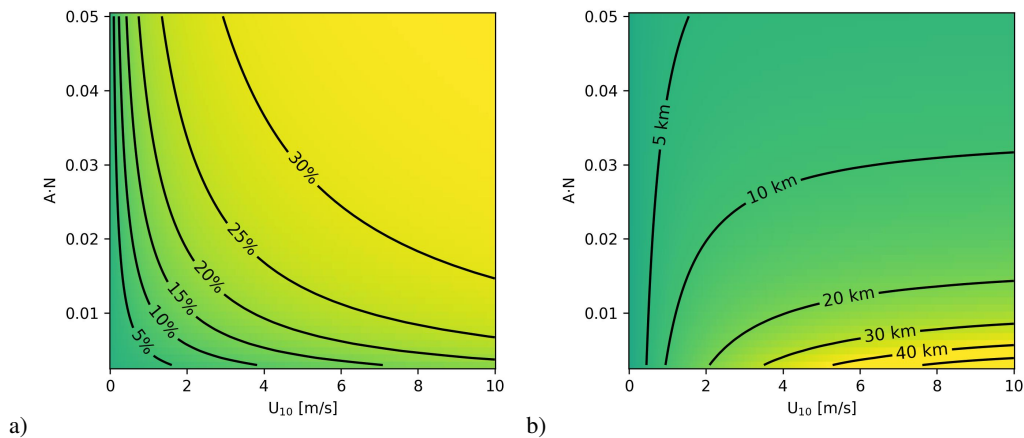


Figure 12. a) Maximum deficit D_{10m}^∞ at 10 m as a function of wind speed and relative normalised rotor area NA in big wind parks. b) The same as a) for the half distance $x^{1/2}$ required to reach D_{10m}^∞ .

450 7 Conclusions and Outlook

A 2D semi-empirical model for wind speed deficits near the sea surface downstream of offshore wind parks was fitted to satellite SAR data. The Wake2Sea model enables the inclusion of OWF wakes a posteriori into existing atmospheric model data sets at significantly lower computational costs compared to complete re-runs of full blown 3D atmospheric models. The optimisation of the method for near surface wind deficits makes Wake2Sea an attractive tool for oceanographers, who need to include
 455 OWFs in the atmospheric forcing for ocean model simulations. The application of the method to wind fields obtained from the operational German Weather forecast system lead to a significantly improved agreement with the satellite observations. Comparisons with independent measurements taken at the FINO-1 measurements confirmed that the empirical model is able

to reduce biases in meteo model data sets not including OWF wake effects. Consideration of turbulence associated with OWF wakes in the extrapolation from 34 m to 10 m height, lead to further improvements in the agreement between wake model results and in-situ observations.

The parameterisation of the empirical model allowed adjustments of the standard Fitch OWF parameterisation in terms of the momentum sink scaling as well as the dependence of the thrust curve on wind speed, however it turned out that the results of the inversion procedure only deviate marginally from the standard formulations found in literature.

The sink term for the wind deficit, which is related to vertical momentum diffusion, was found to exhibit a clear dependency on the air/sea temperature difference with lower deficit diffusion in stable conditions of the ABL. The sink term also showed a slight dependency on the deficit itself with lower diffusion at higher deficits. Further studies have to clarify whether this reflects an actual physical process or if this is an effect associated with the simplified representation of the 3D dynamics in the 2D empirical model.

The inversion results indicated that the wind speed deficit at 10 m height is about 36% of the deficit averaged over the first 200 m of the ABL. The inversions did not show a significant dependency of this ratio on the deficit itself. It is important to emphasize that the inversion scheme can potentially compensate possible deficiencies in the Fitch parameterisation by adjustment of the velocity ratio. A thorough optimisation of the OWF parameterisation would require additional information about vertical wind speed profiles inside and outside of wake regions.

We would like to emphasize that the tuning of the wake model to satellite data in the German Bight leads to limitations with regard to the general applicability. For example, the wind directions in the German Bight are predominantly from the north and west, which means that many situations are characterised by fully developed marine boundary layers and less by intermediate boundary layers associated with the close proximity of land. We think that a more thorough treatment of intermediate boundary layer cases would require an explicit prognostic inclusion of turbulent kinetic energy (TKE) in the model. Because of the substantially higher complexity and the corresponding challenges in the inversion we decided to address this issue in a separate study.

To our knowledge this study represents the first attempt to achieve and demonstrate a quantitative consistency between a physical-based OWF wake model with SAR observations considering a larger variety of wind speed and stability conditions. Despite its simplicity the model is able to capture major characteristics of the observed wakes. The approach has natural limitations because of the 2D treatment of wakes and the missing simulation of turbulence. It will be the subject of follow up studies to remedy some of these deficits with an acceptable growth of computational costs. In this context we think that dedicated insitu measurements in the ABL between the sea surface and hub height would be of great value for the further optimisation of OWF wake models for oceanographic applications.

Author contributions. Johannes Schulz-Stellenfleth implemented and ran the inversion method and coordinated the manuscript writing. Bughsin Djath took care of the pre-processing of the satellite data and contributed to the statistical analysis as well as the manuscript writing and the concept development.

Competing interests. The authors declare that there are no competing interests.

Data availability. The used atmospheric model data from the operational forecast system run at the German Weather Service are available from www.dwd.de. The FINO-1 measurements used for validation can be accessed via the website <https://login.bsh.de/fachverfahren/?localeSelected=en>. The required information on turbine densities, hub heights, and rotor diameters is available for the German wind farms from the data base provided on www.bundesnetzagentur.de. Information about the Dutch windfarms Gemini Buitengaats and Gemini ZeeEnergie was obtained from www.renewable-technology.com. The Sentinel-1 data used in this study are available at <https://scihub.copernicus.eu/>.

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Appendix A: List of used Sentinel-1 SAR data

| No. | Sensor | Acquisition Time [UTC] | ΔT [deg] | U_{10} [m/s] | N_{sp} |
|-----|--------|------------------------|------------------|----------------|----------|
| 1 | S1B | 20170222 17:16 | 1.4 | 14.3 | 64 |
| 2 | S1A | 20170314 05:48 | 1.2 | 7.5 | 73 |
| 3 | S1A | 20170407 05:48 | 1.2 | 8.4 | 94 |
| 4 | S1A | 20170604 17:16 | 0.1 | 7.2 | 106 |
| 5 | S1A | 20180108 05:48 | -4.0 | 8.6 | 124 |
| 6 | S1A | 20180130 17:16 | 0.1 | 8.9 | 65 |
| 7 | S1A | 20180223 17:16 | -3.0 | 7.8 | 122 |
| 8 | S1A | 20180321 05:48 | 0.5 | 4.8 | 65 |
| 9 | S1A | 20180331 17:16 | -0.6 | 7.8 | 65 |
| 10 | S1A | 20180625 05:49 | -1.6 | 9.6 | 65 |
| 11 | S1A | 20190122 05:40 | -3.9 | 10.9 | 130 |
| 12 | S1B | 20190401 17:16 | 0.1 | 7.1 | 118 |
| 13 | S1A | 20190409 05:48 | -1.8 | 8.8 | 118 |
| 14 | S1A | 20190527 05:48 | 0.2 | 9.3 | 119 |
| 15 | S1A | 20190630 17:16 | 0.2 | 7.3 | 130 |
| 16 | S1B | 20190730 17:16 | 0.2 | 5.3 | 119 |
| 17 | S1A | 20200120 17:17 | -0.5 | 8.7 | 121 |
| 18 | S1A | 20200203 05:49 | 0.1 | 10.0 | 121 |
| 19 | S1A | 20200401 17:17 | -0.2 | 8.6 | 119 |
| 20 | S1A | 20200415 05:49 | -0.7 | 8.9 | 121 |
| 21 | S1B | 20200515 05:48 | -0.7 | 8.7 | 98 |
| 22 | S1A | 20200516 05:40 | 0.2 | 7.9 | 81 |
| 23 | S1B | 20200630 17:16 | -2.0 | 12.0 | 81 |
| 24 | S1B | 20201004 17:16 | -2.9 | 12.9 | 104 |
| 25 | S1A | 20210911 17:17 | -0.3 | 7.9 | 70 |
| 26 | S1A | 20240401 05:41 | 0.4 | 4.9 | 64 |
| 27 | S1A | 20240416 17:17 | -1.0 | 10.0 | 64 |
| 28 | S1A | 20240512 05:49 | 0.3 | 7.1 | 64 |
| 29 | S1A | 20240603 17:17 | -0.7 | 5.9 | 70 |
| 30 | S1A | 20240605 05:49 | -2.2 | 10.7 | 106 |

Table A1. Sentinel-1A and Sentinel-1B SAR scenes acquired over the German Bight, which were used in the presented analysis. ΔT denotes the air-sea temperature difference ($T_{2m} - SST$) and U_{10} refers to the 10 m wind speed. The last column gives the number of Bspline basis functions used in the correction of the background wind field (compare Eqs. 16,17).

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