

Author comment in reply to Referee Comment RC2 on wes-2026-138

“Brief communication: On the local disagreement of two engineering models for the radial induced velocity of non-planar wind turbine rotors”

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We thank the referee for placing the radial velocity in a wider theoretical context and for two concrete suggestions. Both could be tested with the models at hand and both are taken up in the revised manuscript (new Sect. 3.4; changes are marked in blue). We note that the manuscript has changed substantially in response to Referee 1 (see our reply to RC1): the cylinder strengths of the vortex-cylinder (VC) model are now formed from the annulus induction a_∞ , and the axial induction change of the non-planar VC model is included. The numbers below refer to the corrected code (KSS-Blade V6). The referee’s comments are summarised in grey italics.

1 Consistency with the relation of Limacher and Wood (2021)

Referee: One of the few theoretical guides to the magnitude of u_r is $\int_0^\infty (a^2 - u_r^2) r \, dr = 0$ in the rotor plane (velocities normalised by the free stream). Any model for the radial velocity should be consistent with it. Neither model is compared with it, nor is sufficient information given to allow a comparison.

Reply. We evaluated both integrals for the planar rotor, integrating $u_r^2 r$ numerically from the axis to 2000 rotor radii, across the (integrable) logarithmic singularities at the cylinder radii.

case	$\int a^2 r \, dr$	$\int u_r^2 r \, dr$ (VC)	ratio	share of $\int u_r^2 r \, dr$ outside the disc
uniform disc, $a = 0.1$ ($R = 1$)	0.005000	0.005000	1.0000	59 %
uniform disc, $a = 0.3$ ($R = 1$)	0.045000	0.045000	1.0000	59 %
NREL 5 MW, $7 \, \text{m s}^{-1}$, $N_{sec} = 70$	163.50	163.50	1.0000	50 %
NREL 5 MW, $11 \, \text{m s}^{-1}$, $N_{sec} = 35$	148.69	148.69	1.0000	51 %
NREL 5 MW, $11 \, \text{m s}^{-1}$, $N_{sec} = 70$	148.76	148.76	1.0000	50 %
NREL 5 MW, $11 \, \text{m s}^{-1}$, $N_{sec} = 140$	148.76	148.76	1.0000	50 %
NREL 5 MW, $15 \, \text{m s}^{-1}$, $N_{sec} = 70$	8.730	8.730	1.0000	43 %

(NREL cases in m^2 , with u_r normalised by U_0 .)

The VC model thus satisfies the relation to at least four significant digits, for the uniformly loaded disc and for the radially varying loading of the turbine. We believe this to be an exact property of a system of semi-infinite right cylinders whose strengths are $\gamma_t = 2U_0 \Delta a$, although we have verified it only numerically. Two remarks:

- About half of $\int u_r^2 r \, dr$ lies *outside* the rotor disc. Inside the disc the area integral of u_r^2 is therefore about half that of a^2 , with u_r smaller than a near the axis and larger towards the edge, as the referee describes.
- The Madsen expression is defined for $r \leq R$ only. Inside the disc its integral is 34 % ($7 \, \text{m s}^{-1}$), 36 % ($11 \, \text{m s}^{-1}$) and 65 % ($15 \, \text{m s}^{-1}$) of the VC value. Measured by this integral the Madsen correction underestimates the radial velocity at high loading; the deficit comes mainly from the tip region, where the VC value is up to twice as large (Fig. 1a of the revised manuscript).

The check is restricted to the planar rotor. As the referee notes, the application of the relation to a coned rotor with a finite number of blades requires further analysis, which is beyond a brief communication.

We agree that u_r is of the order of a and in this sense substantial. For the non-planar rotor, however, the revised manuscript shows that its effect on power is almost exactly compensated by the accompanying change of the axial induction: in 45 cases the radial term alone changes the power by -4.7 to $+7.1$ %, the complete VC model by less than 0.14 %. What remains is a redistribution of in-plane loading along the span.

Changes in the manuscript. New Sect. 3.4 “Consistency checks suggested in the discussion” with the result above; Limacher and Wood (2021), Wood and Limacher (2021) and van Kuik (2022) are cited.

2 Circulation, lift and drag

Referee: Vortex models require the vortex circulation, which in the note and in the references is determined entirely by the lift of the blade element. This is incorrect because, as pointed out by Glauert, the blade element drag reduces the circulation in the equation for conservation of angular momentum applied immediately behind the rotor.

Reply. The remark made us state more precisely which quantity enters where; the discussion paper was not clear about this.

- *Cylinder strengths.* In our implementation the tangential vorticity γ_t , the only source of u_r in a right-cylinder model, is not derived from the bound circulation but from the axial induction of the momentum balance, which uses the full normal force coefficient $C_n = c_L \cos \phi + c_D \sin \phi$. Drag is therefore included. (Li et al., 2022, use the lift-only thrust coefficient.) The difference is small: with lift-only strengths u_r changes by less than 0.07 m s^{-1} for $r > 0.5 R$ (at most 0.32 m s^{-1} for $r > 0.25 R$, and 0.7 m s^{-1} at the innermost cylindrical sections), and the net power change of the complete model for the reference case from -0.7 to $+0.7 \text{ kW}$.
- *Wake rotation.* The tangential induction is obtained from the tangential force coefficient $c_L \sin \phi - c_D \cos \phi$, so that the angular momentum balance contains the drag, in line with Glauert’s remark. The corresponding longitudinal vorticity does not induce a radial velocity for right cylinders.
- *Force term.* The circulation $\Gamma = \frac{1}{2} W c c_L$ in the additional in-plane force $\rho \Gamma u_r \tan \kappa$ is the bound circulation that carries the lift, to which the Kutta–Joukowski theorem applies. If instead the circulation reduced by Glauert’s factor $1 - (c_D/c_L) \cot \phi$ is used — a reduction of 4–8 % for $r > 0.5 R$ in the reference case, not applied to the cylindrical root sections where $c_L = 0$ — the radial term changes from -96.0 to -90.5 kW and the axial term from $+95.3$ to $+88.6 \text{ kW}$; the net change stays small (-1.8 instead of -0.7 kW).

The conclusions are therefore not sensitive to this choice, but the choice is now stated.

Changes in the manuscript. Sect. 2 states that the momentum balance uses the normal-force coefficient; Sect. 3.4 reports the two sensitivities above and cites Glauert (1983).

Use of AI tools

The calculations reported here and the drafting of this reply were carried out with the assistance of Anthropic’s Claude. The authors have checked the results and are responsible for the content.

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