

## Author comment in reply to Referee Comment RC1 on wes-2026-138

“Brief communication: On the local disagreement of two engineering models for the radial induced velocity of non-planar wind turbine rotors”

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We thank the referee for an unusually thorough report, which went down to the source code and to independent calculations with a second code. We have repeated the analysis step by step. **All three main points and the three implementation issues are confirmed**, and the manuscript has been revised accordingly; this includes withdrawing the main quantitative statements of the discussion paper and of our earlier author comment AC1. The corrected code is released as version 6 (V6) of KSS-Blade; V5 remains the version of the discussion paper. In V6 a switch (NPform 0/1 in `Machine.in`) reproduces either formulation. Changes in the revised manuscript are marked in blue. The referee’s comments are summarised in grey italics.

## Summary of what we find

Reference case: modified NREL 5 MW rotor,  $11 \text{ m s}^{-1}$ , 12.1 rpm, 6 m linear downwind tip deflection,  $N_{sec} = 70$ .

|                                                     | discussion paper / AC1        | KSS-Blade V6   | referee (RC1) |
|-----------------------------------------------------|-------------------------------|----------------|---------------|
| deflected geometry – planar / kW                    | −67.9                         | −1.9           | ca. −2        |
| radial term, Madsen / kW                            | +75.8                         | −76.4          | —             |
| radial term, vortex cylinder / kW                   | +78.3                         | −96.0          | −91.8         |
| axial term, vortex cylinder / kW                    | not modelled                  | +95.3          | +92.3         |
| radial + axial, vortex cylinder / kW                | —                             | −0.7           | +0.4          |
| $u_r$ at outermost section (VC) / $\text{m s}^{-1}$ | 6.6, growing as $\ln N_{sec}$ | 4.3, converged | converged     |

The remaining difference between our V6 values and the referee’s is explained by the thrust coefficient used for the cylinder strengths: the referee uses the lift-only (Kutta–Joukowski) thrust coefficient, we use the full normal force including drag. With lift-only strengths we obtain −91.9 kW (radial), +92.6 kW (axial) and +0.7 kW (net), i.e. the referee’s numbers to within 0.4 kW. (The referee used the circulation of the V5 solution; with it and drag-inclusive strengths we obtain −96.5 and +95.9 kW. The referee’s full-model BEVC values, −94.5 and +95.1 kW, lie in between.)

## 1 Radial velocity near the blade tip under grid refinement

*Referee: The growth of the tip value is obtained while the outermost evaluation point moves towards the tip cylinder. KSS-Blade forms the cylinder strengths from the blade induction  $a_B$  instead of the annulus induction  $a_\infty$ ; in the grid study the outer value of  $a_B$  is kept fixed, so the tip cylinder keeps its strength ( $q = 0$ ) and  $u_r$  grows logarithmically. With strengths from  $a_\infty$  following Li et al. (2022) the value converges. The result identifies a numerical issue of the implementation, not a convergence problem of the VC formulation.*

**Reply.** We agree. In V5 the strengths were formed in `Sub3.f` from the converged induction of the BEM iteration, which contains the tip-loss factor and is therefore  $a_B$ . In our grid study  $a_B(r)$  of the  $N_{sec} = 70$  solution was fixed and its outermost value (0.23) held up to the tip, so that the tip cylinder kept the constant strength  $|\gamma_t|/2\pi = 0.81 \text{ m s}^{-1}$ ; this is the origin of the logarithmic growth we reported (fitted slope 0.78).

In V6 the annulus induction is computed from the same local thrust coefficient without tip loss,  $4a_\infty(1 - a_\infty) = 4Fa_B(1 - a_B)$  (with the Glauert branch for  $a > 0.4$ ), and the strengths are  $\gamma_t^{(k)} = 2U_0(a_{\infty,k} - a_{\infty,k-1})$ . The results are:

- re-converged BEM runs with  $N_{sec} = 35, 70, 140, 280$ :  $u_r$  at the outermost midpoint 4.12, 4.33, 4.32,  $4.27 \text{ m s}^{-1}$ ;

- with  $a_B(r)$  and  $\phi(r)$  of the  $N_{sec} = 70$  solution fixed and the tip-loss factor evaluated analytically on the refined grid,  $N_{sec} = 35 \dots 4480$ : 4.12, 4.32, 4.34, 4.32, 4.32, 4.33, 4.34, 4.35  $\text{m s}^{-1}$ , while the tip cylinder strength  $|\gamma_t|/2\pi$  falls from 0.38 to 0.02  $\text{m s}^{-1}$ , roughly as  $N_{sec}^{-1/2}$ , consistent with the referee's exponent  $q \approx 0.59$ ;
- with strengths from  $a_B$ , for comparison: 5.5, 6.5, 7.1, 7.4, 7.9, 8.4, 9.0, 9.5  $\text{m s}^{-1}$  (fit  $0.78 \ln N_{sec} + 3.0$ ).

At a fixed radius ( $r = 61 \text{ m}$ , interpolated between section midpoints) the value converges as well, to 4.40  $\text{m s}^{-1}$  (3.99, 4.25, 4.33, 4.37, 4.39, 4.40  $\text{m s}^{-1}$  for  $N_{sec} = 35 \dots 1120$ ). The numerical cap of the elliptic modulus, which had turned the spurious growth into a plateau, now lowers the tip value by less than 0.02  $\text{m s}^{-1}$  for  $N_{sec} \leq 280$  and by 0.17  $\text{m s}^{-1}$  at  $N_{sec} = 4480$ .

**Changes in the manuscript.** Sect. 3.2 is rewritten ("Tip behaviour and cylinder strengths"). The statement that the VC model is grid-dependent at the tip and requires a regularisation is withdrawn from abstract, text and conclusions. Fig. 2a shows both variants. Sect. 2 states which induction defines the strengths. The comparison of regularisations announced in AC1 is no longer needed and is not included.

*Referee: AC1 states that the sectional loads are grid converged. The supporting scripts and sectional results should be provided.*

**Reply.** The scripts (`study3.py`–`study5.py`, `decomp.py`, `vcfull.py`, `figs2.py`) and their result files are added to the repository together with V6. With the corrected formulation the moment contribution of the outer 5 % of the blade from the radial term is  $-17.8$ ,  $-15.3$  and  $-15.4 \text{ kNm}$  (VC) and  $-9.6$ ,  $-7.9$  and  $-7.9 \text{ kNm}$  (Madsen) for  $N_{sec} = 70$ , 140 and 280; the peak sectional force is  $-35.1$ ,  $-35.4$  and  $-35.7 \text{ N m}^{-1}$  (VC).

## 2 The radial term is not the complete VC result

*Referee: The complete non-planar VC model also changes the axial induction. Its contribution to power is of comparable magnitude and opposite sign and largely cancels the radial contribution. The reported percentages are not the power change predicted by the non-planar VC model.*

**Reply.** We agree, and this is now the main message of the communication. We implemented the axial velocity of the semi-infinite cylinders (closed form of Branlard and Gaunaa, 2015, with the complete elliptic integral of the third kind) in a post-processing script and verified both velocity components against direct Biot–Savart integration of vortex rings to six digits. For the planar rotor the cylinder system returns  $a_\infty$  to machine precision. For the deflected rotor we evaluate the referee's Eq. (11) at the fixed circulation of the V6 solution:

- reference case: radial  $-96.0 \text{ kW}$ , axial  $+95.3 \text{ kW}$ , net  $-0.7 \text{ kW}$  ( $-0.01 \%$ );
- five operating points ( $c_T = 0.78 \dots 0.24$ ), tip deflections 3, 6 and 9 m, three shapes (linear and bending line downwind, cubic prebend upwind), 45 cases: the radial term alone changes the power by  $-4.7$  to  $+7.1 \%$ , the complete model by  $-0.05$  to  $+0.14 \%$ . This agrees with the referee's range for prescribed circulation ( $0.06$ – $0.30 \%$ );
- if the strengths are formed from  $a_B$  as in V5, the cancellation fails (radial  $-77.8$ , axial  $+104.2 \text{ kW}$ ), which is a further consequence of point 1.

One observation goes beyond the integral: the two terms do not cancel locally. The radial term is concentrated near the tip, the axial term has its maximum at mid-span. The complete VC model therefore shifts in-plane loading from the outer 10 % of the blade ( $-30 \text{ kW}$ ) to the inner part ( $+29 \text{ kW}$ ), with a peak change of  $-27 \text{ N m}^{-1}$ , about 6 % of the local aerodynamic in-plane force.

**Changes in the manuscript.** New Eq. (5) and accompanying text in Sect. 2; Table 1 now lists the deflected rotor, the two radial terms and the complete VC result; Fig. 1b shows the spanwise distribution of radial term, axial term and sum; Fig. 2b and Sect. 3.3 give the parametric results. Title, abstract and conclusions are rewritten: the conclusion is that a radial-velocity correction must not be added to a planar momentum balance on its own, which is what happens when the Madsen correction, having no axial counterpart, is used in this way. We state in the manuscript that our evaluation is a one-way post-processing at fixed circulation for one rotor and does not replace the coupled formulation of Li et al. (2022).

### 3 Implementation issues in KSS-Blade

*Referee: Momentum balance: the non-planar section kinematics are used with the planar momentum equation, without the factor  $\cos^2 \kappa$ .*

**Reply.** Confirmed. V6 uses  $a/(1-a) = BcC_n \cos^2 \kappa / (8\pi Fr \sin^2 \phi)$  in the fixed-point iteration, in the Glauert branch and in the Newton fallback.

*Referee: Torque integration: the tangential force per unit blade length is integrated over  $dr$  instead of  $ds = dr / \cos \kappa$ .*

**Reply.** Confirmed and corrected. The thrust integration and the equation for the tangential induction are unaffected, because the factors  $\cos \kappa$  cancel there.

With both corrections the power change of the deflected geometry with respect to the planar rotor is  $-1.9$  kW instead of  $-67.9$  kW (the referee: about  $-2$  kW), and the thrust changes by  $-0.3\%$  instead of  $-0.7\%$ . The “coning effect” of  $-1.4\%$  of power stated in AC1 was an artefact and is withdrawn.

*Referee: Sign of the radial-force contribution: KSS-Blade counts the dihedral angle positive downwind but inserts it into the term  $\rho \Gamma u_r \tan \kappa$  of Li et al., who define  $\kappa > 0$  for upwind dihedral.*

**Reply.** Confirmed. For the downwind deflection of the reference case an outward  $u_r$  reduces the velocity component normal to the blade axis and therefore the driving force. All power changes attributed to  $u_r$  in the discussion paper and in AC1 have the wrong sign. The radial velocity itself is not affected, since  $u_r$  of a semi-infinite cylinder is symmetric with respect to its starting plane. In V6 the input deflection is positive downwind and the sign is corrected; the manuscript adopts the convention of Li et al. ( $\kappa > 0$  upwind) throughout.

While revising the code we found one further inaccuracy: the dihedral angle was evaluated by a backward difference, i.e. half a section inboard of the section midpoint. V6 uses a central difference. This matters only for curved shapes (about  $0.1\%$  of power for the 9 m prebend).

**Changes in the manuscript.** New paragraph “Non-planar BEM” with Eq. (1) in Sect. 2, which also states what was wrong in the discussion paper; sign convention stated with Eq. (2).

### Statements of AC1 that are withdrawn

For clarity we list the statements of our author comment AC1 that do not hold: (i) “coning reduces thrust by  $0.7\%$  and power by  $1.4\%$ ”; (ii) “the  $u_r$  term adds  $+75.8$  kW (Madsen) and  $+78.3$  kW (vortex cylinder)” and “ $u_r$  worth  $+1.6$  to  $+3.9\%$  of power”; (iii) the differences between the two models of  $3\%$ ,  $12$ – $13\%$  and  $27$ – $29\%$  — with strengths from  $a_\infty$  and the correct sign the radial terms differ by  $25$ – $27\%$  (linear),  $32$ – $36\%$  (bending line) and  $46$ – $53\%$  (prebend) at high loading; (iv) the tip values and the comparison of regularisations, which rested on the strengths from  $a_B$ . The statement that the sectional loads are grid-converged holds, and the local disagreement of the two models (factor  $1.8$ – $1.9$  on the outer  $5\%$  of the blade, opposite sign of  $u_r$  between  $r = 6$  and  $31$  m) remains.

### Line-specific comments

1. *Line 25, sweep.* The formulation is restricted to out-of-plane geometry. “Sweep” is removed and the restriction is stated.
2. *Line 28, “local cone angle”.* Replaced by “local dihedral angle of the blade axis with respect to the rotor plane”, with the sign convention ( $\kappa > 0$  upwind, following Li et al.) and coning as the special case of constant  $\kappa$ .
3. *Line 86, “codes of this class will in general contain such guards”.* Removed. The cap is now mentioned only as a detail of our implementation.

4. *Line 109, repository address.* Corrected to <https://github.com/Schaffarczyk/KSS-Blade/>; the code is called KSS-Blade throughout.
5. *Line 109 and Table 1, reproducibility (thrust  $-0.42\%$ , power  $+0.5\%$ ).* We found the cause. The folder NREL-5MW of the repository did not contain the polars of the two cylindrical root sections (CYL1.aer, CYL2.aer). With the cylinder polars of the IEAwind-15MW folder ( $c_D = 0.5$  and  $0.3$ ) instead of the ones we used ( $c_D = 1.0$ ) the V5 code gives 692.2 kN and 4751.8 kW, i.e. exactly the offsets the referee reports ( $-0.42\%$  and  $+0.49\%$  with respect to 695.1 kN and 4728.7 kW). The two files are added to the repository. With them the sources of the repository reproduce Table 1 of the discussion paper to all digits.
6. *Line 62, definition of the 6 m deflection.* Now defined in Sect. 2:  $z(\xi) = df(\xi)$  at unchanged projected radial stations, so that the projected tip radius remains 63 m; the linear case is equivalent to a straight blade coned by  $5.7^\circ$  downwind whose length is increased by  $1/\cos \kappa$ .
7. *Line 61, modified NREL 5 MW rotor.* The rotor is now called “modified” and the deviations are listed: root radius 0.3 m instead of the hub radius of 1.5 m, three additional tip stations (60.0, 62.32, 63.0 m) with the chord tapered to 0.01 m, and — which we add —  $c_D = 1.0$  for the two cylindrical root sections (reference: 0.50 and 0.35).
8. *Lines 9, 30, 47, original sources of the Madsen model.* Madsen (1997) and Madsen et al. (2010) are cited.

## Format

The revised manuscript returns to the limits of a brief communication (one table, two figures, abstract of 100 words, 14 references); the additional figures announced in AC1 are not included. The proposed new title is “Brief communication: On the use of two engineering models for the radial induced velocity of non-planar wind turbine rotors”.

## Use of AI tools

The re-analysis of the source code, the implementation of the corrections (V6), the calculations reported here and the drafting of this reply were carried out with the assistance of Anthropic’s Claude. The authors have checked the results and are responsible for the content.

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